

2.1) (a) We are given $m\ddot{x}_i = f(x_i)g(t)$

If we can separate x -dependent and t -dependent terms on two sides of the equation then we can integrate, but it is not possible in this case.

(b) $m\ddot{x}_i = f(x_i)g(t)$ can be separated using the velocity:

$$m\dot{v}_i = f(v_i)g(t), \text{ i.e.,}$$
$$\frac{m dv_i}{f(v_i)} = g(t) dt$$

(c) $m\ddot{x}_i = f(x_i)g(v_i)$ can be written, using

$$\ddot{x}_i = \dot{v}_i = \frac{dv_i}{dx_i} \frac{dx_i}{dt} = v_i \frac{dv_i}{dx_i}$$

$$\text{as } \frac{m v_i dv_i}{g(v_i)} = f(x_i) dx_i$$

After integration we get $v_i(x_i)$ which can be further integrated using $\frac{dx_i}{dt} = v_i(x_i)$ to obtain $x_i(t)$.