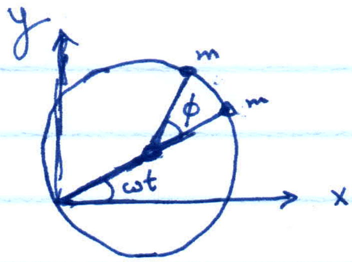
7-11)



$U = 0$ (or constant)

$L = T - U = T$

$$\begin{cases} x = R\cos\omega t + R\cos(\phi + \omega t) \\ y = R\sin\omega t + R\sin(\phi + \omega t) \end{cases}$$

Thus, $L = T = \frac{m}{2}(\dot{x}^2 + \dot{y}^2)$

$$\begin{cases} \dot{x} = R(-\omega\sin\omega t) - R\sin(\phi + \omega t)(\dot{\phi} + \omega) \\ \dot{y} = R(\omega\cos\omega t) + R\cos(\phi + \omega t)(\dot{\phi} + \omega) \end{cases}$$

$$L = \frac{m}{2}R^2\left\{\omega^2 + (\omega + \dot{\phi})^2 + 2\omega(\dot{\phi} + \omega)\cos\phi\right\}$$

Thus, the Euler-Lagrange equation becomes

$$\frac{d}{dt}\left(\frac{mR^2}{2}\left(2(\omega + \dot{\phi}) + 2\omega\cos\phi\right)\right) = mR^2\omega(-\sin\phi)(\dot{\phi} + \omega)$$

$$mR^2\left[\ddot{\phi} - \omega\sin\phi\,\dot{\phi}\right] = -mR^2\omega\sin\phi(\dot{\phi} + \omega)$$

Canceling the $\dot{\phi}$ term we get

$$\ddot{\phi} + \omega^2\sin\phi = 0.$$

For small oscillations, we have the usual simple harmonic motion (when $\sin\phi \approx \phi$).