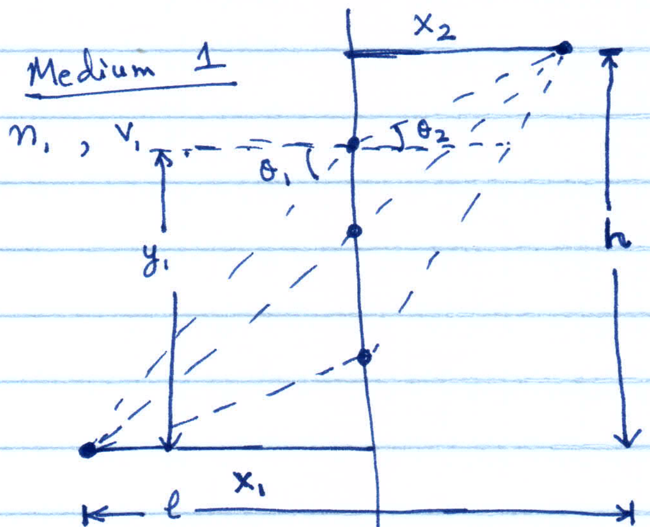
6-7) According to Fermat's principle, the path taken by light within a homogeneous medium is a straight line because it minimizes the flight time. When it crosses a boundary between media we thus only need to know the crossing point:



$$\frac{y_1}{x_1} = \tan\theta_1$$

$$\frac{(h-y_1)}{x_2} = \tan\theta_2$$

Path lengths $= \dfrac{x_1}{\cos\theta_1} = \dfrac{y_1}{\sin\theta_1}$ | Path lengths $= \dfrac{x_2}{\cos\theta_2} = \dfrac{(h-y_1)}{\sin\theta_2}$

Minimize total time $= \dfrac{y_1}{v_1\sin\theta_1} + \dfrac{(h-y_1)}{v_2\sin\theta_2}$ by varying y_1.

$$= \frac{h}{v_2\sin\theta_2} + y_1\left\{\frac{1}{v_1\sin\theta_1} - \frac{1}{v_2\sin\theta_2}\right\}$$

$$T = \frac{\sqrt{x_1^2+y_1^2}}{v_1} + \frac{\sqrt{x_2^2+(h-y_1)^2}}{v_2}.$$

$$\frac{dT}{dy_1} = \frac{2y_1}{2v_1\sqrt{x_1^2+y_1^2}} - \frac{1}{2}\,\frac{2(h-y_1)}{v_2\sqrt{x_2^2+(h-y_1)^2}} = 0$$

Using $\dfrac{y_1}{\sqrt{x_1^2+y_1^2}} = \sin\theta_1$

and $\dfrac{(h-y_1)}{\sqrt{x_2^2+(h-y_1)^2}} = \sin\theta_2$

and also $\dfrac{v_1}{v_2} = \dfrac{n_2}{n_1}$ we get $n_1\sin\theta_1 = n_2\sin\theta_2$.

This is Snell's Law.