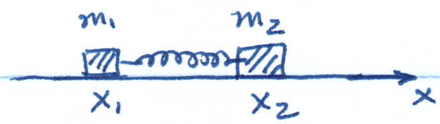
3-6)



The equations of motion if the unstretched spring length is x_0 are

$m_1\ddot{x}_1 = +k((x_2-x_1)-x_0)$ -(1), where the + sign is to ensure a correct direction for $\ddot{x}_1$ when x_1 increases

Similarly,

$m_2\ddot{x}_2 = -k((x_2-x_1)-x_0)$ -(2)

Thus $m_1\ddot{x}_1 + m_2\ddot{x}_2 = 0$ [Momentum conservation – no external force.]

[The RHS of the equations above are the negative of each other.]

Taking the difference of equations (1) and (2) we get $\ddot{x}_2 - \ddot{x}_1 = -k((x_2-x_1)-x_0)\left\{\frac{1}{m_2}+\frac{1}{m_1}\right\}$

Defining the reduced mass m by $\frac{1}{m} = \frac{1}{m_2}+\frac{1}{m_1}$, we find $\frac{d^2}{dt^2}(x_2-x_1) = -\frac{k}{m}((x_2-x_1)-x_0)$

Defining the extended length x by $x \equiv (x_2-x_1)-x_0$ we see that $\ddot{x} = -\frac{kx}{m}$ which leads to the usual solution to an undamped, undriven oscillator: $x(t) = A\cos(\omega t+\phi)$ where

$\omega^2 = \frac{k}{m} = 0.5\,\text{N/m}\left\{\frac{1}{0.2\,\text{kg}} + \frac{1}{0.1\,\text{kg}}\right\} = 7.5\ s^{2}$

and $\omega = 2.74$ rad/s, $\nu = 0.436$ Hz