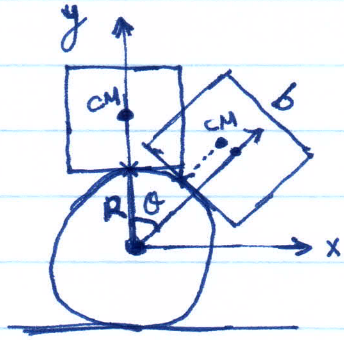
2-42) The idea here is to find the potential energy $U(y)$ for the cube and see if the minimum curves upwards or "holds water" for stability. We're told that the contact is "perfectly rough" meaning there is no slipping.




We can take the cylinder axis as the origin of the 2-D x-y space in which the motion occurs.

Initially, the cube center-of-mass (CM) is at $(0, R+\frac{b}{2})$. When the cube rolls over by an angle $\theta$, the point of contact (poc) moves by a distance $R\theta$. Its location is $(R\sin\theta, R\cos\theta)$. The CM does <u>not</u> lie on the line joining the axis to the poc however. Instead it is "above" the original point on the cube at the original poc. Thus, the CM is at (new location of old poc on cube) + (vector to CM)

$= (R\sin\theta, R\cos\theta) + (-R\theta\cos\theta, R\theta\sin\theta) + \left(\frac{b}{2}\sin\theta, \frac{b}{2}\cos\theta\right)$.

Thus, the new CM height $y = R\cos\theta + R\theta\sin\theta + \frac{b}{2}\cos\theta$

For small $\theta$ (our interest), $y \approx R\left(1-\frac{\theta^2}{2}\right) + R\theta^2 + \frac{b}{2}\left(1-\frac{\theta^2}{2}\right)$

$= R\left(1+\frac{\theta^2}{2}\right) + \frac{b}{2}\left(1-\frac{\theta^2}{2}\right)$

$= \left(R+\frac{b}{2}\right) + \frac{\theta^2}{2}\left(R-\frac{b}{2}\right)$.

Since $U(y) = mgy$ we see that for $R > \frac{b}{2}$ the equilibrium is stable and for $R < \frac{b}{2}$ the equilibrium is unstable.