Scattering Problem

(a) $b = a\cos^{3/2}\theta \qquad \frac{db}{d\theta} = \frac{3}{2}a\cos^{1/2}\theta\,(-\sin\theta)$

$$\frac{d\sigma}{d\Omega} = \left|\frac{b}{\sin\theta}\cdot\frac{db}{d\theta}\right| = \frac{3}{2}\frac{a^2\cos^2\theta\,\sin\theta}{\sin\theta}$$

$$= \frac{3}{2}a^2\cos^2\theta$$

$$\sigma = \int\left(\frac{d\sigma}{d\Omega}\right)d\Omega = \frac{3}{2}a^2\int d\Omega\,(\cos^2\theta) = \frac{3}{2}a^2\cdot 2\pi\cdot\frac{2}{3} = 2\pi a^2$$

$$= 6.28\times 10^{-12}\ m^2$$

(b) The azimuthal angle specified by "directly upwards" is irrelevant since there is no ϕ-dependence in $\frac{d\sigma}{d\Omega}$.

into $5^\circ \times 5^\circ$:

$$N \cong \frac{\left(\frac{d\sigma}{d\Omega}\right)\Delta\Omega}{\sigma}\cdot 10^6$$

$$= \frac{3\cos^2\theta}{4\pi}\cdot\Delta\Omega\cdot 10^6 = \frac{9\times 10^6}{16\pi}\times\Delta\Omega$$

$$= \frac{9\times 10^6}{16\pi}\times\frac{1}{2}\times\left(\frac{5}{180}\right)\left(\frac{5}{180}\right)\pi^2 \cong 682 \text{ particles.}$$