3-38) Let $\frac{F_0}{m} = A_0$. Then the equation is

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A_0 e^{-\gamma t}\sin(\omega t).$$

Assuming the oscillator is initially at rest in its equilibrium position we can use equation (3.116). Then we get

$$x(t) = \int_0^t dt'\, A_0 e^{-\gamma t'}\sin(\omega t') \cdot \frac{1}{m\omega_1} e^{-\beta(t-t')} \sin\omega_1(t-t')$$

$$= A_0 e^{-\beta t}\int_0^t dt'\, e^{-(\gamma-\beta)t'}\left\{\frac{\cos(\omega_1(t-t') - \omega t') - \cos(\omega_1(t-t') + \omega t')}{2}\right\}$$

$$= \frac{A_0}{2} e^{-\beta t}\int_0^t dt'\, e^{-(\gamma-\beta)t'}\, Re\left\{e^{i[\omega_1(t-t') - \omega t']} - e^{i[\omega_1(t-t') + \omega t']}\right\}$$

$$= \frac{A_0 e^{-\beta t}}{2} Re\left\{e^{i\omega t}\int_0^t dt'\, e^{-[(\gamma-\beta) + i(\omega_1+\omega)]t'} - e^{-[(\gamma-\beta) + i(\omega-\omega_1)]t'}\right\}$$

$$= \frac{A_0 e^{-\beta t}}{2} Re\left\{e^{i\omega t}\frac{\left(1 - e^{-[(\gamma-\beta) + i(\omega_1+\omega)]t}\right)}{(\gamma-\beta) + i(\omega_1+\omega)} - \frac{\left(1 - e^{-[(\gamma-\beta) + i(\omega-\omega_1)]t}\right)}{(\gamma-\beta) + i(\omega-\omega_1)}\right\}$$

$$= \frac{A_0 e^{-\beta t}}{2} Re\left\{\frac{\left(e^{i\omega t} - e^{-[(\gamma-\beta) + i\omega_1]t}\right)}{(\gamma-\beta)^2 + (\omega_1+\omega)^2}\left((\gamma-\beta) - i(\omega_1+\omega)\right)\right.$$

$$\left. - \frac{\left(e^{i\omega t} - e^{-[(\gamma-\beta) - i\omega_1]t}\right)}{(\gamma-\beta)^2 + (\omega-\omega_1)^2}\left((\gamma-\beta) - i(\omega-\omega_1)\right)\right\}$$