

5-14) The self-energy can be calculated as the energy required to assemble the sphere, one infinitesimal shell at a time R bringing the new mass in from infinity:

$$U_{\text{self}} = -G \int_0^R \underbrace{M\left(\frac{r}{R}\right)^3}_{\substack{\text{Mass of} \\ \text{sphere when} \\ \text{radius} = r}} \underbrace{\rho 4\pi r^2 dr}_r \quad \leftarrow \text{mass of shell}$$

$$= -\frac{GM\rho \cdot 4\pi}{R^3} \int_0^R r^4 dr = -\frac{GM \cdot 4\pi}{R^3} \left(\frac{M}{\frac{4\pi R^3}{3}} \right) \cdot \frac{R^5}{5}$$
$$= -\frac{3GM^2}{5R}$$