2-15)

Let the dimension <u>down the incline</u> be called x.

Then the equation of motion is $m\ddot{x} = +mg\sin\theta - kmv^2$

Writing this as

$\dot{v} = g\sin\theta - kv^2$ we can try to integrate

$$\frac{dv}{g\sin\theta - kv^2} = dt$$

Integrating using equation (E.4c) we get

$$\frac{1}{\sqrt{kg\sin\theta}} \tanh^{-1}\left(\sqrt{\frac{k}{g\sin\theta}}\, v\right) = t + \text{const.}$$

Since it starts from rest, $v(t=0) = 0$ and thus the constant of integration is zero. We get

$$V = \sqrt{\frac{g\sin\theta}{k}} \tanh\left(\sqrt{kg\sin\theta}\, t\right)$$

Integrating again, (using $\int \tanh(ax)\,dx = \frac{\ln(\cosh(ax))}{a} + \text{const}$) we find that

$$x = \sqrt{\frac{g\sin\theta}{k}} \frac{\ln\left(\cosh\left(\sqrt{kg\sin\theta}\, t\right)\right)}{\sqrt{kg\sin\theta}} + \text{const.}$$

Assume that $x = 0$ at $t = 0$, then again the const $= 0$.

So $x = \frac{\ln\left(\cosh\left(\sqrt{kg\sin\theta}\, t\right)\right)}{k}$.

The time t at which $x = d$ is now clearly given by $t(x=d) = \frac{\cosh^{-1}\left(e^{kd}\right)}{\sqrt{kg\sin\theta}}$