3-12)

Restoring force = $mg\sin\theta \approx mg\theta$.

Torque equation: $\frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r} \times \vec{F}$

$$\frac{d}{dt}(ml^2\dot{\theta}) = -mgl\sin\theta \approx -mgl\theta$$

Thus we get

$$\ddot{\theta} = -\frac{g}{l}\theta$$

Defining $\omega_0^2 \equiv \frac{g}{l}$ we get the natural angular frequency.

If the resistive medium force is $2m\sqrt{gl}\,\dot{\theta}$ then the equation is

$$ml^2\ddot{\theta} = -mgl\theta - 2ml\sqrt{gl}\,\dot{\theta} \quad \text{giving}$$

$$\ddot{\theta} + \frac{g}{l}\theta + 2\sqrt{\frac{g}{l}}\,\dot{\theta} = 0$$

This is a classic critically damped oscillator equation (cf. Eq. (3.53)) with $\omega_0 = \beta = \sqrt{\frac{g}{l}}$.