3-9) We know the initial conditions here:

at $t = 0$ $\quad x = x_0$

$\dot{x} = 0$

The equation is $m\ddot{x} = -k(x-x_0) + F(t)$

where $F(t) = F(u(t) - u(t_0))$.

This problem has initial conditions and a transient force that lasts for a finite time after $t=0$, and thus lends itself to a Laplace Transform solution. Let the Laplace transform of $\begin{pmatrix} x(t) \\ -x_0 \end{pmatrix}$ be $X(s)$.

Transforming the equation we get

$$s^2 X(s) \underset{+sx_0}{-sx(0)} - \dot{x}(0) = \frac{F}{mS}\left[1 - e^{-st_0}\right] - \frac{k}{m}X(s)$$

Thus, we get

$$\left(s^2 + \frac{k}{m}\right)X(s) = sx_0 + \frac{F}{sm}(1 - e^{-st_0})$$

$$X(s) = \frac{F(1 - e^{-st_0})}{mS(s^2 + \omega^2)}$$

Using $\mathcal{L}^{-1}\left(\frac{s}{s^2+\omega^2}\right) = \cos\omega t$, $\mathcal{L}^{-1}\left(\frac{1}{s} \cdot \frac{1}{s^2+\omega^2}\right) = \frac{1}{\omega}\int_0^t dt \sin\omega t$

$$= -\frac{1}{\omega^2}\left[\cos\omega t - 1\right]$$

we get

$$\begin{pmatrix} x(t) \\ -x_0 \end{pmatrix} = \frac{F}{m\omega^2}(1 - \cos\omega t) - \frac{F}{m\omega^2}(1 - \cos\omega(t-t_0))u(t_0)$$

$$\Rightarrow x(t) = x_0 + \frac{F}{k}\left[\cos\omega(t-t_0) - \cos\omega t\right] \quad \text{if } t > t_0.$$