7-30) (a) $\frac{dg}{dt} = \left(\frac{\partial g}{\partial q_k}\right) \dot{q}_k + \frac{\partial g}{\partial p_k} \dot{p}_k + \frac{\partial g}{\partial t}$ [Using the summation convention]

Using $\dot{q}_k = \partial H / \partial p_k$ and $-\dot{p}_k = \frac{\partial H}{\partial q_k}$, we get

$$\frac{dg}{dt} = \frac{\partial g}{\partial q_k}\frac{\partial H}{\partial p_k} - \frac{\partial g}{\partial p_k}\frac{\partial H}{\partial q_k} + \frac{\partial g}{\partial t} = [g, H] + \frac{\partial g}{\partial t}.$$

(b) Thus, $\dot{q}_j = [q_j, H]$ and $\dot{p}_j = [p_j, H]$ since q_j and p_j are generalized coordinates that do not explicitly depend on time

(c) Similarly, $[p_\ell, p_j] = 0$ using symmetry and $\frac{\partial}{\partial t} p_\ell = 0$.
For the same reason $[q_\ell, q_j] = 0$.

(d) $[q_\ell, p_j] = \left(\frac{\partial q_\ell}{\partial q_k}\right)\left(\frac{\partial p_j}{\partial p_k}\right) - \left(\frac{\partial q_\ell}{\partial p_k}\right)\left(\frac{\partial p_j}{\partial q_k}\right)$. The second term is clearly zero (q_k, p_k are independent).
The first term is $\delta_{\ell k}\delta_{jk} = \delta_{\ell j}$.

(e) From part (a), if $[g, H] = 0$ and $\frac{\partial g}{\partial t} = 0$ then $\frac{dg}{dt} = 0$ and g is a constant of motion of the system.