#1-40) We are given $z = 2xy - 3x^2 - 4y^2 - 18x + 28y + 12$

(a) At the top of the hill $\vec{\nabla} z = 0$, i.e., $\partial_x z = \partial_y z = 0$.

$$\left.\begin{aligned} \partial_x z &= 2y - 6x - 18 = 0 \\ \partial_y z &= 2x - 8y + 28 = 0 \end{aligned}\right\}$$

Add 3x the second eqn. to the first eqn. to get $-22y + 66 = 0 \Rightarrow y = 3$.

Thus, $-6x - 12 = 0$ and $x = -2$.

The top of the hill is at $(-2,3)$ and the height there is $z(-2,3) = 72$ m.

(b) $\phi(x,y,z) = z - 2xy + 3x^2 + 4y^2 + 18x - 28y - 12 = 0$

$$\vec{\nabla}\phi \Big|_{(1,1,17)} = (-2y + 6x + 18, -2x + 8y - 28, 1)\Big|_{(1,1,17)} = (22, -22, 1)$$

$$\cos\theta = \frac{1}{\sqrt{2(22)^2 + 1}} = 0.03212 \Rightarrow \theta = 88.16^\circ$$

(c) $\vec{\nabla} z(x,y)\Big|_{(1,1)} = (-22, 22)$ i.e., in the NW direction for steepest ascent.

[Steepest descent is along $-\vec{\nabla} z(x,y)$ = SE.]