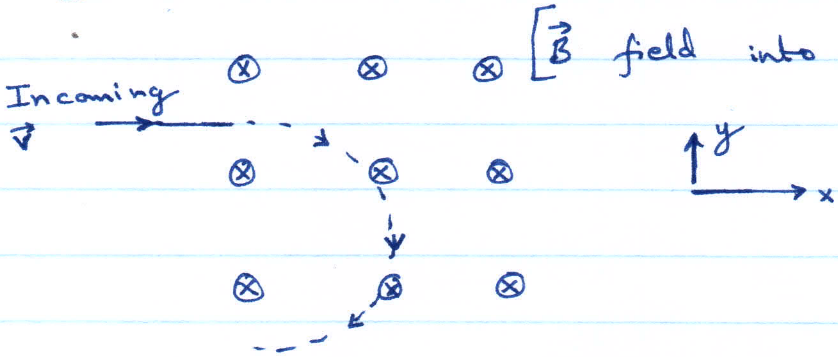
2-22) (a) if $\vec{E}=0$ then $\vec{F}=q\vec{v}\times\vec{B}$.

Since $\vec{v}_{\parallel}$, the component of $\vec{v}$ $\parallel$ to $\vec{B}$ when it enters the field is zero, and since $\vec{v}\times\vec{B}$ is perpendicular to the field, there will be no acceleration parallel to $\vec{B}$ and $\vec{v}_{\parallel}$ will remain zero. Thus the motion is planar and can be described by two Cartesian coordinates which we can call x and y.



Since the acceleration is always perpendicular to $\vec{v}$ as well, the magnitude of $\vec{v}$ never changes, only its direction. Writing $\vec{v}=v\hat{v}$ we see that $mv\dot{\hat{v}}=qv\,\hat{v}\times\vec{B}$. Implicitly stated is the idea that $\vec{B}$ is a uniform vector field: $\vec{B}=B\hat{B}$. Thus, $m\dot{\hat{v}}=qB\,\hat{v}\times\hat{B}$, or $\dot{\hat{v}}=\omega_c\hat{v}\times\hat{B}$ where $\omega_c\equiv\frac{qB}{m}$.

Finally, since $\hat{v}$ is always in a plane perpendicular to $\hat{B}$ we find that the magnitude of $\hat{v}\times\hat{B}$ is a constant and $\hat{v}$ thus changes direction at a uniform rate, i.e., circular motion.

Thus, $\hat{v}=\cos(\omega_c t)\,\hat{i}+\sin(\omega_c t)\,\hat{j}$ where we used the initial direction for $\vec{v}$ and used the magnitude of $\dot{\hat{v}}$ to be ω_c. The magnitude of $\vec{v}$ is $r\omega_c$ where r is the circle radius; thus $r=\frac{v}{\omega_c}=\frac{mv}{qB}$.

2-22 (b) The acceleration along the B-field direction is entirely due to the electric field and is given by

$$a_z = \frac{F_z}{m} = \frac{qE_z}{m}.$$

Thus, along z the motion is just like projectile motion in a uniform gravitational field with $g = \frac{qE_z}{m}$ and we can readily integrate the equation of motion to get

$$V_z = V_{z0} + \frac{qE_z t}{m} \quad \text{and thus}$$

$$z = z_0 + V_{z0}t + \frac{qE_z}{m}\frac{t^2}{2}.$$

(c) Along x and y we have

$$a_x = \frac{qE_x}{m} + \frac{qV_yB}{m} \quad \text{and} \quad a_y = \frac{qE_y}{m} - \frac{qV_xB}{m}$$

$$= \frac{qV_yB}{m} \quad (\text{since } E_x = 0).$$

Setting $\omega_c = \frac{qB}{m}$ we have, with $\alpha_y = \frac{qE_y}{m}$

$$\dot{V}_x = \omega_c V_y \quad \Rightarrow \quad \ddot{V}_x = \omega_c \alpha_y - \omega_c^2 V_x$$

$$\dot{V}_y = \alpha_y - \omega_c V_x \qquad \ddot{V}_y = -\omega_c^2 V_y$$

The solution for V_y is clearly sinusoidal: $V_y = A\cos\omega_c t$ which satisfies the initial conditions given for part (d) and in any case averages to zero:

$$\langle V_y \rangle = 0.$$

The solution for V_x is clearly $-\frac{1}{\omega_c}(\dot{V}_y - \alpha_y)$, i.e.,

$V_x = A\sin\omega_c t + \frac{\alpha_y}{\omega_c} = A\sin(\omega_c t) + \frac{E_y}{B}$, the time average of which is clearly $\langle V_x \rangle = \frac{E_y}{B}$.

(d) Integrating these equations gives us

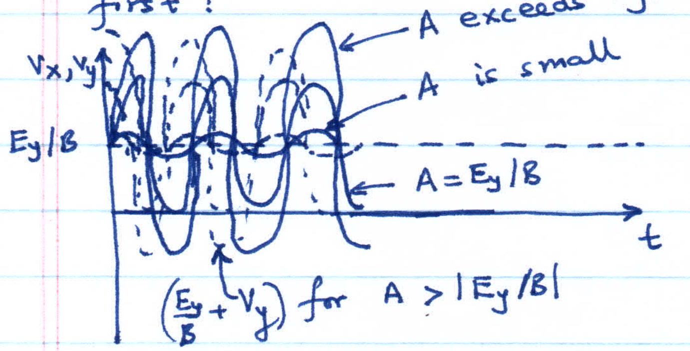
$$y(t) = \frac{A}{\omega_c} \sin(\omega_c t) \quad \text{and}$$

$$x(t) = -\frac{A}{\omega_c} \cos(\omega_c t) + \left(\frac{E_y}{B}\right) t$$

where we chose constants of integration so that

$$x(0) = -\frac{A}{\omega_c} \quad \text{and} \quad y(0) = 0.$$

To make the sketch, let's try to sketch the velocities first :



The actual motion should thus be