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The Adler–Weisberger and Goldberger–Miyazawa–Oehme sum rules as probes of constraints from analyticity and chiral symmetry in dynamical models for pion–nucleon scattering

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Abstract

The Adler–Weisberger and Goldberger–Miyazawa–Oehme sum rules are calculated within a relativistic, unitary and crossing symmetric dynamical model for pion–nucleon scattering using two different methods: (1) by evaluating the scattering amplitude at the corresponding low-energy kinematics and (2) by evaluating the sum-rule integrals with the calculated total cross section. The discrepancy between the results of the two methods provides a measure of the breaking of analyticity and chiral symmetry in the model. The contribution of the Δ resonance, including its dressing with meson loops, is discussed in some detail and found to be small.

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1. Introduction

It is known that much of the hadronic dynamics at low energies is determined by nucleon and meson exchanges while the intermediate-energy region is dominated by various

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resonances. Chiral symmetry is an important physical constraint that governs the meson–nucleon interactions at low energies and unitarity is an essential property at intermediate energies. A natural link between the low- and intermediate-energy regions is provided by various sum rules. In this paper we present a dynamical study of two sum rules for pion–nucleon scattering: the Adler–Weisberger (AW) and Goldberger–Miyazawa–Oehme (GMO) sum rules [1–3]. These relate integrals over the total cross section to the scattering amplitude calculated at a subthreshold kinematic point (in the case of the AW sum rule) and at threshold (in the case of the GMO sum rule), in the isospin-odd channel.

The sum rules follow from the properties of relativistic invariance, unitarity, analyticity and crossing symmetry (from whose combination, and assuming no subtractions, one derives a dispersion relation for the isospin-odd amplitude), augmented by chiral symmetry constraints. The latter may be taken into account either by using the current algebra commutator for axial charges and the PCAC relation for the divergence of the axial current [1,3] or by using a chirally invariant Lagrangian at tree level [2]. In deriving the sum rules one assumes that due to the smallness of the pion mass μ, the non-pole part of the amplitude is a slowly varying function of external pion four-momenta throughout the low-energy region of other variables, thus retaining only the terms of the lowest order in μ. The higher orders corrections come from effects of finite nucleon size (by estimating the pion–nucleon form factor [4] or by evaluating pion loops [5]) and from resonance contributions (of which the Δ is expected to be dominant [4]).

Being based on the very general physical principles, the sum rules can serve as stringent tests of dynamical models of pion–nucleon scattering. Suppose one obtains the low-energy quantities predicted by the sum rules in two ways: first, from the calculated low-energy scattering amplitude itself and, second, by integrating the calculated total cross sections with the appropriate weight. In doing so, it is essential that the two ways of evaluation (called here the "low-energy" and "sum-rule" evaluations) be done within the *same* dynamical model. Then the results of the low-energy and sum-rule evaluations would coincide provided (i) the basic physical properties from which the sum rules are derived hold true, and (ii) the model fulfills these properties exactly. In any practical situation there will be a certain discrepancy between the low-energy and the sum-rule results. This discrepancy allows one to quantify violations of either one or both conditions (i) and (ii). Such a study was first done for nucleon Compton scattering, where the anomalous magnetic moment and polarizabilities of the nucleon are related to the Gerasimov–Drell–Hearn and Baldin–Lapidus sum rules [6].

The purpose of the present paper is to carry out a similar investigation for the Adler–Weisberger and Goldberger–Miyazawa–Oehme sum rules. We use the "Dressed K-matrix Model" [7–9] which is unique in that it provides a good description of pion–nucleon scattering in *both* intermediate- and low-energy regions, while also yielding the nucleon sigma-term in agreement with recent experimental analyses. The model obeys relativistic invariance, unitarity and crossing symmetry exactly, and incorporates constraints from analyticity and chiral symmetry approximately. We interpret the violation of analyticity and chiral symmetry in terms of meson-loop corrections to the free nucleon and Δ propagators and to bare πNN and $\pi N\Delta$ vertices. The calculation of the loops is done up to infinite order within a consistent dressing procedure. As a result of the present study, we find that the dressing improves the agreement between the low-energy and sum-rule calculations. We will focus on the effects of the Δ-resonance dressing in some detail.

2. The Adler–Weisberger sum rule

The sum rules discussed in this paper are formulated in terms of the isospin-odd amplitude $D^-(\nu)$ for forward pion–nucleon scattering.[1] We denote the nucleon and pion masses as m and μ, respectively ($m = 938$ MeV, $\mu = 138$ MeV), and the pion–nucleon coupling constant $g_{\pi N} = 13.02$ [11,12] (this value of $g_{\pi N}$ is also compatible with Ref. [13]). In the following we shall use the invariant energy variable $\nu = (s-u)/(4m)$, with s, u and t being the Mandelstam variables, constrained by $s+u+t = 2m^2+q^2+q'^2$, where q and q' denote the initial and final pion four-momenta, respectively ($q^2 = q'^2 = \mu^2$ for on-shell pions). We shall also use $\omega = (s-m^2-q^2)/(2m)$, the energy of the incoming pion in the laboratory frame. The total cross sections for the $\pi^- p$ and $\pi^+ p$ scattering processes will be denoted as $\sigma_{\pi^- p}$ and $\sigma_{\pi^+ p}$, respectively.

The Adler–Weisberger (AW) sum rule for pion–nucleon scattering [1,2,4] can be written as a relation between the integral over the weighted total cross section and the low-energy limit of the isospin-odd scattering amplitude:

$$C_{\text{AW}} = I_{\text{AW}}, \tag{1}$$

where the left-hand side represents a low-energy limit of the forward scattering amplitude:

$$C_{\text{AW}} = 2F_\pi^2 \lim_{\nu\to 0}\left\{\frac{D^-(\nu)}{\nu} + \frac{g_{\pi N}^2}{2m^2}\right\}, \tag{2}$$

and the right-hand side is given by the sum-rule integral

$$I_{\text{AW}} = 2F_\pi^2\left\{\frac{1}{\pi}\int_\mu^\infty d\omega\,\frac{\sqrt{\omega^2-\mu^2}}{\omega^2}\big[\sigma_{\pi^- p}(\omega) - \sigma_{\pi^+ p}(\omega)\big] + \frac{g_{\pi N}^2}{2m^2}\right\}, \tag{3}$$

where $F_\pi = 92.4$ MeV is the pion decay constant. The presence of the factor $2F_\pi^2$ and of the nucleon pole contribution $g_{\pi N}^2/(2m^2)$ in Eqs. (2) and (3) is a matter of convenience since for a vanishing pion mass chiral symmetry dictates that [1,2]

$$C_{\text{AW}} = 1 + \mathcal{O}\big(\mu^2/m^2\big). \tag{4}$$

Thus the dominant correction to Eq. (1) is of the order $\mathcal{O}(\mu^2/m^2)$, reflecting the fact that the physical pion has a non-vanishing mass whereas the AW sum rule is derived formally for the scattering of a massless pion.

[1] Throughout the paper, the kinematical conventions and definitions for pion–nucleon scattering amplitude follow those of Ref. [10].

3. The Goldberger–Miyazawa–Oehme sum rule

The Goldberger–Miyazawa–Oehme (GMO) sum rule [3,4,14] can be written in the form

$$C_{\mathrm{GMO}} = I_{\mathrm{GMO}}, \tag{5}$$

where the low-energy part is determined by the isospin-odd scattering length a^-:

$$C_{\mathrm{GMO}} = 4\pi\mu a^-, \tag{6}$$

and the sum-rule integral is given by

$$I_{\mathrm{GMO}} = \mu^2 \left\{ \frac{1}{\pi} \int_{\mu}^{\infty} d\omega \, \frac{1}{\sqrt{\omega^2 - \mu^2}} \left[\sigma_{\pi^- p}(\omega) - \sigma_{\pi^+ p}(\omega) \right] + \frac{g_{\pi N}^2}{2m^2} \right\}. \tag{7}$$

Similar to C_{AW} in Eq. (2), C_{GMO} is related to a low-energy value of the scattering amplitude since a^- is proportional to D^- at threshold:

$$a^- = \frac{D^-(\nu = \mu)}{4\pi(1 + \mu/m)}. \tag{8}$$

The last equation also shows that the GMO sum rule Eq. (5) is written up to higher-order terms of $\mathcal{O}(\mu/m)$. This could be compared with the accuracy $\mathcal{O}(\mu^2/m^2)$ of the AW sum rule Eq. (1), as indicated in Eq. (4).

It should be pointed out that some terms in the sum rules might formally appear to be of a lower order in μ/m than they actually are: for example, the leading terms in Eq. (5) might be interpreted as being of $\sim \mathcal{O}(\mu^2/m^2)$, but the explicit evaluation shows that, due to large accompanying coefficients, they are comparable with unity (see Section 5). By the same token, a proper care should be taken in estimating the leading corrections to the sum rules Eqs. (1) and (5). A comprehensive analysis of the intrinsic accuracy of the sum rules was presented recently in Ref. [14], where the GMO sum rule was utilized to deduce the pion–nucleon coupling constant. For the purposes of the present paper, the precise value of $g_{\pi N}$ is not very important since we focus on the comparison of the low-energy and sum-rule evaluations; it is essential however that we keep all parameters (including $g_{\pi N}$) unchanged in evaluating both left- and right-hand sides of Eqs. (1) and (5).

4. Basic features of the model

The objective of our calculation is to check the validity of the AW and GMO sum rules using a dynamical model for pion–nucleon interaction applicable at low and intermediate energies. The crucial point is that the evaluation of the left- and right-hand sides of Eqs. (1) and (5) is done here in the *same* model, rather than utilizing one approach to evaluate the low-energy limits of the amplitudes (the left-hand sides) and another approach to evaluate the integrals (the right-hand sides). Only if Eqs. (1) and (5) are calculated entirely within the same model will their validity reflect the extent to which the basic symmetries are fulfilled in the chosen model, rather than stem from (possible) incompatibility of different approaches.

We use the "Dressed K-matrix Model" whose detailed description can be found in Refs. [7–9]. Here we recapitulate only the principal features of the approach. The properties of relativistic invariance, (two-body) unitarity and crossing symmetry are fulfilled in the model exactly, while those of chiral symmetry and analyticity are implemented partially.

Chiral symmetry constraints are effectively incorporated at low energies through the use of a predominantly pseudovector pion–nucleon coupling in the s- and u-channel nucleon exchange diagrams, plus ρ and σ t-channel exchanges with parameters fitted to provide a good low-energy description of the s- and p-wave phase shifts [8,9]. Effects of the explicit violation of chiral symmetry appear due to the finiteness of the pion mass. In the context of the AW sum rule, these effects manifest themselves in the deviation of C_{AW} from unity, as shown in Eq. (4).

Analyticity is implemented in the model through the dressing procedure for the πNN and $\pi N\Delta$ vertices and the nucleon and Δ propagators. These three- and two-point functions are calculated as a solution of a system of coupled integral equations, which amounts to including meson loop corrections up to infinite order (for a specific description of the class of the loops generated by the dressing, see Refs. [7,9]). The solution is based on the use of cutting rules and dispersion relations in an iteration procedure, which allows us to obtain analytic two- and three-point functions (i.e., the dressed propagators and vertices, respectively). The scattering amplitude is obtained in a K-matrix framework, where the K-matrix is constructed from skeleton diagrams built out of the dressed vertices and propagators. As a result of the dressing, analyticity is partially incorporated into the K-matrix framework (as opposed to the traditional K-matrix models where analyticity is strongly violated because of K-matrices built out of tree diagrams). The full restoration of analyticity could be achieved by dressing the four-point irreducible functions in the amplitude on the same footing with the presently dressed two- and three-point functions (see discussions in Refs. [6–9]).

The calculation presented in this paper provides a useful means of quantifying the extent to which analyticity and chiral symmetry are broken in the model. Indeed, since the sum rules are based on the combination of relativistic invariance, unitarity, crossing symmetry, analyticity and chiral symmetry, and only the latter two properties are not fully implemented in the model, the difference between the evaluation of the low-energy amplitudes (represented by the left-hand sides of Eqs. (1) and (5)) and the evaluation of the sum-rule integrals (the right-hand sides of Eqs. (1) and (5)) will serve as a measure of the violation of analyticity and chiral symmetry constraints. Such a comparison of low-energy and sum-rule evaluations was carried out earlier for the Baldin–Lapidus, Gerasimov–Drell–Hearn and other related sum rules in Compton scattering [6]. The main difference between the present case of the AW and GMO sum rules and that of the sum rules in Compton scattering is that now we test not only the violation of analyticity but also effects of the explicit chiral symmetry breakdown.

5. Results of the calculation

We will study the effects of the dressing of the two- and three-point functions by comparing three different calculations, called the "Dressed", "Bare" and "Dressed, but bare Δ" calculations. The "Bare" calculation corresponds to the use of free propagators and bare vertices in the K-matrix, whereas the "Dressed" one to the use of the fully dressed vertices and propagators. The "Bare" calculation is equivalent to a traditional K-matrix model with a tree-level K-matrix; therefore, the violation of analyticity is maximal in this case. Analyticity is partially restored in the "Dressed" calculation since the dressing of the two- and three-point functions incorporates the use of dispersion relations. The remaining violation of analyticity comes mainly from the lack of the dressing of the four-point contact diagrams. The "Dressed, but bare Δ" calculation highlights effects of the dressing of the Δ resonance and will be described in the next subsection. Since we want to consider the genuine effects of the dressing, the same set of parameters are used in the "Dressed", "Bare" and "Dressed, but bare Δ" calculations (all parameters, including the masses, the coupling constants and the cut-offs of the bare form factors, are given in Ref. [7]).

To investigate convergence of the sum rules, we calculate the integrals in Eqs. (3) and (7) up to a pion energy $\omega_{\rm up}$. The dependence of the calculated AW and GMO sum rules on $\omega_{\rm up}$ is shown in Figs. 1 and 2, respectively. Even though a full convergence is not achieved by $\omega_{\rm up} \approx 1000$ MeV, the major features of the sum rules are obtained due to the nucleon and the Δ-resonance, as can be seen from Fig. 3 where we show the calculated total cross sections. The hump in the cross sections around $\omega - \mu \approx 900$ MeV is not reproduced since we did not include the D15 and F15 resonances in the model at present. We checked however that this higher-energy feature of the cross section is suppressed by the weight factors in Eqs. (3) and (7). This observation is in complete agreement with the conclusion of Ref. [4] that the resonances heavier than the Δ have a negligible effect on the sum rules.

We extract the values of the sum rules at two representative pion energies, $\omega_{\rm up} =$ 750 MeV and $\omega_{\rm up} = 1000$ MeV, and regard them as the results of the sum-rule evaluation, i.e., as $I_{\rm AW}$ and $I_{\rm GMO}$ given by Eqs. (3) and (7), respectively. The low-energy quantity $C_{\rm GMO}$ was calculated at threshold, according to Eq. (6), and the low-energy values of $C_{\rm AW}$,

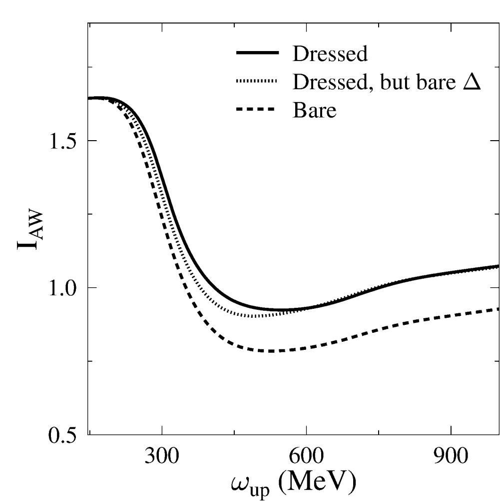

Fig. 1. Dependence of the AW sum-rule integral Eq. (3) on the upper limit of integration.

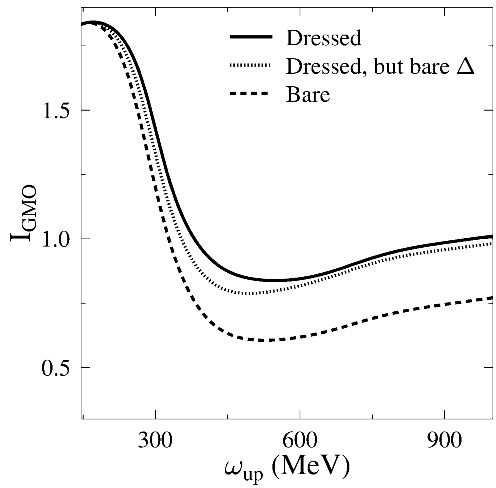

Fig. 2. The GMO sum-rule integral Eq. (7) as a function of the upper limit of integration.

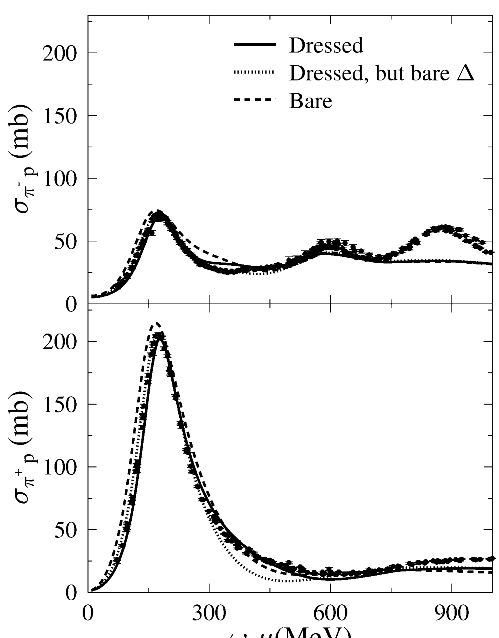

Fig. 3. Total cross sections for the $\pi^+ p$ and $\pi^- p$ scattering processes calculated in the model. The experimental points are taken from the CNS Data Analysis Centre [16].

as given by Eq. (2), were calculated at two subthreshold points: $\{s = u = m^2, t = 2\mu^2\}$ and $\{s = u = m^2, t = 0\}$. The former is the Cheng–Dashen point and the latter is closely related to the Weinberg point, except that at the Weinberg point both pions have a zero

Table 1
The values of the Adler–Weisberger and Goldberger–Miyazawa–Oehme sum rules evaluated from the amplitude at the low-energy kinematical points and from the integrals over total cross sections, as given by Eqs. (2), (3), (6) and (7). The different low-energy values C and sum-rule values I are described in the text, as are the three calculations with varying amounts of dressing

	Dressed	Dressed, but bare Δ	Bare
$C_{\mathrm{AW}}(s=u=m^2, t=2\mu^2)$	1.16	1.14	1.31
$C_{\mathrm{AW}}(s=u=m^2, t=0)$	1.10	1.10	1.25
$I_{\mathrm{AW}}(\omega_{\mathrm{up}}=750\ \mathrm{MeV})$	1.00	1.00	0.85
$I_{\mathrm{AW}}(\omega_{\mathrm{up}}=1000\ \mathrm{MeV})$	1.08	1.08	0.93
C_{GMO}	1.10	1.10	1.22
$I_{\mathrm{GMO}}(\omega_{\mathrm{up}}=750\ \mathrm{MeV})$	0.92	0.90	0.67
$I_{\mathrm{GMO}}(\omega_{\mathrm{up}}=1000\ \mathrm{MeV})$	1.01	0.98	0.77

Table 2
Comparison of the low-energy and sum-rule evaluations of the quantities defined in Eqs. (2), (3), (6), (7) and given in Table 1. The differences are related to the violation of analyticity and the breaking of chiral symmetry

	Dressed	Dressed, but bare Δ	Bare
$C_{\mathrm{AW}}(s=u=m^2, t=2\mu^2) - I_{\mathrm{AW}}(\omega_{\mathrm{up}}=750\ \mathrm{MeV})$	0.16	0.14	0.46
$C_{\mathrm{AW}}(s=u=m^2, t=2\mu^2) - I_{\mathrm{AW}}(\omega_{\mathrm{up}}=1000\ \mathrm{MeV})$	0.08	0.06	0.38
$C_{\mathrm{AW}}(s=u=m^2, t=0) - I_{\mathrm{AW}}(\omega_{\mathrm{up}}=750\ \mathrm{MeV})$	0.10	0.10	0.40
$C_{\mathrm{AW}}(s=u=m^2, t=0) - I_{\mathrm{AW}}(\omega_{\mathrm{up}}=1000\ \mathrm{MeV})$	0.02	0.02	0.32
$C_{\mathrm{GMO}} - I_{\mathrm{GMO}}(\omega_{\mathrm{up}}=750\ \mathrm{MeV})$	0.18	0.20	0.55
$C_{\mathrm{GMO}} - I_{\mathrm{GMO}}(\omega_{\mathrm{up}}=1000\ \mathrm{MeV})$	0.09	0.12	0.45

four-momentum squared whereas in our case the pions are on-shell. All these values of C and I are given in Table 1.

The differences between the values of C and I are summarized in Table 2. The significance of these differences should be assessed in the context of the intrinsic accuracy of the sum rules, which is $\sim \mathcal{O}(\mu^2/m^2) \sim \mathcal{O}(0.02)$ for the AW sum rule and $\sim \mathcal{O}(\mu/m) \sim \mathcal{O}(0.15)$ for the GMO sum rule, as discussed in Sections 2 and 3. We see that the agreement comparable with the intrinsic accuracy is achieved in the "Dressed" calculation when comparing the low-energy evaluation and the sum-rule evaluation with $\omega_{\mathrm{up}} = 1000$ MeV. By contrast, in the "Bare" calculation the discrepancy between the low-energy and sum-rule evaluations is about three to ten times larger than the intrinsic accuracy. This shows the importance of the dressing, mainly because it improves analyticity properties of the amplitudes while maintaining unitarity and crossing symmetry.

5.1. *Effects of the dressing of the $\pi N\Delta$ vertex and Δ self-energy*

Since leading corrections to the sum rules are expected to be due to the Δ-resonance [4], we consider the effects of the Δ dressing separately. The calculation denoted "Dressed, but bare Δ" is based on a K-matrix with the free Δ propagator and the bare $\pi N\Delta$ vertex while all the other vertices and propagators remain dressed as in the full (i.e., "Dressed") calculation.

The Δ dressing is described in detail in Ref. [7]; here we will mention only a few features and present the results. The $\pi N\Delta$ vertex has the following structure:

$$V_\alpha(k,p) \sim G_\Delta\big(p^2\big)(k_\alpha \not{p} - p\cdot k\gamma_\alpha), \tag{9}$$

where p and k denote the four-momenta of the Δ and pion, respectively, and $G_\Delta(p^2)$ is the $\pi N\Delta$ form factor. This vertex has the property $p\cdot V = 0$, which ensures that only the physical spin-3/2 part is retained in the Δ propagator [15]

$$S_\Delta^{\alpha\beta}(p) = \frac{\mathcal{P}_{3/2}^{\alpha\beta}(p)}{\not{p} - m_\Delta - \Sigma_\Delta(p)}, \tag{10}$$

where $\mathcal{P}^{\alpha\beta}(p)$ is the projector on the spin-3/2 states, $m_\Delta = 1232$ MeV is the mass of the Δ-resonance and the Δ self-energy is given by

$$\Sigma_\Delta(p) = A_\Delta\big(p^2\big)\not{p} + B_\Delta\big(p^2\big)m_\Delta - \big(Z_2^\Delta - 1\big)(\not{p} - m_\Delta) - Z_2^\Delta \delta m_\Delta. \tag{11}$$

The invariant self-energy functions $A_\Delta(p^2)$ and $B_\Delta(p^2)$ describe the contribution of the pion loops, and the renormalization constants Z_2^Δ and δm_Δ are fixed to ensure the correct pole location and residue of the dressed propagator. The $\pi N\Delta$ vertex gets dressed with an infinite number of meson loops, i.e., a starting bare form factor $G_\Delta^0(p^2)$ changes into $G_\Delta(p^2)$. Correspondingly, the vertex Eq. (9) changes its normalization from a bare coupling constant to a physical one, the former being adjusted so that the dressed vertex yields the physical $\Delta \to \pi N$ decay rate. It is important that the Δ self-energy $\Sigma_\Delta(p)$ and dressed $\pi N\Delta$ vertex are calculated simultaneously with the nucleon self-energy and dressed πNN vertex; all these two- and three-point functions are obtained as a solution of one system of coupled integral equations (see Ref. [7] for details).

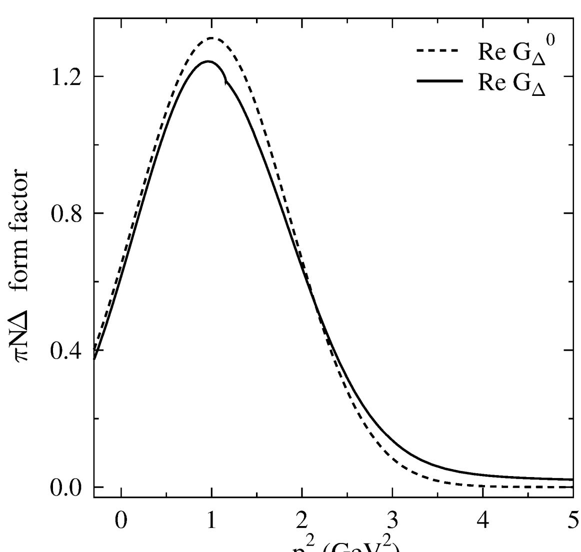

Fig. 4. The bare (superscript 0) and dressed $\pi N\Delta$ form factors, defined in Eq. (9), as functions of the four-momentum squared of the Δ.

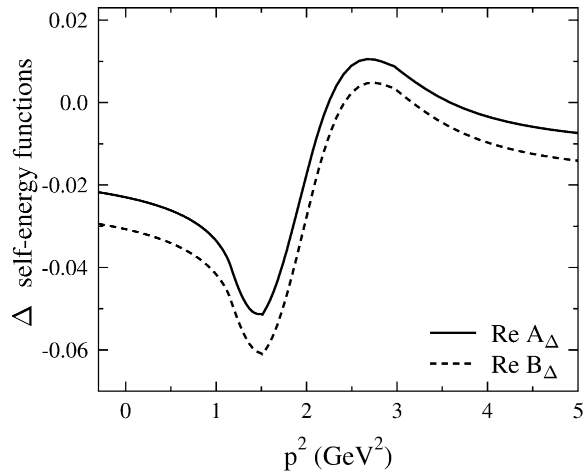

Fig. 5. The Δ self-energy functions, defined in Eq. (11).

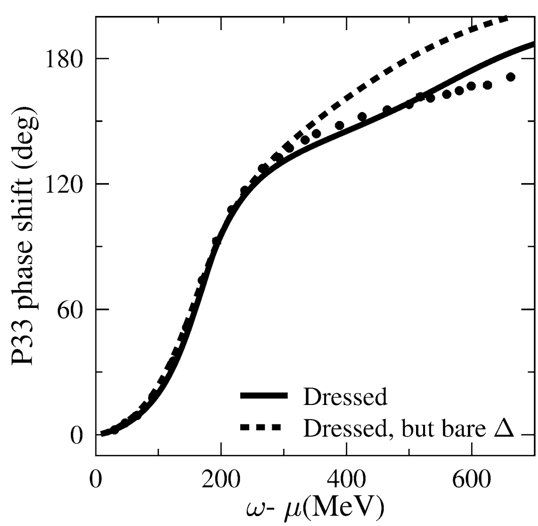

Fig. 6. The effect of the Δ dressing on the P33 phase shift. The data points are from the CNS Data Analysis Centre [16].

The results of the "Dressed, but bare Δ" calculation, presented in Figs. 1, 2 and in Tables 1, 2, show that the Δ dressing has a small effect on the AW and GMO sum rules. Thus while the full ("Dressed") calculation is clearly superior to the "Bare" one, the Δ dressing considered in separation does not improve the agreement between the low-energy and sum-rule evaluations. However, the dressed and bare $\pi N\Delta$ form factors differ noticeably, as shown in Fig. 4. At the Δ mass, $p^2 \approx 1.52\ \text{GeV}^2$, the physical form factor is normalized to one by adjusting the normalization of the bare form factor. The Δ self-energy has a non-trivial structure as shown in Fig. 5. The effects of the Δ dressing manifest themselves quite prominently in the P33 pion–nucleon phase shift, shown in Fig. 6, which is known to be determined mainly by the Δ. As mentioned above, we use the same set of parameters in the "Dressed", "Bare" and "Dressed, but bare Δ" calculations. The only exception is made in presenting the P33 phase shift, where we normalized the bare and physical $\pi N\Delta$ form factor to the same value at the Δ mass to ensure that both curves in Fig. 6 pass through 90 degrees with the correct slope at $\omega - \mu \approx 190$ MeV.

6. Concluding remarks

It is generally accepted that one can derive the Adler–Weisberger and Goldberger–Miyazawa–Oehme sum rules from relativistic invariance, unitarity, analyticity, crossing symmetry and chiral symmetry constraints. Then the difference between the low-energy and the sum-rule calculations (i.e., between the quantities C and I) is a quantitative measure of the extent to which these properties are violated in the present model. Since relativistic invariance, two-body unitarity and crossing are exact in the model, the discrepancy between the low-energy and sum-rule evaluations is due to the partial fulfillment of analyticity and chiral symmetry.

The dressing certainly improves analyticity properties of the amplitude, although as explained in Section 4, it does not restore analyticity completely. Chiral symmetry is explicitly broken in the model due to the finiteness of the pion mass. This is reflected in the difference $C_{\mathrm{AW}}(s = u = m^2, t = 2\mu^2) - C_{\mathrm{AW}}(s = u = m^2, t = 0)$. Notably, the modulus of this difference is of the same order as the discrepancy between either $C_{\mathrm{AW}}(s = u = m^2, t = 2\mu^2)$ or $C_{\mathrm{AW}}(s = u = m^2, t = 0)$ and the sum-rule values I_{AW}. It is also comparable with the measure of convergence of the sum-rule integral, i.e., with $I_{\mathrm{AW}}(\omega_{\mathrm{up}} = 750\ \mathrm{MeV}) - I_{\mathrm{AW}}(\omega_{\mathrm{up}} = 1000\ \mathrm{MeV})$. Thus the extent to which the dressing affects the chiral symmetry constraints is more difficult to quantify than that of analyticity. Nevertheless, our calculations indicate that the explicit chiral symmetry breaking has a small influence on the AW and GMO sum rules. We have also shown that the corrections due to the Δ-resonance, including its dressing, are strongly suppressed.

It would be interesting to conduct comparisons between low-energy and sum-rule evaluations within other approaches to pion–nucleon scattering, such as the Bethe–Salpeter equation [17] or its reductions (see, e.g., [18]), traditional K-matrix models [12,19] or approaches based on chiral Lagrangians (see, e.g., [5,20] and references therein). Such a comparison would however be meaningful only if, similarly to the "Dressed K-matrix Model" used in the present study, a chosen approach is applicable at both low and intermediate energies because one should be able to calculate reliably both the low-energy amplitude and the total cross sections in the *same* framework.

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