Free Fall

Cycle I

1 Introduction

1.1 Objective

The experimenter will measure the acceleration due to gravity on Earth by two methods, for free-falling objects with different masses.

1.2 Equipment

1. Free fall apparatus, mounted on table
2. Grey pad (a switch)
3. Two metal balls of different sizes
4. Photogate (black plastic ‘C’ shape), blue USB controller, and cable
5. ‘Picket fence’ (a rectangular piece of translucent plastic with a pattern of black bars)
6. Cushion for catching falling objects
7. Meter stick, tape measure, or other tool for measuring distance

1.3 Discussion

1.3.1 Galileo Galilei

According to an old story, Galileo dropped spheres of different masses off the leaning tower of Pisa to prove that objects do not, as reported by Aristotle, fall with accelerations proportional to their masses. There is strong evidence[1] that Galileo never did the experiment as described in the story; instead, he probably gathered his data in an equivalent, but much slower, manner

[1] Citation needed

using inclined planes. We will use modern timing equipment to perform the experiment as described in the traditional story.

Galileo showed that an object falling freely in a uniform gravitational field is accelerated at a constant rate. The force[2] that causes the acceleration is the result of the mutual gravitational attraction between the falling mass and the Earth. Of course, if there are other forces present, such as friction or air resistance, the motion of the falling object will not be one of constant acceleration. However, if the distance of fall is not too great and the object is sufficiently dense, the effects of air resistance are very small and may be ignored.

1.3.2 Derivation of an Equation for the Acceleration

An accelerating object feels a force described by Equation 1, Newton's second law, in which $\vec{a}$ is the object's acceleration, and m is its mass.

$$\vec{F} = m\vec{a} \tag{1}$$

This equation can be re-arranged to solve for the acceleration $\vec{a}$ in terms of the mass m and the force, $\vec{F}$:

$$\vec{a} = \frac{\vec{F}}{m} \tag{2}$$

If the force is equated with the gravitational attraction between the Earth and a falling object of mass m[3],

$$\vec{F} = G\frac{mM_{\oplus}}{R_{\oplus}}\hat{r} \tag{3}$$

and this is inserted into Equation 2 to find the object's acceleration[4],

$$\vec{a} = -G\frac{M_{\oplus}}{R_{\oplus}}\hat{y} \tag{4}$$

or

$$\vec{a} = -g\hat{y} \tag{5}$$

[2] It's worth mentioning that Galileo died in 1642, the year Newton was born, so he did not know about forces or Newton's laws of motion.

[3] The symbol "$\oplus$" used here means "Earth", so $M_{\oplus}$ and $R_{\oplus}$ represent the mass and the radius of the Earth, respectively.

[4] Note that in our lab, $\hat{r}$, the unit vector pointing from the object toward the center of the Earth, is the same as $-\hat{y}$, the unit vector pointing down.

This g is called the *acceleration due to gravity (on Earth)*[5]. It is constant near the surface of the Earth, as Galileo discovered, where it has a value of $9.81 m/s^2$.

The path of an object in motion under constant acceleration is described by the kinematic equation[6].

$$\vec{x}(t) = \vec{x_0} + \vec{v_0}t + \frac{1}{2}\vec{a}t^2 \tag{6}$$

where x_0 and v_0 are the position and velocity of the object at the beginning of an experiment. Since we can choose the height h above the ground and drop the object from a rest state in which $v_0 = 0$, Equation 6 becomes

$$y(t) = h - \frac{1}{2}gt^2 \tag{7}$$

If the object lands on the ground at $y(t) = 0$, then Equation 7 can be solved for g in terms of the time t that the object takes to fall to the ground and its initial height h

$$g = \frac{2h}{t^2} \tag{8}$$

In the two experiments described below, you will carefully measure the time t it takes a free-falling object to fall a known distance h in order to calculate the acceleration due to gravity on Earth, g.

2 Exeriment I — Using the Electronic Timer

By dropping balls with different masses a known distance and measuring their fall times, one can calculate their accelerations by Equation 8. About 400 years ago, Galileo showed that the acceleration experienced by all free-falling objects is independent of their mass. In this experiment, you will do the same.

2.1 Procedure

1. Convince yourself that the two metal balls have different masses.

[5]The same equations work just as well on another planet, or an asteroid, or perhaps on the back of a giant turtle, if the correct mass and radius are used.

[6]This is actually *three* equations: one for each of the x-, y-, and z-directions, all described by the vector $\vec{x}$. Since the full vector equation only has a y-component, it reduces to a single equation for y.

2. Assemble the timer with the free fall adapter so the steel ball has an unobstructed path to the grey pad.

3. Push the spring-held rod in slightly, and place the ball between the two metal contacts on the drop mechanism. Turn the thumbscrew so that the ball is held in place. You can then release the ball by loosening the screw.

4. Adjust the height of the ball to about 1.25 m. **Measure the height** from the bottom of the ball when held in the ball-dropping mechanism to the top of the grey pad **and record your measurement.**

5. Set the timer modes to "Stopwatch." Press the "Start/Stop" key. The timer will beep and "*" will appear on the second line of the LCD. Dropping the ball will start the timer. When the ball hits the grey pad the timer will stop and display the elapsed time.

6. Practice dropping the ball several times to ensure that the ball hits the pad near the center before you begin the experiment.

7. Drop the ball five times and **record the time measurements.**

8. **Repeat step 7** recording the fall times for heights of 0.25, 0.50, 0.75, and 1.00 meters. It is not important that the heights be exactly at the specified values, but you should measure these heights as accurately as you can.

9. Repeat steps 4–8 for the second ball.

2.2 Analysis

1. For each height, calculate the average fall time for each ball and square it. **Record the squared times for each height.**

2. Open DataStudio. In the Displays menu, click on the Table icon.

3. A data table should appear. Click on the pencil icon to edit the data. Under the Data button on the Table's toolbar, select Data.

4. Enter the squared average times and heights determined in step 1.

5. A Data icon will appear in the Data menu. Create a graph of this data.

6. Qualitatively describe the position vs. time-squared graph of your data. **What type of curve is it? Is it linear, quadratic, power, or something else?**

7. Click on the graph so that it becomes the active window. Pull down the "Fit" menu and select "Linear." **Do you have a good fit? How do you know?** What are the quantities in the box that describe the fit[7]?

8. Compare the fit parameters to the quantities in Equation 8 and use them to **calculate the acceleration due to gravity for each ball.**

9. When you are finished, close DataStudio.

10. There is probably some spread in the time of fall data for any given height. This indicates there is some uncertainty in the actual value of this time. To estimate this uncertainty, **calculate the deviation from average** for each individual time:

$$\text{deviation} = |t_{avg} - t_i| \tag{9}$$

The deviation that is greatest (neglecting any negatives) can be taken as the uncertainty in your time measurement. Calculate the uncertainty for the time of fall for at least one height.

11. **Compare your values of g for each ball to the standard value of $9.81 m/s^2$ by computing the percent difference.** The percent difference is given by the difference between your result and the standard value divided by the standard value and converted to a percent:

$$\text{percent difference} = 100\% \times \frac{g_{measured} - g_{standard}}{g_{standard}} \tag{10}$$

12. Divide the value from step 11 by the average time for that height. This is the relative uncertainty. If you double this you have an estimate of the relative uncertainty in your measured value of g. If your relative uncertainty is greater than your percent difference, then you can claim that your measured value of g agrees with the standard value, within the limits of your uncertainty. **Does your measured value of g agree with the standard value within your uncertainty?**

[7] These are called *fit parameters*

3 Experiment II — Using the Photogate

This experiment utilizes a piece of equipment that shines a beam of infrared light onto a sensor. When the beam is broken by an object passing through the photogoate, a computer or timer records the length of time that elapses while the beam is interrupted. The photogate speaks to computer software called DataStudio that records and graphs the collected data. By dropping a transparent object with evenly-spaced opaque bands through the photogate, it is possible to measure the time it takes the object to fall the same (small) known distance several times during one fall, and from these data, calculate the object's acceleration.

3.1 Procedure

1. Connect the Photogate port to the USB link. Connect the USB link to your computer. Plug the photogate into the #1 location on the photogate port.

2. Open DataStudio. It will open a window asking you to "Choose a timing or counting mode to add to the activity." Choose the "Photogate and Picket Fence" option[8] and click OK.

3. Use a meter stick to measure the leading edge-to-leading edge[9] distance between opaque strips on the 'picket fence.' Record this measurement in "Band Spacing." Make sure the units are correct.

4. Connect the photogate to a stand. Notice that the photogate is in the shape of a "C." Arrange the gate so that the C is parallel to the floor.

5. Place a cushion on the floor underneath the photogate.

6. Press "Start" to begin timing and drop the picket fence (the strip of clear plastic with the evenly spaced black bands) through the photogate. Press "Stop" after the photogate hits the cushion. It is not imperative that you drop the fence immediately after starting the program; data collection will only start once the photogate sees the picket fence.

7. **Record the slope of the resulting velocity plot. What does the slope represent? What are the units?**

8. **Repeat the steps 6 and 7** to collect a total of 7 data runs.

[9]The leading edge of something is the edge in the direction of the motion.

[9]Some of the computers are confused and want you to pick "Collision Timer" instead.

3.2 Analysis

1. **Compute the average value of g from your data.**

2. Use Equation 9 to **compute the devation from the average** for the values of g obtained from your graphs.The result will be your uncertainty for Experiment II.

3. **Calculate the percent difference** between your average value for g and the standard value of $9.81 m/s^2$ as in Experiment I, using Equation 10.

4 Questions

Answer the following questions in your report.

1. **What is your measured result for g for each experiment?** Report it as *measured value* $\pm$ *uncertainty* m/s^2, making note of the uncertainty in your measurements.

2. How well do your measurements compare with the standard value of $9.81 m/s^2$ for the gravitational acceleration? **Did you successfully measure the standard value, within your uncertainties?**

3. What factors contribute to any differences between what you have found and what you expected?

4. How do the two methods for measurement of g compare?