10-10) Taking into account that in the previous problem we have an upward as well as westerly acceleration (in addition to the southward one) we recognize that the range will differ from the $\omega = 0$ range R_0' because

(a) The total time spent $T' > T_0'$ because we effectively reduce the downward acceleration seen by the projectile from g to $(g - 2\omega V_0 \cos\alpha \cos\lambda)$. Thus, $T' \approx \dfrac{2V_0 \sin\alpha}{(g - 2\omega V_0 \cos\alpha \cos\lambda)}$, and

(b) The eastward velocity is not simply a constant $V_0 \cos\alpha$ but instead is $V_0 \cos\alpha - \left\{ V_0 \sin\alpha \cos\lambda\, t - \cos\lambda \dfrac{g t^2}{2} \right\} \omega$

Thus, the range R' is given by

$$R' = V_0 \cos\alpha\, T' - \omega V_0 \sin\alpha \cos\lambda \frac{T'^2}{2} + \omega \cos\lambda \frac{g T'^3}{6}$$

Thus, the change of range

$$\Delta R' \equiv R' - R_0' = V_0 \cos\alpha (T' - T_0') - \omega V_0 \sin\alpha \cos\lambda \frac{T'^2}{2} + \omega \cos\lambda \frac{g T'^3}{6} \quad -(1)$$

Since the problem gives a result to first order in ω, which is a small quantity of order 10^{-4}, we attempt to do the same.

$$T' = \frac{2V_0 \sin\alpha}{g\left\{1 - 2\dfrac{\omega V_0}{g} \cos\alpha \cos\lambda\right\}} \cong T_0' \left\{1 + 2\frac{\omega V_0}{g} \cos\alpha \cos\lambda\right\}$$

Using this in equation (1) above we get

$$\Delta R' = V_0 \cos\alpha \cdot \frac{2\omega V_0}{g} \cos\alpha \cos\lambda \left(\frac{2V_0 \sin\alpha}{g}\right) - \frac{\omega V_0 \sin\alpha \cos\lambda\, T'^2}{2} + \frac{\omega \cos\lambda\, g T'^3}{6}$$

The last 2 terms can be combined since $gT' \approx 2V_0 \sin\alpha$, giving

$$\Delta R' = \frac{4\omega V_0^3}{g^2}\cos^2\alpha \sin\alpha \cos\lambda - \frac{\omega V_0 \sin\alpha \cos\lambda\, T'^2}{2} + \frac{2V_0 \sin\alpha \cos\lambda\, \omega T'^2}{6}$$

$$\approx \frac{4\omega V_0^3}{g^2}\cos^2\alpha \sin\alpha \cos\lambda - \frac{V_0 \sin\alpha \cos\lambda}{6}\left(\frac{2V_0 \sin\alpha}{g}\right)^2 \omega$$

$$= \frac{4\omega V_0^3 \sin\alpha \cos\lambda}{g^2}\left\{\cos^2\alpha - \frac{\sin^2\alpha}{3}\right\}$$

The problem uses $\sqrt{\frac{2R_0'^3}{g}} = \sqrt{\frac{2}{g}\left(\frac{V_0^2 \sin 2\alpha}{g}\right)^3} = \frac{4V_0^3}{g^2}\sin^{3/2}\alpha \cos^{3/2}\alpha$

Thus, $\Delta R' \approx \sqrt{\frac{2R_0'^3}{g}}\, \omega \cos\lambda \frac{(\cos^2\alpha - \sin^2\alpha/3)}{\sqrt{\sin\alpha \cos^3\alpha}}$

$$= \sqrt{\frac{2R_0'^3}{g}} \cdot \omega \cos\lambda \left(\cot^{1/2}\alpha - \frac{1}{3}\tan^{3/2}\alpha\right)$$

Thus, for $\alpha = 60°$ $\quad \Delta R' \cong 0$.