The nucleon-nucleon force and the quark degrees of freedom

Fred Myhrer

Department of Physics and Astronomy, University of South Carolina, Columbia, South Carolina 29208

J. Wroldsen*

TH-Division, CERN, 1211, Geneva, Switzerland

This review gives a detailed overview of the current status of our understanding of the nucleon-nucleon forces. The authors review the known long-range meson exchange forces and explain how these forces originate from an underlying quark model constrained by chiral symmetry, a symmetry that is very well satisfied in low-energy nuclear phenomena. These effective meson exchange forces describe the large-impact-parameter nucleon-nucleon scattering. The authors show how the small-impact-parameter nucleon-nucleon scattering can be explained by the quark structure of the nucleons and why this quark model is successful in reproducing the energy dependence of the "measured" *S*- and *P*-wave nucleon-nucleon phase shifts.

CONTENTS

I. PRELUDE—THE YUKAWA POTENTIAL

The atomic nucleus is built up of nucleons (protons and neutrons). We know, as a result of the electron scattering experiments at Stanford in the late 1960s, that the nucleon itself consists of pointlike particles (partons/quarks and "gluons"; Panofsky, 1968). The strong forces between the quarks result in the comparatively weaker nuclear forces, and the question is how we can understand these nuclear forces based on our knowledge of the structure of the nucleons and the strong confining quark interactions.

More than fifty years ago (1935) the first understanding of the size of the atomic nucleus came with Yukawa's meson theory of the nuclear force (Yukawa, 1935). He based his theory on the analogy with the electric force (Coulomb force) between two charged particles. The electric force is mediated by the exchange of photons between charged particles. The Coulomb force has infinite range ($\sim r^{-1}$) because the photon has no mass. Yukawa postulated a new meson field, the pion, which has a mass μ and through exchange yields a finite-range force between nucleons ($\sim e^{-\mu r}/r$).

This pion exchange produces a strong tensor force which, at shorter distances, is weakened due to the heavy vector-meson (rho meson) exchange. The existence of the vector mesons, the ρ and the ω, was postulated by Nambu (1957) in order to understand the measured electromagnetic form factors of the nucleons. The combined π plus ρ tensor force gives the deuteron a *D* state, quadrupole moment, etc. (Brown and Jackson, 1976). The ω exchange is strongly repulsive and results in a strong spin-orbit nucleon-nucleon interaction. Based on nucleon-nucleon scattering data Breit (1960a, 1960b) and Sakurai (1960) estimated the ω meson's mass, an estimate that was later confirmed when the ω meson was found.

The ρ and ω vector mesons have masses ($\sim$780 MeV) that are almost as large as the nucleon mass ($\simeq$940 MeV). Whereas ω has a relatively long lifetime ($\Gamma\simeq10$ MeV) and can be thought of as a single meson exchange, the ρ has a large width $\Gamma\simeq120$ MeV (decaying to two pions) and has to be treated carefully, as will be discussed in Sec. IV where we present the medium-range two-pion exchange forces. This short discussion of the long- and medium-range nucleon-nucleon forces is illustrated in

*Present address: Gjøvik College of Engineering, P.O. Box 191, 2801, Gjøvik, Norway.

Fig. 1, which shows exchanges of one pion, two pions, and the repulsive ω meson. Using Fig. 1 we shall write the NN long-range potential as

$$V = V_\pi + V_{2\pi} + V_\omega \ ,$$

where each term corresponds to each diagram in Fig. 1. Here the precise definition of $V_{2\pi}$ from Fig. 1(b) is given in Sec. IV (see Fig. 11). From Fig. 2 we see that the short-ranged ω exchange is effective at NN distances of 1 fm. We stop at the ω meson because a heavier meson has too short a range, and at short NN distances r it is no longer meaningful to consider the nucleon as a point particle, which this picture requires. The measured rms radius of the proton is 0.9 fm (Borkowski *et al.*, 1974). As will be clear from the discussion of the chiral bag model in Sec. III.C.3, this extended charge distribution has two components, one from the quark "core" and one from the pion cloud surrounding this "core." The quark core's rms radius is 0.5–0.6 fm (Myhrer, 1984; Weise, 1984). Therefore we expect a sizable quark "core" overlap when two nucleons are about 1 fm or less apart. For larger separation distances this nucleon size can result in vertex form factors, which we shall discuss in Sec. IV.B.3.

We know the nuclear forces are (almost) charge independent. This allows us to introduce an (almost perfect) symmetry called isospin. The proton and neutron are then two different isospin states of one particle, the nucleon, and isospin is (almost) conserved in nuclear interactions. The very weak electromagnetic forces, possible charge-dependent nuclear forces, and the proton-neutron (or the u and d quark mass) difference violate this symmetry by about 3%. This means that if we calculate to only 10% accuracy, we can completely neglect this symmetry violation.

The pion has an extremely low mass ($\mu \sim 140$ MeV) compared to the other mesons and nucleons (baryons), and this low mass indicates another fundamental symmetry of strong interactions, chiral symmetry. Chiral symmetry was first used in current-algebra calculations, and there gave rise to important results such as the Goldberger-Treiman relation linking the strong pion-nucleon coupling constant $g_{\pi NN}$, the axial coupling of neutron decay g_A and the pion decay constant f_π (see

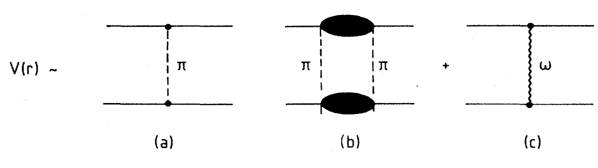


FIG. 1. Illustration of the nucleon-nucleon meson exchange potentials of shorter and shorter ranges. The two-pion exchange potential (b) involves nucleon excitations, crossed box diagrams, and $\pi\pi$ interaction. It contains the scalar σ or ε exchange as well as the ρ-meson exchange of the one-boson exchange potentials (OBEP).

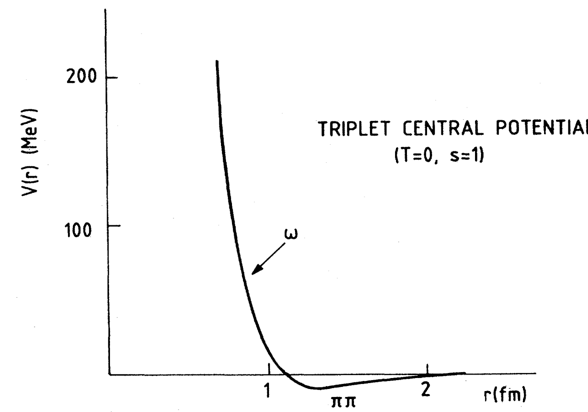


FIG. 2. The spin triplet central potential ($T=0$) as calculated by Lacombe *et al.* (1980), illustrating the attractive two-pion and repulsive ω contributions.

derivation in Sec. III.C.3), the Adler-Weisberger sum rule for πN cross sections, and the πN and $\pi\pi$ scattering lengths (Adler and Dashen, 1968; Llewellyn Smith, 1980). If the chiral symmetry is "exact," which means that the axial current is conserved, the pions are massless. For confined quarks this implies that the helicity of a spin-$\frac{1}{2}$ particle (a quark) is conserved (the helicity is the projection of the particle's spin along its direction of motion). When the quark's momentum is reflected from a solid wall, the quark's spin is also turned around by some dynamics at the wall in order to conserve helicity (see Fig. 3 and discussion in Sec. III.C.3). The proposed Lagrangian of the strong interaction, quantum chromodynamics (QCD), is both isospin symmetric and chiral invariant. This latter symmetry requires zero quark masses in the Lagrangian (see Sec. III.C.3). The degree to which the chiral symmetry is explicitly broken is reflected in a small quark mass in the Lagrangian, which leads to a nonzero but small pion mass (Gell-Mann, Oakes, and Renner, 1968). The divergence of the axial current is a pseudoscalar, and if the pion has a small mass it is postulated (Adler and Dashen, 1968) that the divergence of the axial current is proportional to the pion field (PCAC, partial conserved axial current). This idea has been used to introduce successfully the meson exchange currents in nuclei (Riska, 1985), as will be discussed in Sec. IV.B.3. The chiral symmetry predictions are accurate to 7% when compared to experiments (Pagels, 1975), which means this symmetry is very relevant for low-energy nuclear phenomena.

On the basis of quarks as *the* fundamental particles, we shall in this article argue that the nuclear forces can now be understood from a more fundamental perspective as follows.

(a) The long- and medium-range nuclear forces illustrated in Fig. 1 and the strength of the coupling constants involved can be understood on the basis of the underlying chiral symmetry of the strong forces, as argued

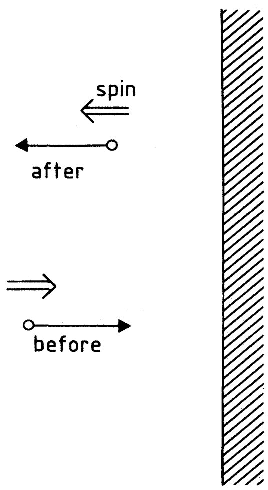


FIG. 3. Illustration of how helicity of a quark reflected from a solid cavity wall is conserved by flipping the quark's spin direction. This has implications for the confinement models of quarks, as will be discussed in Sec. III.

by Brown and Rho (1979) and Brown, Rho, and Vento (1979). We shall go through these arguments in detail in Sec. III where we shall present our model for the understanding of the nuclear forces based on quarks and the underlying chiral symmetry which generates the one-pion exchange potential (OPEP), illustrated in Fig. 1(a), with the correct coupling constant. In other words, we shall *derive* the pion-nucleon coupling constant from the quark model. For the long-range potential which describes the large-angular-momentum *NN* scattering we shall *then* use the pion and nucleon as entities.

These simple chiral quark models also give (roughly) the correct low-energy pion-nucleon and pion-pion scattering phase shifts (see Sec. IV.B.3). This is important if we want to understand the two-pion exchange force in terms of quark models. One reason is that these "measured" phase shifts are input to the dispersion calculations of the two-pion exchange (TPE) potential, Fig. 1(b). The resulting *NN* potential is successful in describing the peripheral *NN* phase shifts, as discussed in Sec. IV (for details, see Brown and Jackson, 1976). This two-pion potential [Fig. 1(b)] has also been calculated directly in an effective meson-baryon model by the Bonn group (Machleidt, Holinde, and Elster, 1987), a calculation that will be compared with the dispersion calculation in Sec. IV.B. The Bonn group emphasizes the intermediate Δ excitations of the nucleon (N) in their $V_{2\pi}$ calculation, and, as we shall discuss, this is in line with the chiral quark model of the N and the Δ (see Sec. IV.B.3).

(b) To describe the short-range nucleon-nucleon forces we also need to present some details about the quark structure of the nucleon itself. That this is necessary can be illustrated by an analogy with molecular forces.

To calculate the (electromagnetic) molecular forces between two electrically neutral hydrogen atoms H, it is necessary to know some properties of the atomic structure and the forces among the constituents (here we really know the forces and the atomic spectra). Then one can calculate the scattering of two H atoms, deduce the effective interatomic HH force, and calculate the binding energy of the hydrogen molecule H_2. Similarly, if we scatter two nucleons (*NN*), we have to know the (quark) structure of the nucleon (N), its excitations Δ, N^*, and Δ^*, and something about the forces between the quarks to deduce the effective short-range *NN* forces.

Thus, in order to understand the short-range nuclear forces, one has to have a reasonable quark model that explains the static properties of the nuclear constituents, i.e., protons and neutrons and their low-energy excitations. In the next two sections (II and III) we shall, therefore, sketch our present understanding of the quark-quark forces (QCD) and then present different QCD-inspired quark models. These sections are meant to introduce the mathematical arguments that will be used in our later discussion of the short-range nuclear forces. Here we want to stress that different confinement models (potential models, including "flip-flop" models) give the same energy dependence of the *S*- and *P*-wave *NN* phase shifts, as we shall show in Sec. V.C.

In Sec. V the small-impact-parameter *NN* scattering calculations (really *NN S*- and *P*-wave phase shifts) are presented. There (Sec. V.D), as we shall argue, it is the antisymmetry requirement of the six quarks of the two interacting nucleons, with the help of the color hyperfine interaction, which results in an effective short-range repulsive (but nonlocal) interaction between nucleons [see a recent discussion by Fiebig and Harvey (1987)]. We try to keep a critical eye on the model calculations performed so far and to focus on some problems still remaining. We conclude in Sec. VI with a short discussion of some open questions that have to be examined further. But, as we shall argue, we have *for the first time* a microscopic model on which to base our understanding of low-energy *NN* scattering.

II. AN INTRODUCTION TO THE QUARKS AND THEIR INTERACTIONS (QCD)

The quark concept was introduced by Gell-Mann (1964) and Zweig (1964) [for details see, for example, the textbook of Close (1979)]. At that time a large number of elementary particles had been found. These particles grouped themselves into irreducible multiplets of the group SU(3). By the introduction of a fundamental SU(3) triplet (the quark flavors u,d,s) one was able to explain the multiplet structure of the observed baryons and mesons. Although only a mnemonic in their early days, quarks are today thought to be physical entities. The earliest experimental support for this came from a number of deep inelastic scattering (DIS) experiments of leptons and neutrinos off protons. In these experiments, at Stanford in the late sixties, high-energy electrons were scattered off protons, and one observed that there were more scattered electrons with high transverse momenta than if the proton had been a diffuse distribution of matter (Panofsky, 1968). Furthermore, the energy and angular distributions of the scattered electrons indicated scattering from "pointlike" particles or partons (Bjorken, 1967; Bjorken and Paschos, 1969; Feynman, 1969). Thus the quarks (and "gluons") were discovered in experiments analogous to the famous Rutherford experiments that revealed the atomic nucleus.

Another important series of experiments was stimulated by the discovery of the J/Ψ particle in 1974 (Aubert *et al.*, 1974; Augustin *et al.*, 1974). This discovery led to the recognition of a fourth flavor of quarks. In addition to flavor, spin, and charge, the quarks have another intrinsic quantum number: $SU_c(3)$ "color." The "color" charges have three different values within the fundamental representation of $SU_c(3)$. The color hypothesis resolves a number of puzzles in quark models. For instance, the three-quark wave function of the delta particle Δ^{++} is built from three u quarks in spatial S states. The three-spin $S=\frac{1}{2}$ u quarks couple to total spin $S=\frac{3}{2}$, which is a symmetric spin coupling. Thus the flavor$\times$spin$\times$space wave function for Δ^{++} is totally symmetric, which is contrary to the Pauli principle. The introduction of a totally antisymmetric wave function [SU(3) singlet] in color-space resolves this puzzle.

Another puzzle was the experimental cross-section ratio

$$R=\frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)}\ , \tag{2.1}$$

where the denominator is a pure electrodynamic process. The process $e^+e^-\to$hadrons is thought to go via $q\bar{q}$ creation (see Fig. 4 and Quigg, 1983). For energies much greater than the participating quark masses, the ratio R can be written as

$$R=\sum_q e_q^2\ , \tag{2.2}$$

where the sum runs over "active" quarks, meaning quarks with effective masses smaller than the center-of-mass energy at which R is measured. For energies below the charm production threshold, one gets (see Table I below for properties of quarks) $R=\frac{2}{3}$, rising to $R=\frac{10}{9}$ above the threshold if the color degree of freedom does not exist. This disagrees with the experimental R values in Fig. 5. With the introduction of color $SU_c(3)$, the

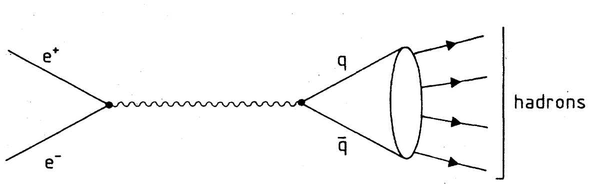


FIG. 4. Nonresonant production of hadrons in e^+e^- annihilation.

TABLE I. Some properties of the five known quark flavors.

Name (flavor)	Symbol	Charge in units of e	Effective mass (GeV)
Up	u	$\frac{2}{3}$	0.3–0.4
Down	d	$-\frac{1}{3}$	0.3–0.4
Strange	s	$-\frac{1}{3}$	~0.5
Charm	c	$\frac{2}{3}$	1.5–1.85
Bottom	b	$-\frac{1}{3}$	5.0–5.3

theoretical values in Eq. (2.2) should be multiplied by 3, giving good agreement with experiment.

Another evidence for colored quarks is the neutral pion decay ($\pi^0\to\gamma\gamma$). The dominant Feynman diagram for this decay is the one in which the pion and the two photons couple to a quark loop (the pion's $q\bar{q}$ constituents radiate two photons). This gives the decay rate (Close, 1979; Llewellyn Smith, 1980)

$$\Gamma(\pi^0\to\gamma\gamma)=\left[\frac{\alpha}{2\pi}\right]^2(N_c/3)^2\mu^3/8\pi f_\pi\sqrt{2}\ , \tag{2.3}$$

where the fine-structure coupling constant $\alpha=\frac{1}{137}$, N_c is the number of quark colors, μ is the pion mass, and f_π is the pion decay constant. For $N_c=3$ this gives the measured decay rate.

Finally the quarks are indirectly seen in electron-positron collisions producing two energetic jets of hadrons ($e^+e^-\to2$ jets). The two jets' angular dependence indicates that they originate from two spin-$\frac{1}{2}$ particles (Hanson *et al.*, 1975; Wu, 1984) similar to the angular dependence of $e^+e^-\to\mu^+\mu^-$. This strongly suggests that two quarks are created and that they subsequently produce the two jets.

Today quantum chromodynamics (QCD) is thought to be the correct theory of strong interactions (Close, 1979; Quigg, 1983). This renormalizable gauge theory with SU(3) color gauge symmetry of spin $S=\frac{1}{2}$ quarks and spin $S=1$ gluons should in principle give just as accurate a description of hadronic phenomena as quantum electrodynamics (QED) gives for the interactions of leptons. However, QCD is a non-Abelian theory, i.e., the gauge bosons of QCD, the gluons, carry color charges. This means that gluons, unlike photons in electrodynamics, can couple to themselves, as will be evident in the following. The postulated QCD Lagrangian based on color gauge invariance is (in analogy to QED) given by

$$L=\bar{\Psi}i\not{D}\Psi-\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}\ , \tag{2.4}$$

where the (gauge) covariant derivative is

$$\not{D}=\gamma^\mu(\partial_\mu-ig\tfrac{1}{2}\lambda^a G^a_\mu) \tag{2.5}$$

and the gluon field tensor is

$$F^a_{\mu\nu}=\partial_\mu G^a_\nu-\partial_\nu G^a_\mu+gf^{abc}G^b_\mu G^c_\nu\ . \tag{2.6}$$

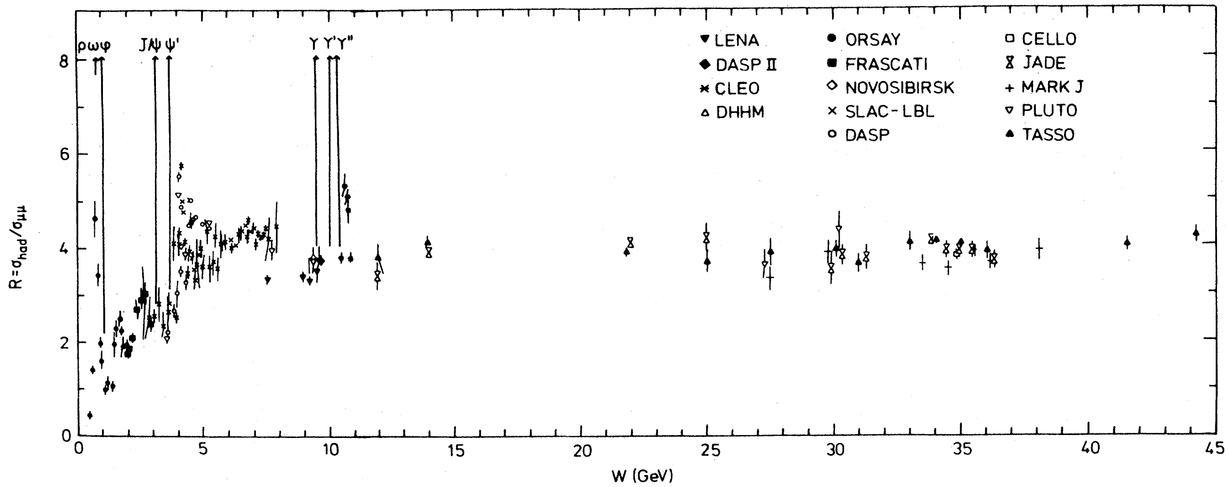


FIG. 5. The ratio R [Eq. (2.1)] vs $E_{\rm CM}$ (Tasso Collaboration, 1984).

The covariant derivative $\not{D}$, Eq. (2.5), contains the colored gluon field G_μ^a, where the superscript a denotes the color charge of the gluon. The SU(3) matrices λ^a ($a=1,2,\ldots,8$) are the SU(3) analogs of the three Pauli spin matrices σ^k ($k=1,2,3$) of SU(2). As in the well-known commutation relation (Lie algebra) of SU(2) with the structure constant ε^{klm},

$$[\sigma^k,\sigma^l]=2i\varepsilon^{klm}\sigma^m \ , \tag{2.7}$$

the eight 3×3 λ matrices also satisfy a Lie algebra,

$$[\lambda^a,\lambda^b]=2if^{abc}\lambda^c \ , \tag{2.8}$$

where the SU(3) structure constants f^{abc} are completely antisymmetric in the indices a,b,c ($=1,2,\ldots,8$). However, the field tensor $F_{\mu\nu}^a$ differs from the electromagnetic one by the last (non-Abelian) term in Eq. (2.6), which in L describes gluon-gluon interaction with a coupling constant g the same as in Eq. (2.5). The possible interaction terms of the Lagrangian, Eq. (2.4), are illustrated in Fig. 6. Diagram 6(a) is due to the gluon field in the covariant derivative equation (2.5), as in QED. Diagrams 6(b) and 6(c) come from the non-Abelian term in the field tensor, Eq. (2.6).

The strong force between two quarks is mediated by gluon exchanges between the interacting particles. The nonlinear couplings [Figs. 6(b) and 6(c)] give rise, one believes, to the confinement of the colored quarks at low energies. All observed particles carry zero color quantum number [really color $SU_c(3)$ singlets], and color is, in QCD-based models, postulated to be confined. Nobody has proven this aspect of QCD, but lattice calculations seem to confirm the confinement conjecture (Rebbi, 1983; Kogut, 1984). All low-energy QCD models discussed here include the confinement property of quarks and gluons.

The nonlinear gluon couplings discussed above also lead to what is called asymptotic freedom of QCD. This means that as the four-momentum transfer Q of a process becomes very large, the QCD fine-structure constant $\alpha_s=g^2/4\pi\to0$. The effective coupling constant $\alpha_s(Q^2)$ is a function of Q^2 and vanishes logarithmically as Q^2 grows. In the leading-logarithm approximation (Quigg, 1983) for $Q^2\gg\Lambda_{\rm QCD}^2$,

$$\alpha_s(Q^2)=\frac{\alpha_s(\Lambda_{\rm QCD}^2)}{1+\alpha_s(\Lambda_{\rm QCD}^2)\beta/4\pi\ln(Q^2/\Lambda_{\rm QCD}^2)} \ , \tag{2.9}$$

where $\beta=11-\frac{2}{3}n_f$, and $\Lambda_{\rm QCD}$ is a cutoff parameter giving the scale of QCD. The number n_f is the number of quark flavors contributing to the quark loop (virtual $q\bar{q}$ intermediate states) in the gluon propagator in the calculation of Eq. (2.9) (Quigg, 1983). The number 11 contributing to β comes from the gluon loop (virtual gluon-gluon intermediate states of the gluon propagator) and the quark-gluon vertex corrections, i.e., it comes from the non-Abelian property of QCD. For $\beta>0$ it can be shown from Eq. (2.9) that $\alpha_s(Q^2\to\infty)=0$. This behavior is in contrast to that of an Abelian gauge theory like QED, in which one gets contributions from the fermion

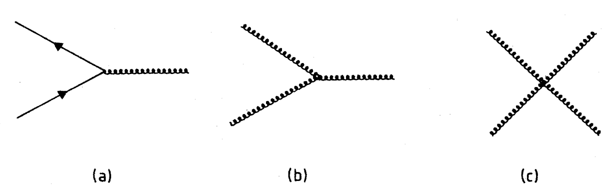


FIG. 6. The elementary interactions between quarks (straight lines) and gluons (curly lines) as prescribed by the Lagrangian, Eq. (2.3).

loops (virtual particle-antiparticle intermediate states of the photon propagator) and $\alpha_{\rm QED}(Q^2 \to \infty) \to \infty$. The asymptotic freedom of QCD is of course a highly welcome feature, since in the large-Q^2 limit one should now be able to do perturbation theory. We know from the SLAC experiments on the nucleon structure functions that the partons, or constituents of the nucleons, are almost free (Panofsky, 1968). Using QCD perturbation theory, we can calculate the deviation from free partons to find the momentum-transfer (Q^2) dependence of the processes (only when $Q^2 \to \infty$ does $\alpha_s \to 0$). This Q^2 and energy dependence given by the QCD perturbation calculations has been confirmed by several experiments, e.g., a recent CERN experiment (Arnison *et al.*, 1986). This means that for high-energy and high-momentum-transfer (very-short-distance) phenomena, perturbative QCD is a tractable theory. However, if one is interested in low-energy, low-momentum-transfer phenomena in which the quark-quark forces become extremely strong (confining), the nonlinear aspects of QCD make the calculations very hard and have prevented a solution so far. In this region one has to rely on different QCD-based models. As will be clearer later on, the specific confinement forces are not crucial to an understanding of the nuclear forces, so we shall next present different quark models that incorporate confinement and later use these models to discuss the various aspects of the short-range nuclear forces.

III. QUARK MODELS

There have been many different quark models, from the earliest nonrelativistic models of the sixties (Kokkedee, 1969), and potential models (Isgur and Karl, 1977, 1978, 1979a, 1979b, 1980; Isgur, 1978; Close, 1979), to a variety of relativistic models, e.g., the bag models that have been popular during the last ten years (Chodos *et al.*, 1974a, 1974b; Bardeen *et al.*, 1975; Hasenfratz, Kuti, and Szalay, 1975; Hasenfratz and Kuti, 1978).

Below we shall sketch the most important features of some of these quark models, concentrating on the aspects that will be most important in applications to nuclear properties to be discussed.

A. The SU(6) quark model

From the previous section we learned that quarks (so-far discovered) come in five different flavors, each belonging to the fundamental representation of $\mathrm{SU}_c(3)$ color. From Table I we see that the flavor symmetry $\mathrm{SU}_F(n_f)$ (n_f is the number of flavors) is badly broken by nature if $n_f=4$ or 5, because the masses of the b and c quarks are very different from the masses of the s, d, and u quarks ($m_b \gg m_c \gg m_s > m_d \simeq m_u$). For $n_f=3$, however, we have fairly good $\mathrm{SU}_F(3)$ flavor symmetry, as indicated by the effective masses of the s and u,d quarks not being too different. For $n_f=2$ we have the $\mathrm{SU}_F(2)$ isospin symmetry (charge independence of nuclear forces). In nuclear physics one is interested mostly in baryons built only from u and d quarks, allowing one to use the almost exact $\mathrm{SU}_F(2)$ isospin symmetry.

The early quark model in the sixties used the $\mathrm{SU}_F(3)$ flavor symmetry mainly to classify the known hadron multiplets. In this model the fundamental quark (flavor) triplet **3** contains the three quarks u, d, and s. From elementary group theory one knows that three fundamental multiplets (representations) of $\mathrm{SU}_F(3)$ flavor can be coupled together in the following way (also illustrated by Young tableaux; for an easy introduction to this technique see, for example, Close, 1979):

$$\begin{array}{l}\square\times\square\times\square=\square\square\square+\begin{smallmatrix}\square\square\\\square\end{smallmatrix}+\begin{smallmatrix}\square\square\\\square\end{smallmatrix}+\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}\\ \mathbf{3}\times\mathbf{3}\times\mathbf{3}=\mathbf{10}+\mathbf{8}_1+\mathbf{8}_2+\mathbf{1}\end{array} \quad (3.1)$$

In the equation above, the upper line is the Young tableaux description of the given multiplets and the lower line is the number of states in the corresponding multiplet when we have SU(3). The Young tableaux contain (among other things) information on the permutation symmetry of the quarks (more correctly the quark wave functions) belonging to the multiplets. A Young tableau with only *one* horizontal row (vertical column) of boxes has a symmetric (antisymmetric) wave function under the interchange of any pair of quarks. Since we have three possible quark flavors (u,d,s), the symmetric flavor decuplet **10** in Eq. (3.1) accommodates the ten observed $J^P=\frac{3}{2}$ baryons [see Fig. 7(a)]. These wave functions can be found in Kokkedee (1969), and we give only two examples, $|\Delta^{++}\rangle = uuu$ and $|\Delta^{+}\rangle=(uud+udu+duu)/\sqrt{3}$. The antisymmetric multiplet, the SU(3) singlet **1** in Eq. (3.1), has the wave function

$$|\,\text{singlet}\rangle=(uds-dus+sud-sdu+dsu-usd)/\sqrt{6}\ .$$

The other two Young tableaux in Eq. (3.1) have mixed symmetries (no specific symmetry when one interchanges two arbitrary quarks). For example, a proton (isospin $\frac{1}{2}$) contains the quarks u and d, which form an isospin dou-

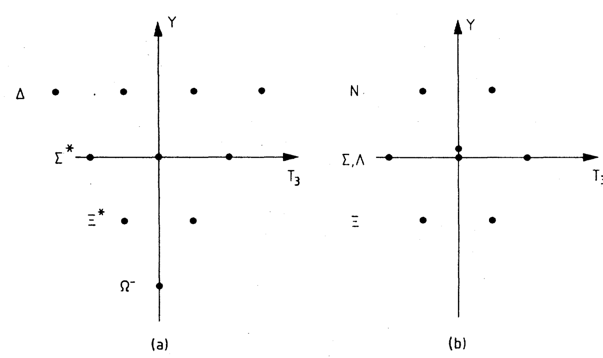
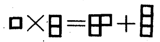


FIG. 7. Weight diagrams for the ground-state baryon decuplet (a) and octet (b) where I_3 is the third component of isospin. In this figure $Y=1+S$ where S is the strangeness quantum number.

blet. One way of constructing two linearly independent wave functions with the quark content *uud* is (where the Clebsch-Gordan coefficients are explicitly given)

$$\Psi_1=\sqrt{2/3}(uu)d-\frac{1}{\sqrt{3}}\frac{1}{\sqrt{2}}(ud+du)u$$
$$=(2uud-udu-duu)/\sqrt{6}\ ,$$
$$\psi_2=(udu-duu)/\sqrt{2}\ .$$

These wave functions belong to the multiplets $\mathbf{8}_1$ and $\mathbf{8}_2$ in Eq. (3.1) and are orthogonal to the decuplet and singlet wave functions of Eq. (3.1); ψ_1 is symmetric and ψ_2 is antisymmetric under the interchange of the two first quarks. The eight observed $J^P=\frac{1}{2}^+$ baryons [see Fig. 7(b)] are given by a *specific* linear combination of $\mathbf{8}_1$ and $\mathbf{8}_2$, requiring the flavor-spin product wave function to be symmetric under the interchange of any two quarks (see Close, 1979). In the baryon ground states, the space wave function is symmetric (if all three quarks are in the lowest S states). Thus the flavor singlet **1** in Eq. (3.1) does not exist in nature, due to the Pauli principle [because this would require a completely antisymmetric $\mathrm{SU}_S(2)$ spin wave function of three particles, which is impossible]. This nicely explains the observed **8** and **10** multiplets of baryons as three-quark states. The $q\bar{q}$ states are given by the product of a triplet and an antitriplet,

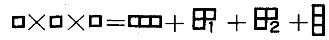


$$3\times\bar{3}=\mathbf{8}+\mathbf{1}\ , \tag{3.2}$$

which gives the observed pseudoscalar $J^P=0^-$ and vector $J^P=1^-$ meson nonets. Here we have an example of $\mathrm{SU}_F(3)$ breaking, because the $I=0$ state of the **8** above mixes with the singlet **1** state to give the physical particles. Also due to (almost perfect) chiral symmetry on the $\mathrm{SU}_F(2)$ (isospin) level, the pion is (almost) massless, which means it cannot be the pure $q\bar{q}$ state of Eq. (3.2), but rather is a collective state, which means the pion wave function also contains $qq\bar{q}\,\bar{q}$, etc., components (Brodsky and Lepage, 1981; Brodsky, 1982; Weise, 1984).

Since the quarks are classified by the fundamental $\mathrm{SU}_F(3)$ flavor and $\mathrm{SU}_S(2)$ spin multiplets, one can postulate that the six quarks $(u\uparrow,u\downarrow,d\uparrow,d\downarrow,s\uparrow,s\downarrow)$[1] transform according to the fundamental representation of $\mathrm{SU}_{FS}(6)$ flavor-spin (Gursey, Pais, and Radicati, 1964; Gursey and Radicati, 1964; Pais, 1964; Sakita, 1964). This $\mathrm{SU}_{FS}(6)$ proposal for the quarks was a generalization of Wigner's idea (Wigner, 1937) that the protons and neutrons should belong to an approximate $\mathrm{SU}_{FS}(4)$ [from $\mathrm{SU}_F(2)\times\mathrm{SU}_S(2)$] isospin-spin fundamental representation. Imposing $\mathrm{SU}_{FS}(6)$ symmetry for the quarks is much stronger than just imposing $\mathrm{SU}_F(3)$ and $\mathrm{SU}_S(2)$ separately. Thus, to the extent that $\mathrm{SU}_F(3)$ is broken in nature, $\mathrm{SU}_{FS}(6)$ will also be broken. Furthermore, an objection to the $\mathrm{SU}_{FS}(4)$ scheme of Wigner is that this symmetry is not valid at large energies because a Lorentz boost will inevitably mix spin and orbital angular momentum, thus distinguishing between the two aspects of the $\mathrm{SU}_{FS}(4)$ [or $\mathrm{SU}_{FS}(6)$] group, i.e., isospin (or flavor) and spin.

As a classification scheme for different hadron states, the $\mathrm{SU}_{FS}(6)$ wave functions are often used. As an example consider the product of three $\mathrm{SU}_{FS}(6)$ quarks:



$$6\times6\times6=\mathbf{56}+\mathbf{70}_1+\mathbf{70}_2+\mathbf{20}\ . \tag{3.3}$$

If all the quarks are in the ground state (S state), they have a symmetric space wave function. The color wave function is antisymmetric because the baryon must be an $\mathrm{SU}_c(3)$ color singlet, and therefore the flavor-spin wave function must be symmetric [i.e., belong to the **56**-plet of Eq. (3.3)] to give a totally antisymmetric baryon wave function. The $\mathrm{SU}_{FS}(6)$ **56**-plet contains the earlier mentioned $\mathrm{SU}_F(3)$ **10** with $J^P=\frac{3}{2}^+$ (40 states) and $\mathrm{SU}_F(3)$ **8** with $J^P=\frac{1}{2}$ (16 states). The $\mathrm{SU}_{FS}(6)$ **56** wave functions are very often used in practical calculations in various quark models, and are often referred to simply as SU(6) wave functions. Explicit expressions for these can be found in the literature (Thirring, 1965). If the quarks are allowed to be in excited states, the spatial wave functions of the three quarks can have mixed symmetry and the flavor-spin **70** and **20** multiplets of Eq. (3.3) will contribute. This is the case for excited baryons (N^*,Δ^*,Σ^*, etc.)—see Isgur and Karl (1977, 1978, 1979a).

We should keep in mind that $\mathrm{SU}_{FS}(6)$ is broken, so these wave functions are not physical eigenstates, but they give a convenient basis to work with when constructing eigensolutions of a particular Hamiltonian.

B. Potential models for the baryons

The QCD-motivated nonrelativistic quark models employ a strong, long-range potential V^{CONF}, which confines the quarks within the hadrons. In addition one adds a Fermi-Breit interaction V^{HYP} to describe the hyperfine splitting. The latter is the nonrelativistic reduction of the one-gluon exchange force between two quarks illustrated in Fig. 8 (De Rújula, Georgi, and Glashow, 1975), which at short distances should be a weak perturbative effect due to the asymptotic freedom of QCD. These models have been very successful in calculating different properties of hadrons (masses, magnetic moments, decay rates, etc.).

The Hamiltonian is taken to be

$$H=\sum_i\left[m_i+\frac{P_i^2}{2m_i}\right]+\sum_{i<j}(V_{ij}^{\mathrm{CONF}}+V_{ij}^{\mathrm{HYP}})-E_{\mathrm{c.m.}}\ , \tag{3.4}$$

[1] ↑ and ↓ are the $m=\pm\frac{1}{2}$ states with $S=\frac{1}{2}$.

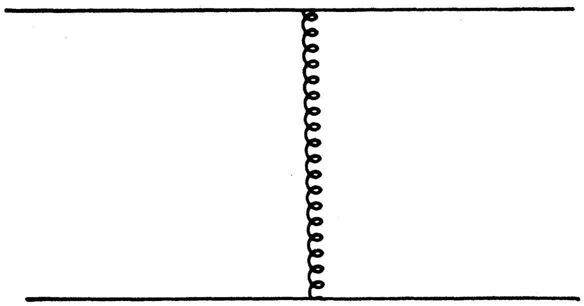


FIG. 8. The gluon exchange diagram.

where m_i is the mass of quark i, $E_{\text{c.m.}}$ is the kinetic energy associated with the center-of-mass motion that should be subtracted, and the confining potential is

$$V_{ij}^{\text{CONF}}=-C_n\lambda_i^a\lambda_j^a r_{ij}^n, \quad n=1,2 \ . \tag{3.5}$$

Here λ^a are the (3×3) Gell-Mann SU(3) matrices normalized as $\lambda^2=\frac{16}{3}$, r_{ij} is the distance between quarks i and j, and C_n is some coupling constant.

The hyperfine (Fermi-Breit) interaction is given by (De Rújula, Georgi, and Glashow, 1975)

$$V_{ij}^{\text{HYP}}=\lambda_i^a\lambda_j^a\frac{\alpha_s}{4}\left[\frac{1}{r_{ij}}-\frac{\pi}{m_im_j}(1+\tfrac{8}{3}\mathbf{S}_i\cdot\mathbf{S}_j)\delta(\mathbf{r}_{ij}) -\frac{4}{m_im_j}S_{ij}r_{ij}^{-3}\right], \tag{3.6}$$

where we have dropped the spin-orbit (LS) term of the Fermi-Breit interaction (see below), α_s is the strong-coupling constant, e.g., Eq. (2.9), and S_{ij} is the tensor operator given by

$$S_{ij}=\frac{3(\mathbf{S}_i\cdot\mathbf{r}_{ij})(\mathbf{S}_j\cdot\mathbf{r}_{ij})}{r_{ij}^2}-\mathbf{S}_i\cdot\mathbf{S}_j \ . \tag{3.7}$$

The advantage of the harmonic-oscillator model, i.e., $n=2$ in Eq. (3.5), is that one can easily separate the center-of-mass motion *exactly*. For any translationally invariant potential of two-body or three-body type, introducing the Jacobi coordinates [Eq. (3.8)] separates *exactly* the center-of-mass motion, and for the harmonic-oscillator potential the resulting potential [Eq. (3.9)] is very simple. This center-of-mass separation is very important for describing the forces between two nucleons in terms of six quarks. If we choose the two quark masses (m_2 and m_3) to be equal, then a convenient choice of internal coordinates is

$$\mathbf{r}_{1,23}=\mathbf{r}_1-\tfrac{1}{2}(\mathbf{r}_2+\mathbf{r}_3), \quad \mathbf{r}_{23}=\mathbf{r}_2-\mathbf{r}_3 \ , \tag{3.8}$$

where $\mathbf{r}_i$ is the position of quark i. Then it is possible to write the confining harmonic oscillator potential in a baryon [including $\langle\lambda_i^a\lambda_j^a\rangle=-\frac{8}{3}$ $(i\neq j)$ for a color singlet three-quark state] as

$$V^{\text{CONF}}=\sum_{i<j}V_{ij}^{\text{CONF}}=\tfrac{16}{3}C_2(\mathbf{r}_{1,23}^2+\tfrac{3}{4}\mathbf{r}_{23}^2) \ , \tag{3.9}$$

showing that the confining potential contains only two independent oscillators and that the center-of-mass coordinate does not enter V^{CONF}. Note that Eq. (3.9) follows from Eq. (3.5) even if $m_1\neq m_2=m_3$, which covers the possibilities in light baryon spectroscopy. Isgur and Karl (1977, 1978, 1979a, 1979b, 1980) used this model to calculate masses of ground-state and excited baryons. The theoretical results agree very well with experimental values. As an example, we give their numbers for the ground-state baryons in Table II.

In calculations of the ground-state masses the tensor part of V^{HYP} is not very important. The only way in which the tensor force operates in the ground-state baryons is through the small admixture of higher oscillator functions in the ground-state wave functions (Isgur and Karl, 1977, 1978, 1979a, 1979b). In calculations of masses of excited baryons the tensor part turns out to be of some importance (Isgur and Karl, 1977). However, this tensor interaction is unimportant in calculations of the baryon-baryon interaction (Oka and Yazaki, 1984). Thus we shall use the model of Eqs. (3.4)–(3.6) without the tensor part of Eq. (3.6) in our discussion of the short-range NN interaction, in Sec. V. In addition, one is forced to ignore the spin-orbit forces, which also should be an integral part of Eq. (3.5). They will give the wrong ordering of the energy levels of negative-parity N^* and Δ^* states as compared to experiments (Isgur and Karl, 1977). However, this neglect of $\mathbf{L}\cdot\mathbf{S}$ forces in potential model calculations of the N^* and Δ^* states is supported by a bag model calculation (Fiebig and Schwesinger, 1983; Myhrer and Wroldsen, 1984a, 1984b) in which it has been shown phenomenologically (for $\alpha_s\simeq0.7$) that the apparent $\mathbf{L}\cdot\mathbf{S}$ force due to one-gluon exchange is almost canceled by the effective $\mathbf{L}\cdot\mathbf{S}$ force due to the MIT model's confinement condition (see Sec. V.E for further comments).

The orbital part of the quark wave function for the three-cluster in the baryon (used in calculating baryon-

TABLE II. Masses (in MeV) of ground-state baryons compared to the nonrelativistic quark model (Isgur and Karl, 1979b).

	N	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω
Expt.	940	1115	1195	1320	1240	1385	1535	1670
Theory	940	1110	1190	1325	1240	1390	1530	1675

baryon scattering) is assumed to be Gaussian,

$$\Phi(r_i)=\pi^{-3/4}b^{-3/2}e^{-r_i^2/2b^2}, \tag{3.10}$$

where b is the oscillator length related to the rms charge radius of the proton by

$$\langle r^2\rangle_p^{1/2}=b . \tag{3.11}$$

In the first applications of this model to describe baryon-baryon scattering (Oka and Yazaki, 1980, 1981; Harvey, 1981b) b was simply fixed by the relation (3.11). The other parameters of the theory were determined by fitting the magnetic moments of the nucleon and the N-Δ mass difference; see, for instance, Harvey (1981b); Faessler *et al.* (1982, 1983). Later it was realized (see, for instance, Faessler *et al.*, 1982; Liu, 1982; Ohta *et al.*, 1982; Harvey and LeTourneux, 1984) that Eq. (3.11) is inadequate in determining the oscillator length b. One has to impose the stability condition for the nucleon mass

$$\frac{\partial}{\partial b}\langle N\mid H\mid N\rangle=0 \tag{3.12}$$

to separate the nucleon from its excited states (the Roper resonance, etc.). The oscillator length determined by the stability condition is usually in the range 0.5–0.6 fm. This is somewhat lower than the value of $b=0.8$ fm preferred in earlier work (Oka and Yazaki, 1980, 1981; Harvey, 1981b). The new, lower value of b does not necessarily come in conflict with the rms charge radius, Eq. (3.11), because one should remember that Eq. (3.11) takes into account only the root-mean-square radius of the charge distribution of the *quarks*. In addition chiral symmetry arguments say we have to add a correction to the quarks' rms radius due to the meson cloud surrounding the nucleon (see Sec. III.C.3).

C. Bag models

1. Introduction and motivation

The quark models discussed so far have been nonrelativistic models in which u and d quarks have an effective mass of the order $\sim$300 MeV (see Table I). However, the quarks are confined to a small volume, which means that they acquire an appreciable momentum $\sim$300 MeV/c. Thus the nonrelativistic treatment is doubtful. Despite the phenomenological successes of the quark potential models, one should try to construct relativistic quark models. Several such alternatives exist. Two of the first relativistic quark models were discussed by Bogoliubov (1968) and by Feynman, Kislinger, and Ravndal (1971). The latter model is a relativistic generalization of the harmonic-oscillator model. These potential models have been refined by the Regensburg group (Weise, 1984), but have not been used to calculate NN scattering. In the following we shall discuss the MIT bag model (Chodos *et al.*, 1974a, 1974b; DeGrand *et al.*, 1975), which will be used to understand some of the results of NN low-impact-parameter scattering in Sec. V.

The MIT bag model is like a free-Fermi-gas model for quarks. This model was later modified to incorporate chiral invariance (Chodos and Thorn, 1975; Inoue and Maskawa, 1975; Brown and Rho, 1979; Brown, Rho, and Vento, 1979; Callan, Dashen, and Gross, 1979). A typical ingredient in the bag models is the small u and d quark masses ($\sim$10 MeV), which should be compared to the masses used in nonrelativistic models (Table I). The effective u and d quark masses used in the nonrelativistic models can be understood as quark energies due to confinement, as can be seen, for example, in comparing expressions for magnetic moments in the two models. In bag models one understands why isospin, $SU_F(2)$, is a good quantum number. Since the major part of the u and d effective masses used in the nonrelativistic models are due to confinement forces that are flavor independent, the small isospin breaking is measured as the u and d mass difference relative to their effective masses. Since $m_u\simeq m_d\ll m$(effective), the difference is also small. However, the bag models suffer from lack of translational invariance, and the corresponding center-of-mass motion causes problems (which can easily be solved for the nonrelativistic harmonic-oscillator models). Below we shall discuss the main features of the bag models, before we discuss in some detail the MIT model and the necessary modifications and consequences of chiral symmetry.

The basic ideas of the bag models are simple: Quarks and gluons exist inside a "bubble" in the complicated physical vacuum of QCD. Inside the small bubble the quarks and gluons have "high" momenta, and asymptotic freedom of QCD then says that they interact very weakly (perturbatively) with one another. The so-called perturbative vacuum inside the bubble is thought of as an excited state of the true QCD vacuum, and the bubble has a constant energy density B, which in principle can be determined from models of the QCD vacuum, e.g., that of Hansson (1985). Color quantum numbers are excluded from the true QCD vacuum in this model, which means that the quark and gluon fields inside the bag obey certain boundary conditions on the surface of the "bubble" to prevent color from leaking into the surrounding QCD vacuum. Thus asymptotic freedom and color confinement (discussed in Sec. II) are postulated in the bag models.

The MIT model enforces a sharp boundary condition (confinement) on the quark wave functions. This is certainly an approximation to a more realistic model in which we expect some spatial transition region between the QCD vacuum outside the bubble and the perturbative vacuum inside. If this transition region is very small compared to the bubble (bag) volume, the MIT bag is a reasonable model. However, if the transition region to the interior, where quarks move freely, is a very large fraction of the bag volume, then we have the SLAC bag (Bardeen *et al.*, 1975). Hasenfratz and Kuti (1978) also allowed the surface to have dynamic degrees of freedom (surface tension in addition to the constant bag pressure

B), which allows for collective bag states to enter the model. For a review see, for example, Myhrer (1984).

2. The MIT bag model

The first version of the bag model used for explicit calculations was the static, spherical MIT bag (DeGrand, Jaffe, Johnson, and Kiskis, 1975). In this model the quarks and gluons move inside a sphere of radius R. Inside the sphere the quarks obey the free Dirac equation

$$H\Psi=(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m)\Psi=\omega\Psi, \quad r<R \ , \tag{3.13}$$

where

$$\gamma^k=\begin{bmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{bmatrix}, \quad \beta=\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}=\gamma^0 ,$$

$$\gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

and $\boldsymbol{\alpha}=\beta\boldsymbol{\gamma}$ are the Dirac 4×4 matrices (we use the metric $g_{00}=-g_{kk}=1$, $k=1,2,3$). Equation (3.13) is solved with the bag boundary condition (BBC)

$$-i\hat{\mathbf{r}}\cdot\boldsymbol{\gamma}\Psi=\Psi, \quad r=R \ . \tag{3.14}$$

Equations (3.13) and (3.14) are equivalent to a Dirac particle in an infinite square-well potential; see Bogoliubov (1968). Furthermore, the BBC in Eq. (3.14) implies that, for $r=R$,

$$\hat{\mathbf{r}}\cdot\mathbf{j}=i\hat{\mathbf{r}}\cdot\Psi^\dagger\boldsymbol{\alpha}\Psi=0 \ , \tag{3.15}$$

i.e., no quark vector current $\mathbf{j}$ escapes the bag. Solutions to these equations can be found in many places in the literature; see DeGrand *et al.* (1975) or Chodos and Thorn (1976). We write here only the lowest possible mode, i.e., the $S_{1/2}$ state (j_0 and j_1 are spherical Bessel functions):

$$\Psi(\mathbf{r})=\frac{N}{\sqrt{4\pi}j_0(x_s)} \times \begin{bmatrix} \left[\dfrac{\omega_s+m}{\omega_s}\right]^{1/2} i j_0(x_s r/R) \\ -\left[\dfrac{\omega_s-m}{\omega_s}\right]^{1/2} \boldsymbol{\sigma}\cdot\hat{\mathbf{r}} j_1(x_s r/R) \end{bmatrix} \chi \ , \tag{3.16}$$

where the quark energy $\omega_s=[m^2+(x_s/R)^2]^{1/2}$ is determined by the BBC equation (3.14). For quark mass $m=0$ one finds $x_s=2.043$, and the normalization $\int d^3x\, \Psi^\dagger\Psi=1$ gives $|N|^2=x_s/2R^3(x_s-1)$.

To illustrate the model (and arguments to be used in Sec. V), we shall calculate the nucleon mass taking into account only the energy terms we have already discussed, i.e., the volume energy due to the density B and the energy that the three quarks acquire due to the BBC equation (3.14) (the MIT bag does not have a surface energy). We get for three $S_{1/2}$ quarks

$$E_{3q}=\tfrac{4}{3}\pi BR^3+3\omega_s \ . \tag{3.17}$$

The u and d quarks are very light, and to a good approximation we can assume their masses to be equal to zero. Thus we get

$$E_{3q}=\tfrac{4}{3}\pi BR^3+3x_s/R \tag{3.18}$$

for massless u and d quarks. The bag radius is determined by requiring bag stability, which for spherical bags equals

$$\frac{\partial E}{\partial R}=0 \ . \tag{3.19}$$

(The QCD pressure B requires smaller bag radii, whereas the fast-moving u and d quarks exert an outward-directed pressure on the surrounding bag surface.) This gives

$$R=(3x_s/4\pi B)^{1/4}, \quad E_{3q}=4x_s/R \ . \tag{3.20}$$

Having only one free parameter, B, we can fit this to the average of the proton and Δ masses of 1085 MeV and get the rough estimate $B^{1/4}\simeq110$ MeV and $R=1.5$ fm (these values are modified in the final version of the model).

The model above would give the same masses to the N and the Δ. However, there are two important corrections to the above discussion which must be added. One is the short-distance perturbative hyperfine interaction, similar to Eq. (3.6), which is due to the one-gluon exchange forces between quarks (Fig. 8). This will result in different N and Δ masses and is calculated as the color magnetic interaction energy of the quarks, which here are confined in the bag. As shown in Akhiezer and Berestetskii (1965), solving Maxwell's equations or calculating the interaction energy due to one-gluon exchange between two quarks give the same result. [In the MIT bag model one can show that only exchanging the lowest gluon mode gives an answer only 0.5% off the correct value for quarks in the $S_{1/2}$ mode (Close and Horgan, 1980).] We calculate the color magnetic energy (sum over repeated color index a) as

$$\Delta E_{\rm CM}=-\tfrac{1}{2}g^2\int d^3\mathbf{r}\,\mathbf{B}^a(\mathbf{r})\cdot\mathbf{B}^a(\mathbf{r}) \ , \tag{3.21}$$

where $\mathbf{B}^a(\mathbf{r})=\sum_i \mathbf{B}_i^a(\mathbf{r})$, and where the sum runs over all the quarks i in the bag. The color magnetic field $\mathbf{B}_i^a(\mathbf{r})$ is found by solving Maxwell's equations inside the bag with appropriate boundary conditions given the quark current $\mathbf{j}_i^a=\frac{1}{2}\lambda^a\Psi_i^\dagger\boldsymbol{\alpha}\Psi_i$. The color magnetic field must satisfy the boundary condition (no color quantum number allowed outside the bag)

$$\mathbf{r}\times\mathbf{B}^a=0, \quad r=R \ . \tag{3.22}$$

Using the minimal MIT prescription of not including self-interactions in $\Delta E_{\rm CM}$ thus gives (DeGrand *et al.*, 1975)

$$\Delta E_{\rm CM}=-g^2\sum_{i<j}\int d^3\mathbf{r}\,\mathbf{B}_i^a(\mathbf{r})\cdot\mathbf{B}_j^a(\mathbf{r}) \ . \tag{3.23}$$

Equation (3.23) is the MIT bag analog to the nonrelativistic hyperfine splitting of Eq. (3.6). By performing the calculation outlined above for two massless quarks in the lowest $S_{1/2}$ mode (for other modes, see DeGrand and Jaffe, 1976, and Wroldsen and Myhrer, 1984) one gets

$$\Delta E_{\rm CM} = -\alpha_s \frac{0.177}{R} \sum_{i<j} \lambda_i^a \mathbf{S}_i \cdot \lambda_j^a \mathbf{S}_j \ , \qquad (3.24)$$

where $\alpha_s = g^2/4\pi$ is the strong fine-structure constant, and the number 0.177 comes from integrating the space part of the quark wave functions in the bag.

The second correction to the original MIT bag mass formula is the so-called zero-point energy, which also includes center-of-mass effects. This is parametrized as $E_0 = -Z_0/R$. A simple estimate of the center-of-mass energy contribution to Z_0 from three quarks in a proton gives $Z_0^{\rm c.m.} \simeq 0.8$ (Jaffe, 1982).

Adding all these pieces together we finally arrive at the original MIT bag energy formula,

$$E = \tfrac{4}{3}\pi B R^3 + \sum_i \omega_i + \Delta E_{\rm CM} - Z_0/R \ . \qquad (3.25)$$

With this fairly simple model one is able to calculate reasonably well a large quantity of experimental data, such as masses, ratio of magnetic moments, etc. (DeGrand *et al.*, 1975). In particular, this is the first quark model to give a reasonable value for the axial-vector coupling g_A (see Sec. III.C.3). The favored parameter set for the original bag model is (masses of u,d quarks equal zero)

$$B^{1/4} = 0.145 \text{ GeV}, \quad Z_0 = 1.84, \quad \alpha_s = 2.2 \ ,$$

$$m_s = 0.279 \text{ GeV} \ ,$$

giving typical baryon radii of $\sim 5 \text{ GeV}^{-1}$. In this bag model the "effective mass" of the massless u and d quarks ($x_s/R \simeq 400$ MeV) is due only to the confinement condition [Eq. (3.14); see confinement values for quark masses in Table I].

3. The bag, chiral symmetry, and the theoretical determination of the pion-nucleon coupling constant

Although appealing in its simplicity and in its reasonable success in predicting the static baryon properties, the original MIT bag model does not provide any mechanism for the well-known long-range meson interaction between nucleons, thus making it quite useless in the context of nuclear physics (DeTar, 1978, 1979; Thomas, 1983). However, as briefly mentioned in the prelude, chiral symmetry is important in low-energy nuclear and particle physics (Lee, 1972; Pagels, 1975; Rho, 1984); requiring the MIT bag model to conform with chiral symmetry immediately gives us a way to understand the long-range interaction between nucleons from a quark viewpoint (Brown and Rho, 1979; Brown, Rho, and Vento, 1979). The implementation of chiral symmetry in the quark model and how this generates the correct pion-nucleon coupling constant will be the subject of this subsection.

Chiral symmetry is, like the bag model, reasonable for low-energy phenomena. In the chiral symmetric limit both the vector current j^ν and its parity partner, the axial current A^ν, are conserved ($\partial_\nu j^\nu = 0$ and $\partial_\nu A^\nu = 0$). Consider now the matrix element of the axial current for a neutron of momentum p to decay into a proton of momentum p'. This matrix element involves (from Lorentz invariance; see a more general discussion in Marshak, Riazuddin, and Ryan, 1969) two possible couplings: g_A and f_p,

$$\langle p(p') \mid A^\nu \mid n(p) \rangle = \overline{U}(p')(-g_A \gamma^\nu \gamma_5 + f_p q^\nu \gamma_5) U(p) \ .$$

Here the momentum transfer $q^\nu = p'^\nu - p^\nu$. Taking the derivative of this expression gives from axial current conservation

$$q_\nu \langle p(p') \mid A^\nu \mid n(p) \rangle = 0 = \overline{U}\gamma_5 U(-2Mg_A + q^2 f_p) \ ,$$

where we have used the Dirac equation, Eq. (3.13), for the nucleon of mass M to replace $(p'_\nu - p_\nu)\gamma^\nu$ by $-2M$ when operating on $U(p)$ and $\overline{U}(p')$. Since $\overline{U}\gamma_5 U \neq 0$, the right-hand side (rhs) above can only be zero as $q^2 \to 0$ if one of the following conditions applies: (i) $M = 0$, which means we have manifest chiral symmetry on the nucleon level and nucleons come in energy-degenerate parity doubled (Wigner mode, not seen), or (ii) $f_p \sim q^{-2}$ and $M \neq 0$, which means we have a hidden chiral symmetry (Goldstone mode). In neutron decay the second alternative is illustrated in Fig. 9, where a pion (Goldstone boson) is propagating between the n-p vertex and the vertex where we take the derivative of A^ν. The πpn coupling is $g_{\pi N}$, and the coupling of the pion of momentum q_ν to the axial current A^ν is $f_\pi q^\nu$. (The pion decay constant f_π is measured by $\pi \to$ muon + neutrino via the axial current.) This diagram can be written as

$$\frac{g_{\pi N} f_\pi q^\nu}{q^2 - \mu^2} \ ,$$

where μ is the pion mass. This gives $f_p q^\nu$ above, which means $q_\nu A^\nu = 0$ provided $\mu^2 = 0$ and $-2Mg_A + q^2 f_p = -2Mg_A + q^2 g_{\pi N} f_\pi / q^2 = 0$. From here follows the Goldberger-Treiman relation,

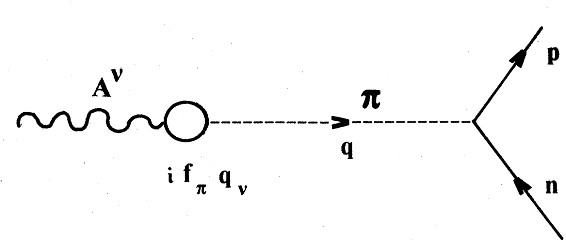


FIG. 9. The pion-pole dominance of the axial current in neutron decay.

$$2Mg_A = f_\pi g_{\pi N} \ ,$$

which experimentally is very well satisfied:

$$1 - 2Mg_A / f_\pi g_{\pi N} = 0.08 \pm 0.02 \ .$$

For other relations see, for example, Adler and Dashen (1968). The introduction of chiral symmetry in the bag models forces us to couple pseudoscalar meson fields to the bag (Chodos and Thorn, 1975; Inoue and Maskawa, 1975; Brown and Rho, 1979; Brown, Rho, and Vento, 1979; Callan, Dashen, and Gross, 1979). Here it is really postulated that the QCD vacuum surrounding the bag spontaneously breaks chiral symmetry and therefore contains massless Goldstone bosons (pions). We say that the QCD vacuum is realized in the Goldstone mode. If, in addition, we require that the bag interior has $\langle \overline{\Psi}\Psi \rangle = 0$, the bag interior is realized in the Wigner-Weyl mode (each state comes in parity doublets), and massless pions do not exist inside the bags. (As mentioned, some type of transition region between the exterior and interior is expected. The sharp boundary of bag models is hopefully a reasonable approximation.) As long as the momentum transfer to these mesons (pions) is small, we can justify treating the pions as inert, elementary boson fields. [The Regensburg group (Weise, 1984) has implemented chiral symmetry in a relativistic potential model using similar arguments.] Since we know that these mesons are the mediators of the long-range nuclear force, the first question is whether these chiral quark models can give the *correct* meson-nucleon coupling constant. In order to answer this question, we shall first go through some details of the chiral bag model.

For a massless quark, the Dirac equation (3.13) is chiral invariant, $[\gamma_5, H] = 0$ (γ_5 does not commute with a mass term $m\bar{\psi}\psi$). Therefore, if we have one solution Ψ_+ of Eq. (3.13), the chirally rotated wave function (rotated an angle δ)

$$\Psi_+ \to \Psi^\delta = e^{i\delta\gamma_5}\Psi_+ = (\cos\delta + i\gamma_5 \sin\delta)\Psi_+ \qquad (3.26)$$

will have the same energy. However, it is obvious that the BBC equation (3.14) is not chiral invariant. As an example, it is easy to show that if we take Ψ_+ to be the $S_{1/2}$ state discussed earlier, the chirally rotated state $\Psi^{\pi/2}$ will be [using $\Psi_+ = \Psi$ of Eq. (3.16) with $m = 0$ and Eq. (3.26) with $\delta = \pi/2$]

$$\Psi^{\pi/2} = i\gamma_5 \psi \propto \begin{bmatrix} ij_1 \sigma \cdot \hat{\mathbf{r}} \\ j_0 \end{bmatrix} \chi \ . \qquad (3.27)$$

This is the $P_{1/2}$ MIT wave function, which has an energy different from that of $S_{1/2}$; both are found by imposing the BBC of Eq. (3.14). To get the same energy for Ψ^δ and Ψ_+ we have to impose the chiral boundary condition

$$-i\hat{\mathbf{r}} \cdot \gamma \Psi = e^{i\theta\gamma_5}\Psi, \quad r = R \ , \qquad (3.28)$$

where one introduces a field $\theta(R)$ that transforms as $\theta_+ \to \theta^\delta = \theta_+ - 2\delta$ on the bag surface $r = R$ when Ψ is chirally transformed according to Eq. (3.26). Using Eq. (3.28) for ψ^δ and θ^δ, we find

$$-i\hat{\mathbf{r}} \cdot \gamma e^{i\delta\gamma_5}\Psi_+ = e^{i(\theta_+ - 2\delta)\gamma^5} e^{i\delta\gamma_5}\Psi_+$$
$$= e^{-i\delta\gamma_5} e^{i\theta_+\gamma_5}\Psi_+ \ . \qquad (3.29)$$

The factor $e^{-i\delta\gamma_5}$ drops out because on the left-hand side γ and γ_5 anticommute, and Eq. (3.28) gives the same eigenenergies for Ψ_+ and Ψ^δ. Extending $\theta(r)$ outside the bag surface and demanding chiral symmetry in all space leads to the U(1) σ model. From this it is clear that the Lagrangian of the quark and θ fields is chirally invariant. By extending this to the SU(2) case, we see that the pseudoscalar field $\theta(r)$ becomes the Goldstone pion field $\vec{\pi}$, and the extension of Eq. (3.28) becomes the *central equation* of this section, the chiral boundary condition equation:

$$-i\gamma \cdot \hat{\mathbf{r}} \Psi = e^{i\vec{\pi} \cdot \vec{\tau}\gamma_5 / f_\pi}\Psi, \quad r = R \ . \qquad (3.30)$$

This equation is the basis for the following discussion, and it is this chiral-invariant boundary condition which, as will be shown, will give us the correct pion-nucleon coupling constant $g_{\pi N}$. It should be stressed that this requirement on the quark wave function is due to chiral symmetry alone. In Eq. (3.30) $\vec{\tau}$ is the isospin operator, $\vec{\pi}$ the classical pion field, and $f_\pi = 93$ MeV the pion decay constant. This equation shows how the pions couple to the quarks at the bag surface, i.e., the quarks in the bag act as the source for the pion field. Since chiral symmetry requires a continuous axial current, we assume that the pions carry this current outside the bag and that the source of the pion field is the divergence of the quark axial current on the bag surface. This model of a pion field surrounding a cavity is an approximation to the physical picture of a hadron, in which we have quarks inside the cavity and these quarks strongly polarize the QCD vacuum outside in $J^P = 0^-$ $q\bar{q}$ pairs in order to preserve chiral symmetry. However, these massless pions are then no longer pure $q\bar{q}$ states, but collective states, meaning that the 0^- state is $|\pi\rangle = a\,|q\bar{q}\rangle + b\,|qq\bar{q}\,\bar{q}\rangle + \cdots$, as discussed by Brodsky and Lepage (1981), Brodsky (1982), and the Regensburg group (Weise, 1984).

The chiral-invariant pion Lagrangian is highly nonlinear (Weinberg, 1968) but, in the following, we shall use the linear approximation to the pion field equation of motion, which means that we assume the pion field to be a small perturbation of the MIT bag, and we shall treat the pion effects only to lowest order. This is justified for a bag radius $R \sim 1$ fm or larger (Hulthage, Myhrer, and Xu, 1981; Thomas, 1983). This way of treating the pion field is the one used in the different chiral or "cloudy bag" models (Jaffe, 1982; Thomas, 1983; Miller, 1984; Myhrer, 1984). In the "little bag" of Brown and Rho (Brown and Rho, 1979; Vento *et al.*, 1980) the bag radius is so small that the nonlinearities of the chiral Lagrangian are dominant. However, one should keep in mind that both treatments have the same origin, i.e., the bag model with chiral symmetry requirements imposed. The hedgehog solution (Vento, 1980; Vento *et al.*, 1980) of this nonlinear chiral Lagrangian is connected with a consideration of nucleons as Skyrmions and arguments as to how nucleon-nucleon interactions can be described as Skyrmion-Skyrmion interactions (Jackson, Jackson, and Pasquier, 1985; Kutschera and Pethick, 1985; Lacombe *et al.*, 1985; Zahed and Brown, 1986).

Below we shall now sketch the calculations in the case of a "perturbative" pion field outside the bag, i.e., we neglect the nonlinear pion-pion interaction terms of the chiral Lagrangian and get the free Klein-Gordon equation for the pion outside the bag (see, for example, Myhrer, 1984, for a derivation). However, we keep the chiral-invariant nonlinear pion-quark coupling given by Eq. (3.30) to keep chiral symmetry on the quark level. This difference in treatment is not unreasonable, since the pion field in this chiral model is a classical field, as opposed to the quark fields. Our presentation will be very brief, but detailed enough to give the reader a feeling for the steps needed in such a bag model calculation. We shall discuss only the coupling of the pion to the nucleon, which is a special case of the general pion-baryon coupling discussed by Høgaasen and Myhrer (1983a).

In this case the pion field will satisfy the free Klein-Gordon equation (the overarrow indicates a vector in isospin space)

$$(\nabla^2-\mu^2)\vec{\pi}(\mathbf{r})=0, \quad r\neq R \ , \tag{3.31}$$

where μ is the pion mass introduced for later convenience ($\mu=0$ in the chiral limit). This equation gives the free, static, classical pion field outside the bag,

$$\vec{\pi}^{\,0}(\mathbf{r})=\frac{g_0}{8\pi M}\frac{1+\mu r}{r^2}e^{-\mu r}\vec{\tau}\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}, \quad r>R \ , \tag{3.32}$$

where we define g_0 as the pion-quark coupling constant and where $\vec{\tau}$ and $\boldsymbol{\sigma}$ are quark operators, $\hat{\mathbf{r}}=\mathbf{r}/r$, and M is the nucleon mass. In the cloudy bag model (Théberge, Thomas, and Miller, 1980) the pion field is also allowed to enter the bag, and it has the form

$$\vec{\pi}^{\,i}(\mathbf{r})=\frac{3C}{\mu i}j_1(i\mu r)\vec{\tau}\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}, \quad r<R \ . \tag{3.33}$$

Note that this is a free (noninteracting) pion field, the solution of Eq. (3.31) inside a cavity of radius R. The constant C in this equation is zero if we do not allow pions inside the bag. Otherwise, it is determined by requiring $\vec{\pi}^{\,i}=\vec{\pi}^{\,0}$ on the bag surface ($r=R$).

In the chiral limit ($\mu=0$), the axial current has to be continuous in all space. Demanding the radial component of the axial current to be continuous on the surface gives

$$\hat{\mathbf{r}}\cdot\vec{\mathbf{A}}^{Q}-f_\pi\frac{\partial}{\partial r}\vec{\pi}^{\,i}\Big|_{r=R-\varepsilon}=-f_\pi\frac{\partial}{\partial r}\vec{\pi}^{\,0}\Big|_{r=R+\varepsilon}\ , \quad \varepsilon\to 0^+ \ , \tag{3.34}$$

where $\vec{\mathbf{A}}^{Q}$ is the axial isospin current carried by the quarks:

$$\vec{\mathbf{A}}^{Q}=\bar{\Psi}\gamma\gamma_5\frac{\vec{\tau}}{2}\Psi \ . \tag{3.35}$$

The axial isospin current carried by the pion, in our "perturbative" approach, is to lowest order in the pion field (see Myhrer, 1984)

$$\vec{\mathbf{A}}^{\pi}=-f_\pi\nabla\vec{\pi} \ . \tag{3.36}$$

This has already been used to get Eq. (3.34). Using the $S_{1/2}$-state solution to the Dirac equation (3.16),

$$\psi=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} iF \\ G\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\end{bmatrix}\chi \ , \tag{3.37}$$

gives

$$\hat{\mathbf{r}}\cdot\vec{\mathbf{A}}^{Q}=\frac{1}{4\pi}\chi^\dagger\frac{\vec{\tau}}{2}(|F|^2+|G|^2)\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\chi \ . \tag{3.38}$$

This, together with the pion fields of Eqs. (3.32) and (3.33) in the continuity equation (3.34), gives the quark-pion coupling constant in terms of the quark wave-function components F and G:

$$g_0=\frac{MR^3}{2f_\pi}\{|F|^2+|G|^2\}_{r=R}\frac{e^{\mu R}}{1+\mu R+\frac{1}{2}(\mu R)^2}\frac{1}{1+X} \ . \tag{3.39}$$

If the pions are allowed inside the bag, C is determined as discussed and X is given by

$$X=\frac{1+\mu R}{2+2\mu R+(\mu R)^2}\times\frac{-2\mu R\cosh\mu R+[2+(\mu R)^2]\sinh(\mu R)}{\mu R\cosh\mu R-\sinh\mu R} \ . \tag{3.40}$$

If $\mu=0$ we have $X=\frac{1}{2}$, and although we can suppress this free (noninteracting) pion field inside, $\vec{\pi}^{\,i}$, by increasing μ in Eq. (3.33), the value of X above is $X\simeq 0.5\pm 0.02$ for $\mu R\leq 2$. For increasing $\mu R>2$, X will increase towards 1. Here $X=0$ if pions are excluded from the bag [$C=0$ in Eq. (3.33)]. Furthermore, g_0 also enters the chiral boundary condition [Eq. (3.30)] through the pion field. To first order in the pion field this gives the energy quantization for a single nonstrange quark in the nucleon:

$$F(kR)+G(kR)(1+\rho)=0 \ , \tag{3.41}$$

where ρ for the nucleon (see Høgaasen and Myhrer, 1983a, and Myhrer, Brown, and Xu, 1981, for the calculation of the expectation values needed here) is given by

$$\rho\simeq\frac{16g_0}{24\pi M}\frac{1+\mu R}{f_\pi R^2}e^{-\mu R} \ . \tag{3.42}$$

By using the SU(6) wave functions discussed in Sec. III.A one can relate the pion-quark coupling g_0 used above to the pion-nucleon coupling constant $g_{\pi N}$. One finds

$$g_{\pi N}=5g_0/3 \ . \tag{3.43}$$

Including second-order terms in the pion field, one finds that these contribute less than 1% to the baryon's total quark energy for $R > 1$ fm (Høgaasen and Myhrer, 1983a). [However, for smaller bags the nonlinearities in the pion Lagrangian will certainly be important, and this perturbative treatment cannot be used. Perry and Rho (1986) argue on the basis of the two-dimensional model of Nadkarni, Nielsen, and Zahed (1985) that observables might be independent of the bag size.]

The two equations (3.39) and (3.41) will give us the correct pion-nucleon coupling constant if treated in a self-consistent, iterative way as follows: First compute the trial wave functions F_0 and G_0 satisfying the boundary condition of the MIT bag [i.e., Eq. (3.30) with $\vec{\pi}=0$ or Eq. (3.41) with $\rho=0$], with an energy ω_s (3.16) for the quarks. Then compute the pion-quark coupling constant with these functions from the continuity of the axial current [Eq. (3.39)]. The resulting pion field is then put into the boundary condition of Eq. (3.41) through ρ, and one finds new solutions F and G with an energy ω_1. This procedure is repeated until convergence. Explicit numerical calculations show this convergence to be rapid (see Table III). Furthermore, it can be shown that $g_{\pi N}$ will increase monotonically towards convergence in this procedure. This means that only models that for the lowest order in the pion coupling have a value of $g_{\pi N}$ smaller than the experimental value have a chance to converge towards the correct value.

The simplest model discussed in the literature is that in which the pion field is continuous across the bag surface (Théberge, Thomas, and Miller, 1980). For certain combinations of the parameters of this model it can indeed be shown to converge to solutions that have the correct $g_{\pi N}$ (see Table III, in which values for a similar calculation of the axial charge g_A are also included for illustration).

If one excludes the pions from the interior of the bag by an abrupt cutoff in the pion field (the sharp-boundary approximation), one is not able to get correct values for $g_{\pi N}$ and g_A unless one introduces some mechanism to lower $g_{\pi N}$ and g_A. This can be done, for instance, by introducing a color Coulomb-type potential in the bag, as is done in the LAPP (Lab. d'Annecy-le-Vieux de Physique des Particules) bag (Høgaasen, Richard, and Sorba, 1982).

TABLE III. Convergence of iteration in the cloudy bag model, in which interacting pions are allowed inside the bag (Høgaasen and Myhrer, 1983b), with the parameters $B=(125\text{ MeV})^4$, $Z_0=0.6$, and $\alpha_s=3.0$, and in which the pion mass $\mu=140$ MeV is used.

No. of iterations	Three-quark energy 3ω MeV	$g_{\pi N}^2/4\pi$	g_A
0th	992.8	10.744	1.088
1st	840.5	13.983	1.242
2nd	821.7	14.387	1.259
3rd	819.6	14.434	1.261
4th	819.3	14.439	1.262
5th	819.3	14.439	1.262

Within the chiral bag model, the nucleon form factor for coupling to a pion, $g_{\pi N}(q^2)$, has also been calculated. It is

$$g_{\pi N}(q^2)=3j_1(qR)/qR \ . \tag{3.44}$$

At very low q^2, where one can justify this coupling, this (for $R\simeq 0.8$ fm) is in agreement with the calculations in the Regensburg model (scalar confinement potential model; Weise, 1984), which gives

$$g_{\pi N}(q^2)=e^{-q^2/\Lambda^2}, \quad \Lambda\simeq 0.8 \text{ GeV} \ . \tag{3.45}$$

This form factor, or its monopole form $(1+q^2/\Lambda^2)^{-1}\simeq 1-q^2/\Lambda^2$ with $\Lambda\simeq 0.8$ GeV, is too soft compared to $\Lambda\geq 1$ GeV, which is required by both theoretical dispersion considerations (Durso, Jackson, and VerWest, 1977) and an effective meson exchange nucleon-nucleon model's fit to *NN* data (Holinde, 1981). This will be discussed in the next section. However, once the form factors in the effective meson exchange description of *NN* forces become important, they indicate that the effective meson-nucleon description is inadequate. Especially for *NN* scattering in which the short-distance nuclear forces are dominant, one has to recast the description of nucleon-nucleon forces into a quark description (see, for example, Guichon and Miller, 1984), as will be discussed in detail in Sec. V. For large-impact-parameter *NN* scattering (angular momentum $l\geq 2$), one comes close to describing *NN* data without form factors, as was shown earlier by the Stony Brook (Brown and Durso, 1971; Chemtob and Riska, 1971) and the Paris (Vinh Mau *et al.*, 1973) groups. We shall discuss this case and the need for meson-nucleon form factors next. Here we should emphasize once more that we have *calculated* the pion-nucleon coupling from a quark model in which chiral symmetry requirements were crucial. In the next section we shall use the mesons and nucleons as "elementary" particles when treating the long-range *NN* interactions and forget that their coupling has been calculated from a quark model.

For smaller impact parameters (or $l\leq 1$), the meson exchange picture will partially break down. So far a parametrization has been used to describe the short-range *NN* potential (Lacombe *et al.*, 1975, 1980). Others (e.g., Holinde, 1981; Machleidt, Holinde, and Elster, 1987) have used form factors in order to be able to describe *S*- and *P*-wave phase shifts. However, by using a quark model, one can explain the short-range *NN* forces, as will be discussed in Sec. V. This picture of long-distance meson exchanges and a short-range quark description is very much in line with the coupled-channel picture of Lomon (1984) and Simonov (1981, 1984). In this description the meson exchange forces operate only at large distances $r>r_b$ in one channel, and the six quarks of the two nucleons interact at $r<r_b$ in the other channel. The coupling potential going from the nucleon-nucleon channel to the six-quark channel acts only at $r=r_b$. In this

coupled-channel analysis the nucleon and quark wave functions exist in all space without cutoffs. Henley, Kisslinger, and Miller (1983) and Kim and Orlowski (1984a, 1984b) have also separated the NN wave function into two parts, a long-distance part in which mesons are exchanged between nucleons and a short-distance part described by six quarks. However, their splitting of the NN wave function is somewhat unfortunate, as is carefully discussed by Yamauchi and Wakamatsu (1986a, 1986b). In an unrelated development Weber and collaborators (Weber, 1980; Bakker *et al.*, 1982; Beyer and Weber, 1984, 1987; see also Elster and Holinde, 1984) have argued for a short-range quark description, stressing the similarities between the quark exchange amplitudes and the normal meson exchange amplitudes of, for example, Nagels, Rijken, and deSwart (1975).

IV. THE LONG-RANGE NUCLEAR FORCE

A. Introduction to dispersion calculations of *NN* potentials

The bag model and chiral symmetry say that nucleons are quark cores or bags surrounded by pion clouds, as discussed in Sec. III.C.3. For large-impact-parameter nucleon-nucleon (NN) scattering, we expect only the tail of the nucleon clouds to overlap. Therefore pion exchanges dominate the long-range nuclear potential. The pion-nucleon coupling constant is experimentally determined from pion-nucleon (πN) dispersion relations (Hamilton and Woolcock, 1963); this coupling constant has been calculated in the chiral bag model as due to the underlying chiral symmetry (Sec. III.C.3). In this section we shall suppress the quark origin of this coupling constant and effectively consider pions and nucleons as elementary particles. The one-pion exchange potential is well established, and the strong tensor force from this one-pion exchange (OPE) potential has been tested very accurately by the deuteron's asymptotic ($r \to \infty$) D-state/S-state ratio (Ericson and Rosa-Clot, 1985).

For NN distances $r \simeq 2-4$ fm the nucleons' pionic clouds will overlap considerably, and therefore, in this medium range, we expect the two-pion exchange (TPE) forces to become important. In Fig. 10 we have illustrated some contributions to the TPE forces as well as some effective three-pion exchange forces [Figs. 10(h)–10(j)]. The intermediate excitations of the quark core are the nucleon resonances Δ, N^*, Δ^*. [The quark models give the $\pi N\Delta$, πNN^*, and $\pi N\Delta^*$ coupling constants, as well as the masses of these states (see Sec. IV.C).] These diagrams have been calculated in the sharp-resonance approximation (Chemtob, Durso, and Riska, 1972) to generate the TPE force in a model to test the dispersion calculations of the Paris group (Vinh Mau, 1979). The Bonn group has also calculated the N and Δ intermediate excitations diagrams to describe the TPE forces (Holinde, 1981; Machleidt, 1984; Machleidt, Holinde, and Elster, 1987). To this the Bonn group adds a zero-width, scalar-meson (σ') exchange between the nucleons to account for the strong $\pi\pi$ S-wave interaction [illustrated in Fig. 10(g) and similar diagrams with intermediate Δ] which can be thought of as a *very broad* S-wave pion-pion resonance (Durso, Jackson, and VerWest, 1980). Durso and co-workers find that a large part ($\sim\frac{2}{3}$) of the strength of the NN scalar exchange potential is due to this pion-pion correlation (see a more detailed discussion later in this section).

To see how these two-pion exchange forces enter the NN potential (for the moment we treat nucleons as point particles), we refer to Fig. 1, where the NN potential is given by

$$V = V_\pi + V_{2\pi} + V_\omega \ , \tag{4.1}$$

for which the correct definition of $V_{2\pi}$ is illustrated in Fig. 11. The angular momentum structure of V is written as

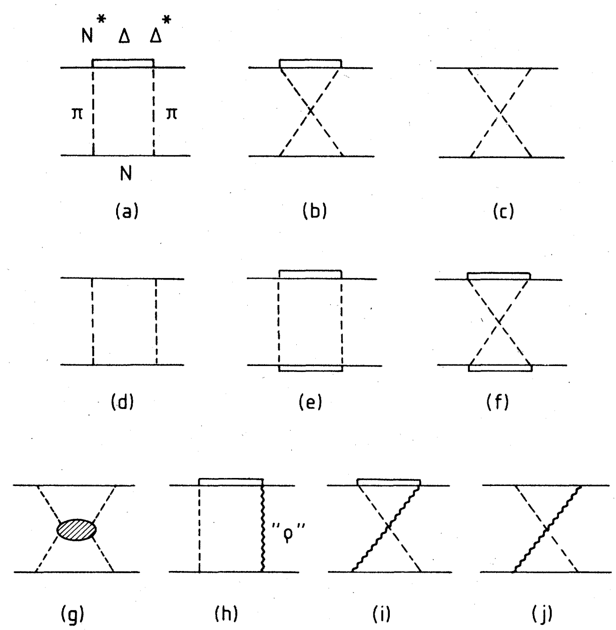


FIG. 10. Several contributions to TPE forces. Diagrams (a)–(g): the nucleon resonances Δ, N^* (Roper and higher), and Δ^* (1530 and higher). Diagram (g) (where intermediate nucleon lines can also be Δ) indicates the very important (Durso, Jackson, and VerWest, 1980) $\pi\pi$ interactions (mainly S- and P-wave interactions at low energies), which have to be projected out of the other diagrams before being added, as explained later in the text. Diagrams (h)–(j): some effective three-pion exchange contributions, whose importance is discussed in the text. From Green and Haapakoski (1974; Haapakoski, 1974) we know that, at short and medium NN ranges, diagrams (a) and (h) partly cancel for the central potential at shorter distances ($r < 1.5$ fm), a result recently confirmed by the Bonn group (Holinde, 1981; Machleidt, 1984). The rho meson is here in quotation marks, since it cannot, strictly speaking, be treated as a stable particle exchange, as discussed in the text.

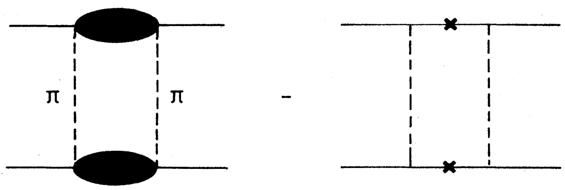


FIG. 11. The two-pion exchange diagram of Fig. 1(b) minus the once-iterated OPE diagram, in which the crosses on the nucleon propagators indicate that this is not a Feynman diagram but an illustration of the second-order term $V_\pi G V_\pi$ (see text).

$$V = V_0 + V_{LS}\mathbf{L}\cdot\mathbf{S} + V_T S_{12} + V_Q Q \ . \tag{4.2}$$

Here, for example,

$$V_0 = V_c^0 + V_{ss}^0 \sigma_1\cdot\sigma_2 + V_\tau^1 \vec{\tau}_1\cdot\vec{\tau}_2 + V_{GT}^1 \vec{\tau}_1\cdot\vec{\tau}_2 \sigma_1\cdot\sigma_2 \tag{4.3}$$

is the central potential and V_{LS}, V_T, and V_Q (the quadratic spin-orbit potential; Brown and Jackson, 1976) can also be written as the sum of an isoscalar plus an isovector component. The OPE potential V_π can be written schematically as

$$V_\pi = (V_{GT}^1 \sigma_1\cdot\sigma_2 + V_T^1 S_{12})\vec{\tau}_1\cdot\vec{\tau}_2 \ . \tag{4.4}$$

The strong nuclear tensor force V_T^1 of the OPE potential is partly canceled at shorter distances by the isovector 2π exchange $V_{2\pi}^1$, which contains the ρ-meson exchange.

The isoscalar 2π exchange $V_{2\pi}^0$ gives the medium-range attraction of V_0, but V_0 is dominated by the strong repulsion at shorter distances due to ω exchange V_ω (see Figs. 1 and 2). The isoscalar 2π exchange and the ω exchange also contribute to the spin-orbit force V_{LS}. These are the major parts of the long-range NN potential, and at low energy both the $\mathbf{L}\cdot\mathbf{S}$ and the tensor force will contribute to the polarization phenomena in NN and proton-nucleus scattering. Nucleon-nucleon scattering experiments with polarized nucleons can to some extent isolate the different contributions to the long-range NN potential.

The complete two-pion exchange contribution to the nucleon-nucleon force has been calculated by the Paris (Vinh Mau, 1979) and the Stony Brook (Brown and Jackson, 1976) groups. They used dispersion relation techniques that relate πN and $\pi\pi$ scattering data to the (NN) two-pion exchange potential. This can quickly be seen from Fig. 1(b) or Fig. 11, where the upper black area can be seen as (i) reading from left to right, $\pi N \rightarrow$black area$\rightarrow \pi N$, or (ii) reading from the top and down, which is the NN momentum-transfer channel or t channel, $N\bar{N} \rightarrow$black area$\rightarrow \pi\pi$. In this latter case, since momentum transfer in nucleon-nucleon scattering is small, neither the pions nor the $N\bar{N}$ will be physical particles, which means we have to know this reaction amplitude (black area) for unphysical energies. This dispersion calculation should give basically a parameter-free meson exchange nuclear potential. We shall briefly present the calculation of this potential to show some of its weaker points and to focus on open questions. For a complete description see Brown and Jackson (1976) or Vinh Mau (1979).

To calculate the TPE potential $V_{2\pi}$ we consider the process illustrated in Fig. 1(b) or better in Fig. 11. The dispersion relation calculations consider the nucleon-nucleon scattering amplitude M, which theoretically is calculated by solving an equation of motion with the potential V of Eq. (4.1). As we shall show (Sec. IV.B), the dispersion approach considers all possible processes that can contribute to M. Apart from the one-pion exchange pole (V_π) in M, it also has two-pion, three-pion, etc., exchanges, as will be discussed. Let us designate as M' the two-pion exchange part of M. Then to avoid double counting when we iterate V_π the TPE potential has to be defined as

$$V_{2\pi} = M' - V_\pi G V_\pi \ .$$

This equation is illustrated in Fig. 11; here G is the Green's function of the equation of motion (Schrödinger or Blankenbecler-Sugar or another) used to iterate the OPE potential. When we discuss the NN amplitude M below we shall concentrate on the two-pion exchange contribution M' [after Eq. (4.14)].

We can always write a dispersion relation for the NN amplitude M as

$$M(s,t) = \frac{1}{\pi}\int_{4\mu^2}^{\infty} \frac{\mathrm{Im}M(s,t')dt'}{t'-t} + \text{pole terms} \ , \tag{4.5}$$

where s is the square of the nucleon-nucleon center-of-mass energy and t is the four-momentum transfer squared. If the imaginary part of the amplitude $M(s,t)$ is dominated by a sharp resonance at some point, one has

$$\mathrm{Im}M(s,t') \simeq \rho(t_0)\delta(t'-t_0) \ . \tag{4.6}$$

If we use this imaginary amplitude $M(s,t)$ also for $t<0$ describing NN scattering where $t^2 = -\Delta^2 < 0$ is the square of the momentum transfer, then we get from Eq. (4.5)

$$M(s,t) \simeq \frac{1}{\pi}\rho(t_0)/(t_0+\Delta^2)$$
$$\sim \int \frac{e^{-\sqrt{t_0}r}}{r} e^{i\Delta\cdot r} d^3 r \ .$$

This means that $M(s,t)$ is just the Fourier transform of a Yukawa potential corresponding to the exchange of the particle of mass t_0. We can therefore think of the integral in Eq. (4.5) as being a sum of exchanges of heavier and heavier mesons of mass $\sqrt{t'} \geq 2\mu$, and the corresponding TPE potential can be written as

$$V_{2\pi}(r) = \int \rho(t') \frac{e^{-\sqrt{t'}r}}{r} dt' \ , \tag{4.7}$$

where $\rho(t')$ is a spectral function giving the strength of the exchange of the $\pi\pi$ system of mass equal to $\sqrt{t'}$ (Brown, 1979). However, $\mathrm{Im}M(s,t')$ is not proportional to the "coupling constant," since part of the exchange in Eq. (4.5) contains, for example, the iterated one-pion exchange potential, as discussed above. To clarify the physics input to the technically involved calculations, we shall present some details next. We want to derive a *NN* potential V to describe *NN* scattering, and this V is then to be used in nuclear calculations. This potential, when used in a wave equation [Schrödinger equation or Blankenbecler-Sugar (BS) equation], will give the *NN* scattering matrix, which is to be compared with *NN* scattering data. The simplest and longest-range ($=\mu^{-1}$) potential is the Yukawa potential V_π. The potential of shorter range [$\leq(2\mu)^{-1}$] is the TPE potential, ideally to be calculated directly from the diagrams of Fig. 10 in a microscopic model, as will be outlined in Sec. IV.C. However, in the dispersion calculations of two-pion exchange, the diagram in Fig. 1(b) is considered. In both approaches we have to subtract the OPE potential [Fig. 1(a)] once iterated, which instead of the two shaded boxes in Fig. 1(b) has a two-nucleon propagator, Fig. 11, where the intermediate two-nucleon propagator G is given by the wave equation used (Schrödinger or BS equation). The resulting TPE potential can then only be used in the corresponding wave equations. In order to calculate the TPE potential we first discuss the structure of the *NN* scattering matrix $M(s,t)$, Eq. (4.5).

B. Calculation of the two-pion exchange amplitude

1. Preliminaries

The on-shell elastic *NN* scattering amplitude (for distinguishable nucleons) can be written as (Amati, Leader, and Vitale, 1960, 1963)

$$M(s,t,u)=\sum_{j=1}^{5}[3h_j^+(s,t,u)+2\vec{\tau}_a\cdot\vec{\tau}_b h_j^-(s,t,u)]P(j) . \tag{4.8}$$

Here the factors 3 and 2 are isospin factors, and the relativistic invariant kinematical Mandelstam variables s,t,u are defined as

$$\begin{aligned} s&=-(p_a+p_b)^2, \quad t=-(p_a'-p_a)^2 ,\\ u&=-(p_a'-p_b)^2 , \end{aligned} \tag{4.9}$$

where p_a (p_a') and p_b (p_b') are initial (final) four-momenta of nucleons a and b, respectively, as indicated in Fig. 12. The isospin operators for nucleons a and b are $\vec{\tau}_a$ and $\vec{\tau}_b$, and the five Lorentz invariants $P(j)$ in Eq. (4.8) are defined through the Dirac γ matrices as

$$\begin{aligned} &P(1)=1^a1^b ,\\ &P(2)=i\tfrac{1}{2}[\gamma^a(p_b+p_b')]1^b+i[\gamma^b(p_a+p_a')]1^a ,\\ &P(3)=i[\gamma^a(p_b+p_b')]i[\gamma^b(p_a+p_a')] ,\\ &P(4)=\gamma^a\gamma^b, \quad P(5)=\gamma_5^a\gamma_5^b . \end{aligned} \tag{4.10}$$

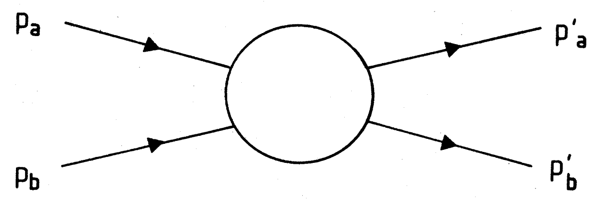


FIG. 12. Kinematical variables for *NN* elastic scattering.

The scalar amplitudes $h_j^\pm$ in Eq. (4.8) satisfy the dispersion relations (Amati, Leader, and Vitale, 1960)

$$h_j^\pm(s,t,u)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}dt'\frac{\rho_j^\pm(s,t')\pm(-1)^j\rho_j^\pm(u,t')}{t'-t}+\text{poles} . \tag{4.11}$$

Here μ is the pion mass, and the spectral functions $\rho_j^\pm$ contain any number ≥ 2 of meson exchanges between two nucleons. The relevant pole contributions in Eq. (4.11) are (i) the isovector and isoscalar one-pion exchange (OPE) contributions, respectively,

$$\begin{aligned} h_j^-(s,t,u)&=-\tfrac{1}{2}g_{\pi N}^2(\mu^2-t)^{-1}\delta_{j5} ,\\ h_j^+(s,t,u)&=0 , \end{aligned} \tag{4.12}$$

where $g_{\pi N}^2/4\pi=14.5$ is the πN coupling constant, and (ii) the isoscalar and isovector ω-meson exchange contributions, respectively,

$$\begin{aligned} h_j^+(s,t,u)&=-g_\omega^2/3(m_\omega^2-t)\delta_{j4} ,\\ h_j^-(s,t,u)&=0 , \end{aligned} \tag{4.13}$$

where we have used the knowledge that the ωN tensor coupling is almost zero. This means $\kappa_s\simeq 0$ in the effective interaction Lagrangian

$$L_{\omega NN}=ig_\omega\bar{N}\left[(1+\kappa_s)\gamma_\mu+i\frac{\kappa_s}{M}p_\mu\right]\Phi^\mu N , \tag{4.14}$$

where N ($\bar{N}$) is the nucleon (adjoint) field, M the nucleon mass, and Φ^μ the ω field. We can write the ω as a pole contribution because it has a relatively long lifetime ($\Gamma_\omega\simeq 10$ MeV), in contrast to the ρ meson of the same mass ($\Gamma_\rho\sim 140$ MeV). This pole term is treated as a separate contribution to the *NN* potential V_ω [see Eq. (4.2) and Fig. 1].

We now consider only the TPE contribution to the amplitudes $h_j^\pm$ in Eq. (4.11). Then the TPE potential is the amplitude $h_j^\pm$ minus $r_j^\pm$ (4.16), the iterated V_{OPE} amplitude [minus $r_j^\pm$ for reasons of double counting already given (see Fig. 11)]. We define the TPE potential through the spectral function

$$v_j^\pm(\mathbf{p}',\mathbf{p};s)=h_j^\pm(s,t,u)-r_j^\pm(s,t) . \tag{4.15}$$

This spectral function ν gives the strength or "coupling constant" of the TPE potential [see discussion below Eq. (4.7)]. Chemtob, Durso, and Riska (1972) have shown that the iterated OPE amplitude $r_j^{\pm}$ can also be written on a dispersion relation basis,

$$r_j^{\pm}(s,t)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}dt'\frac{q_j^{\pm}(s,t')}{t'-t} \quad . \tag{4.16}$$

The spectral functions $q_j^{\pm}$ can be found in Chemtob, Durso, and Riska (1972) or in Brown and Jackson (1976). We have to calculate the spectral function $\rho_j^{\pm}$, and we have required that it comes from two-pion exchange between nucleons only. Then $\rho_j^{\pm}(s,t')$ can be written in terms of the $N\bar{N}\rightarrow\pi\pi$ partial-wave reaction amplitudes $\lambda_J^{\pm}$, where J is the $\pi\pi$ angular momentum (i.e., we consider only the lower or upper half of each of the diagrams in Fig. 10, so it is $|\lambda_j^{\pm}|^2$ that contributes to $\rho_j^{\pm}$). These latter amplitudes $\lambda_j^{\pm}$ we have to know from the $\pi\pi$ threshold $(t'\geq 4\mu^2)$ for positive values of t', as indicated by the integral of Eqs. (4.11) and (4.16). Furthermore, for the nuclear potential, it is the values of t' below the physical $N\bar{N}$ threshold that are relevant (Durso, Jackson, and VerWest, 1980) [the higher the energy $\sqrt{t'}$ in the $N\bar{N}\rightarrow\pi\pi$ reaction, the shorter the range of the corresponding nuclear force, as argued by Eqs. (4.6) and (4.7)]. This means that the dispersion integrals above are cut off at some high $t'<4M^2$. This is reasonable because, at some point, due to the physical extension of the quark core of the nucleons and mesons as discussed in Sec. III, this dispersion picture will no longer be valid. Therefore it is very reasonable to leave out very-short-range meson exchanges (high t'), which will necessarily take place mainly within two overlapping nucleons. This means that for high t' we should replace this picture with a more microscopic model (e.g., a quark model) for NN force, which will be discussed in detail in Sec. V.

2. The calculation

To calculate $\rho_j^{\pm}$ we first observe that the $N\bar{N}\rightarrow\pi\pi$ amplitude $\lambda_J^{\pm}$ is related to the elastic πN scattering amplitude by crossing symmetry. However, to extrapolate from the physical πN scattering amplitude with $t'\leq 0$ to the $N\bar{N}\rightarrow\pi\pi$ amplitude (for $t'\geq 4\mu^2$) requires an analytic continuation in both the invariant energy and the πN invariant momentum transfer squared (t') variables. The extrapolation in πN square energy s' can be performed via a fixed-t' dispersion relation for the πN amplitude (Chew *et al.*, 1957), in which the $N\bar{N}\rightarrow\pi\pi$ amplitude is given as a sum of the nucleon pole term and a fixed-t' dispersion integral over the imaginary part of the πN amplitude. Then, this $N\bar{N}\rightarrow\pi\pi$ amplitude has to be extrapolated from negative t' values to positive values of $t'\geq 4\mu^2$, assuming the imaginary part of the πN amplitudes to be analytic (Frazer and Fulco, 1960). An additional point in calculating $\rho_j^{\pm}$ is that for low $\pi\pi$ energies (small $t'\gtrsim 4\mu^2$) S- and P-wave $\pi\pi$ scattering are important. Amati, Leader, and Vitale (1960, 1963) suggested that the $l=0$ and 1 partial waves for $N\bar{N}\rightarrow\pi\pi$ should be treated separately. In the first calculations (Amati, Leader, and Vitale, 1960, 1963; Cottingham and Vinh Mau, 1963; Durso, 1966; Kapadia, 1967) this separation was attempted with sparse $\pi\pi$ data. Partovi and Lomon (1970) assumed the "nucleon pole" graph [Figs. 10(c) and 10(d)] dominance to calculate $\rho_j^{\pm}$, while others (Brown and Durso, 1971; Chemtob and Riska, 1971; Chemtob, Durso, and Riska, 1972) calculated $\rho_j^{\pm}$ by including the higher πN resonances Δ and N^*, which were treated in the sharp-resonance approximation. The Paris group (Cottingham *et al.*, 1973) used the complete measured πN amplitudes (πN phase shifts) as well as S- and P-wave $\pi\pi$ scattering phase shifts as input in their dispersion calculations.

To summarize, the important inputs in the 2π exchange potentials are the $N\bar{N}\rightarrow\pi\pi$ S- and P-wave spectral functions. These have been calculated from $\pi N\rightarrow\pi N$ and $\pi\pi$ data not only by the Paris group (Cottingham *et al.*, 1973), who constrained their dispersion relations calculations with known high-energy Regge behavior of the amplitudes, but also by Nielsen and Oades (1972) [see the review of Höhler (1983)]. Brown and Durso (1971) used instead soft-pion theorems to constrain the dispersion calculations of the $N\bar{N}\rightarrow\pi\pi$ spectral functions. This is reasonable, since one needs these amplitudes for low-energy pions ($t\gtrsim 4\mu^2$ or low momentum transfer in Fig. 10). These latter spectral functions were different from those of the Paris group (Cottingham *et al.*, 1973), resulting in different NN phase shifts (compare Brown and Durso, 1971; Chemtob and Riska, 1971; and Vinh Mau *et al.*, 1973). Later Jackson, Riska, and VerWest (1975) discussed what changes had to be made in the spectral functions to fit NN data. Durso, Jackson, and VerWest (1980) then calculated the spectral functions using a model for $\pi\pi$ scattering including a discussion of form factors at the vertices in Fig. 10. Their model for $2\pi\rightarrow N\bar{N}$ isovector amplitude included the nucleon pole graph plus a Breit-Wigner ρ-meson exchange of $\Gamma_\rho\sim 125$ MeV. They used a parametrization for the isoscalar 2π spectral function of Jackson, Riska, and VerWest (1975) shown in Fig. 13, necessary to reproduce the NN phase shifts, and they stressed the importance of the $\pi\pi$ rescattering for understanding the dispersion-calculated spectral functions $v_J^{\pm}$ of Eq. (4.15).

The Achilles heel of these approaches is the use of the fixed-t dispersion relations and the analytic expressions used for the extrapolation in t' from the $\pi N\rightarrow\pi N$ amplitudes to the $N\bar{N}\rightarrow\pi\pi$ amplitudes, together with the subtraction of the large iterated pion exchange amplitudes to finally find $v_J^{\pm}$ of Eq. (4.15). The model calculations of the Stony Brook/Copenhagen group (Brown and Durso, 1971; Chemtob and Riska, 1971; Chemtob, Durso, and Riska, 1972; Durso, Jackson, and VerWest, 1980) give some physical insight into the calculations of the Paris group (Cottingham *et al.*, 1973; Lacombe *et al.*, 1975, 1980). The latter are the most complete calculations, but a better understanding of the extrapolations made in

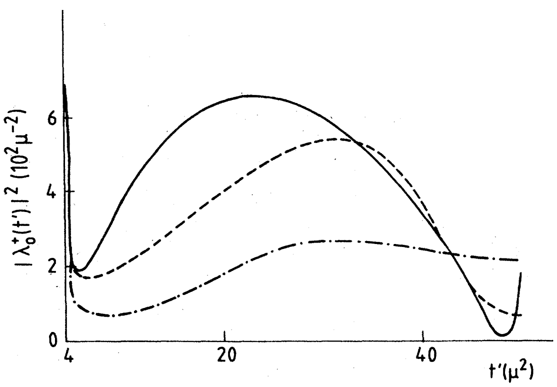


FIG. 13. Comparison of the $N\bar{N}\rightarrow\pi\pi$ S-wave amplitude spectral function $|\lambda_0^+|$, where $\lambda_0^+(t)=4\pi/(M^2-t/4)f_+^0(t)$ and where f_+^0 is the $N\bar{N}\rightarrow\pi\pi$ S-wave helicity amplitude, the argument of which equals the S-wave $\pi\pi$ phase shift for $t\leq 16\mu^2$. Solid curve, Nielsen and Oades (1972) analysis; dot-dashed curve, Brown and Durso (1971) soft-pion constrained calculation; dashed curve, model used in Jackson, Riska, and VerWest's (1975) calculation, from which the figure is taken.

these calculations is still lacking. This means we still do not have a complete model understanding of the Paris group's results.

3. A model understanding?

The dispersion calculations outlined above are very involved and include extrapolations of amplitudes to the unphysical regions which carry some uncertainties. In addition the iterated OPE potential [see Fig. 11 or Eq. (4.15)] is comparable in magnitude to the calculated M' term (the NN amplitude with only $\pi\pi$ exchange). This means that each of the terms subtracted in Eq. (4.15) has to be calculated very precisely in order to give a reliable TPE potential. It is therefore desirable to calculate $V_{2\pi}$ directly, using a model, in order to have an independent check of the dispersion theory (which, in principle, is parameter-free) calculation. The Bonn group (see the reviews by Holinde, 1981; Machleidt, 1984; Machleidt, Holinde, and Elster, 1987) has used an effective meson-baryon theory, illustrated in Fig. 10, to calculate the two-pion exchange potential directly (no extrapolations in energy and momentum variables, which are necessary in a dispersion calculation). They add a fictitious (zero-width) scalar-meson exchange σ' to simulate the broad $\pi\pi$ S-wave interaction of Durso, Jackson, and VerWest (1980) mentioned earlier. The parameters of this σ', its mass $m_{\sigma'}$, and coupling constant $g_{\sigma' N}$, are determined so that this σ', together with the "box" diagrams of $V_{2\pi}$ [Figs. 10(a)–10(f)] plus V_π and V_ρ, reproduces the higher NN phase shifts when σ' is replaced by the broad $\pi\pi$ mass distribution of Durso, Jackson, and VerWest (1980) illustrated by Fig. 10(g). [The isovector $V_{2\pi}$ is dominated by the ρ-meson exchange V_ρ where the width of ρ is included (Durso, Jackson, and VerWest, 1980).] However, this broad $\pi\pi$ "resonance" accounts for $\frac{2}{3}$ of the strength of the isoscalar $V_{2\pi}$ interaction ($\frac{1}{3}$ comes from the "box" diagrams), according to Durso, Jackson, and VerWest (1980). This scalar exchange NN force also has a mass that is distributed over several hundred MeV by a spectral function $\rho(t)$ which becomes nonzero for $t\simeq(2\mu)^2$, which means a long-distance tail [see Eqs. (4.6) and (4.7)]. To replace this by $\rho(t)=\rho(t_0)\delta(t-t_0)$, where $t_0=m_{\sigma'}^2$, is a very rough approximation. The imperfect agreement of the higher NN partial waves between these two treatments of the isoscalar $V_{2\pi}$ might be due to the loss of the long-range tail of the broad $\pi\pi$ "resonance" [see the discussion in Sec. 5 and especially Fig. 7 in Machleidt, Holinde, and Elster (1987)]. It is clear that introducing σ' is convenient, but this aspect of the Bonn potential has to be improved. Despite these critical remarks it is necessary to do a model calculation like the one of the Bonn group to test the analytic continuations of the dispersion calculations and to build up a better physical understanding of the main physical processes of the two-pion exchange potential.

The Bonn group also introduces form factors at each meson-nucleon vertex with a cutoff mass $\Lambda\geq 1$ GeV. This latter is consistent with dispersion calculations of the meson-nucleon (πNN) vertex, but it can modify the OPE potential for $r\approx 1.5$ fm (see Holinde, 1981, for a discussion). However, if three-meson exchanges between nucleons are included [for examples, see Figs. 10(h)–10(i), and similar graphs with "σ" instead of "ρ"], the Paris group (Vinh Mau, 1980) has found that these three-pion exchange contributions together with the πNN form factors are significant only for internucleon distances $r\leq 0.8$ fm [see also a remark by Holinde (1981), p. 135], a point which warrants further investigation (see below). However, this is consistent with the finding of meson exchange current effects measured in deuteron breakup, $ed\rightarrow e'pn$, where the energy of the pn pair is less than 3 MeV but the momentum transfer from the electron is large ($q^2\simeq 12$ fm^{-2}). A minimal calculation prescribed by chiral symmetry involving pions and nucleons only, with nucleon and pion electromagnetic form factors but no meson-nucleon vertex form factors, predicts the cross section for this process (for a review see Riska, 1979, or Brown, 1982a), which was accurately measured by a Saclay group (Bernheim *et al.*, 1981). This nice experimental confirmation of the presence of exchange currents says that one has a high number of cancellations between heavier meson exchanges and meson-nucleon form factors and Δ exchange current contributions (Riska, 1979). [Recently Riska (1985) and Gross and Riska (1987) have argued that instead of considering meson exchange diagrams, *ad hoc* meson-baryon form factors, and heavier meson exchanges, one should (carefully) link the meson exchange current operators to the NN meson exchange potential. Whether this will give some insight into the meson-baryon form factors is an open question.] We know that the complete Paris (OPE and TPE) potential plus the ω exchange potential with $g_\omega^2/4\pi \simeq 4.5$, a value that is expected from SU(3), does reproduce many NN partial waves with $l \geq 2$ (Vinh Mau *et al.*, 1973), and later versions (Lacombe *et al.*, 1975, 1980) with a better calculation of $V_{2\pi}$ do better. For all meson exchange potentials the effective strength $g_\omega^2/4\pi$ of the V_ω [Fig. 1(c)] is of the order 10–12, whereas the elementary $g_\omega^2/4\pi$ coupling should be 4.5 (Brown and Jackson, 1976). This discrepancy can be understood, according to Durso, Saarela, Brown, and Jackson (1977), in an effective meson-nucleon theory if the isoscalar exchange of processes like those illustrated in Figs. 10(h) and 10(i) are considered. These are then ω-like exchanges, the effective coupling from such three-pion exchange processes giving an effective ω-like coupling of the order 5–8 (Durso, Saarela, Brown, and Jackson, 1977), which when added to the normal ω coupling of 4.5 gives the large $g_\omega^2/4\pi$ of meson exchange (or boson exchange) models required to "fit" NN phase shifts. However, here we have to be careful, since we know from Haapakoski (1974) that diagrams like 10(a) and 10(h) partly cancel each other for short NN distances. In fact, the Bonn group (Machleidt, Holinde, and Elster, 1987) confirms this and goes further, stating explicitly that diagrams of the type shown in Figs. 10(h)–10(j) (with none, one, or two intermediate Δ states) counterbalance the corresponding isoscalar 2π exchange contributions. In other words, the Bonn group says that the effective $\pi\rho$ exchange does not explain the "enhanced" ω coupling necessary in NN meson exchange potentials as argued by Durso, Saarela, Brown, and Jackson (1977). This point obviously necessitates further investigations. If the finding of the Paris group (Vinh Mau, 1980), is correct that meson-baryon form factors together with other three-meson exchanges (e.g., $\pi\rho$, π"σ") are important only for NN distances $r \lesssim 0.8$ fm, then one needs only some cutoff prescription at short distances $r_0 < 0.8$ fm to describe $l \leq 1$ NN phase shifts. One possible reason for this "cancellation" is that a dispersion relation description of the πNN form factor in an effective meson-nucleon theory says that (1) the pion couples directly to the point nucleon via $g_{\pi N}$, or (2) the pion couples to similar intermediate $\pi\rho$ or π"σ" states (or "heavier" states) where these virtual particles couple to the nucleon. In fact, dispersion considerations (e.g., Durso, Jackson, and VerWest, 1977) say that the πNN form factor should have a cutoff mass $\Lambda \gtrsim 1$ GeV, as stated below Eq. (3.45). One short-distance parametrization, which describes all NN partial waves, is proposed by the Paris group (Lacombe *et al.*, 1975, 1980). However, this does not help us to understand the physics involved. We expect that at short distances $r < 1$ fm the inner structure of the nucleons plays an important role. It is therefore necessary to repeat the Bonn calculation with a chiral bag model, using calculations in which the pionic cloud surrounds the quark core of the nucleons. This model, as we argued in Sec. III and above, can describe correctly the *large*-impact-parameter NN scattering and give a theoretical basis for the meson exchange potential or the older boson exchange potentials (see for examples, Nagels, Rijken, and de Swart, 1975). In addition it will have some "effective" NN form factors built into the model.

The chiral bag model has a natural cutoff, and one can calculate directly the $N\bar{N} \to \pi\pi$ S- and P-wave amplitudes, including $\pi\pi$ interactions for low pion energies, to compare with the amplitudes of Jackson, Riska, and VerWest (1975), Durso, Jackson, and VerWest (1980), and the similar Paris amplitudes. With this model one can also examine the strong cancellations between the large Δ and N^* diagrams in Fig. 10 found by Chemtob, Durso, and Riska (1972). However, these latter diagrams might not be too important if Δ and N^* are not treated in the sharp-resonance approximation (Durso, Jackson, and VerWest, 1980). The chiral quark model can be used to examine these diagrams, since it describes well the baryon ground-state mass spectrum, Fig. 14(a), the excit-

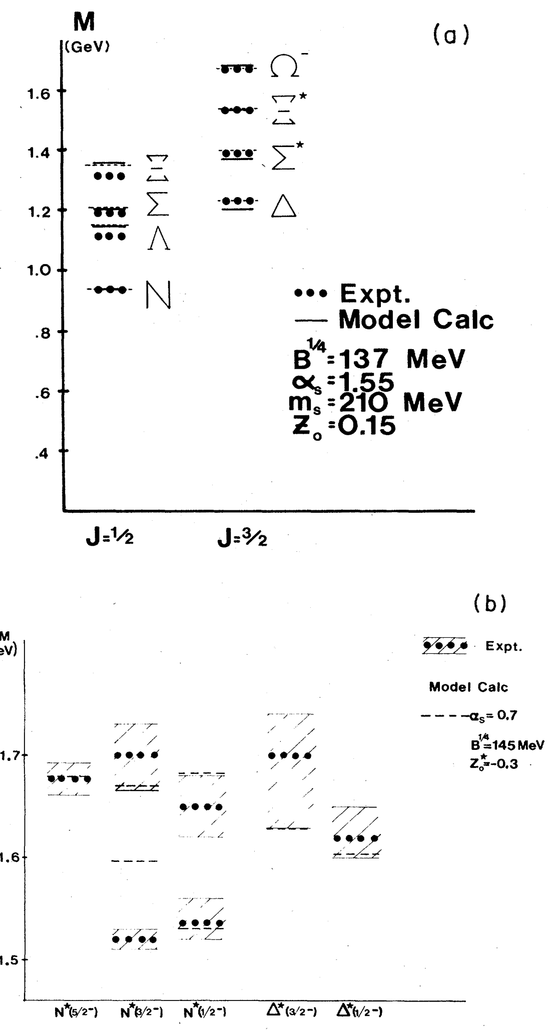


FIG. 14. Mass spectra for baryons. (a) The octet mass spectrum (Myhrer, Brown, and Xu, 1981); (b) the N^*, Δ^* mass spectrum (Myhrer and Wroldsen, 1984a).

ed mass spectrum, Fig. 14(b), and the symmetry mixing angles of the wave functions seen in the decays of the N^* and Δ^* states (Myhrer and Wroldsen, 1984a, 1984b).

In addition, as shown by Théberge, Thomas, and Miller (1980) and others (Israilov and Musakhanov, 1981; Rinat, 1982; Suzuki, Nogami, and Ohtsuka, 1983; Jennings and Maxwell, 1984; Jennings, Veit, and Thomas, 1984; Kälbermann and Eisenberg, 1984; McLeod and Ernst, 1984; Cooper and Jennings, 1986; Veit, Jennings, and Thomas, 1986), this model can describe the pion-nucleon phase shifts and has the correct low-energy properties to describe the soft-pion πN scattering length (Adler and Dashen, 1968; Szymacha and Tatur, 1981; Thomas, 1981). [It should be remarked that an effective meson-nucleon theory can also describe the meson-nucleon scattering; see, for example, Büttgen, Holinde, and Speth (1985).] Furthermore, this chiral model contains the correct low-energy $\pi\pi$ scattering in its nonlinear Lagrangian (Szymacha and Tatur, 1981), and it describes the decay of the rho meson ($\rho \rightarrow \pi\pi$) (Vento, 1980; Miller and Singer, 1983; Maxwell and Jennings, 1985) as well as ρ-meson coupling to nucleons (Oset, 1984). Since this chiral model reproduces the input to the dispersion calculations, it could provide us with a "model extrapolation" as well as a test of the dispersion calculations of $V_{2\pi}$ involving subtraction of large terms. It can also be used to test the cancellation (Vinh Mau, 1980) of πNN form factors and multiple-pion exchanges at larger distances. In the following section we shall use a primitive version of this nucleon structure model and explain in simple physical terms why the nucleon's quark structure might produce the necessary repulsion to explain the S-wave NN phase shifts, which as stated are fitted by *ad hoc* form factors or short-distance parametrizations in the meson exchange models.

V. THE SHORT-RANGE NUCLEAR FORCE

A. Introduction

As we have learned in the previous sections, the mesonic clouds of the nucleons dominate the NN scattering for large impact parameters (or $l \geq 2$), and the quark cores of the nucleons only contribute at most some spatial extensions (form factors) to the pion-baryon couplings. For $l \geq 2$ one needs only the OPE and TPE potentials plus ω exchange with the correct SU(3) coupling constant to describe reasonably well most of the NN scattering phase shifts, as shown by the Paris (Vinh Mau *et al.*, 1973) and Stony Brook (Brown and Durso, 1971; Chemtob and Riska, 1971) groups. However, the short-distance forces between nucleons, which are probed for low (or zero) impact parameters (S- and P-wave scattering), have only been parametrized in the Paris potential (Lacombe *et al.*, 1975). The Bonn group (Machleidt, Holinde, and Elster, 1987) has used meson-nucleon form-factor parametrizations to fit the same low phase shifts. A physical understanding of this can come only from the structure of the nucleons themselves, a topic we shall now discuss. We shall here first take the extreme position and say that a nucleon is made only of three quarks (neglecting chiral symmetry and consequently the pionic cloud), meaning we will not be able to reproduce NN phase shifts, since we neglect the long-range forces. However, this will allow us to show how the quark structure influences the NN S-wave scattering. Then we shall discuss how these results are modified when we allow for pion exchanges, i.e., pionic cloud effects (OPE plus TPE) or possible extensions of the Fock space in the quark description to include $qqqq\bar{q}$, etc., components in the N and Δ wave functions (Fujiwara and Hecht, 1986a, 1986b, 1986c, 1987).

We will now discuss the recent work of Skyrmion-Skyrmion interactions and their possible short-range repulsion (Jackson, Jackson, and Pasquier, 1985; Lacombe *et al.*, 1985), because this approach is still limited to finding a reasonable NN potential, and one is not ready to calculate NN S- and P-wave phase shifts and compare to data. Second the nonlinear equations of the Skyrmions make the calculations very hard and, unlike the quark model (see later), these equations do not point to some simple quantitative physical understanding of the results. The long-range attraction from TPE, which should be included in the Skyrmion picture, is best treated by corrections to the zeroth-order calculation, according to arguments presented by Zahed and Brown (1986). For some recent work on this see Eisenberg *et al.* (1986), Lacombe *et al.* (1986), or Nyman and Riska (1986), and a review of the Skyrmions by Zahed and Brown (1986). We also shall not discuss the work of Schuh *et al.* (1986), who treat NN interaction in soliton models, nor the possible formation of six-quark states or dibaryons, which has recently been reviewed by Locher, Sainio, and Švarc (1986), nor the six-quark effects discussed by Kisslinger (1982) or Henley, Kisslinger, and Miller (1983), already mentioned at the end of Sec. III. In short, we concentrate on descriptions that are reasonably successful in describing measured NN phase shifts.

Our main discussion below will be based on the extensive calculations of Oka and Yazaki (1980, 1981), Harvey (1981a), and Faessler *et al.* (1982, 1983). The reader is referred to the recent review by Oka and Yazaki (1984) for complete technical details and references. Here we shall only make a few observations relevant to our considerations, in order to gain a physical understanding of the results of these complicated calculations. These six-quark calculations use the technique of generator coordinate methods (see, for example, Wong, 1975) or resonating group methods to solve the problem of, say, NN S-wave scattering as the scattering of two three-quark clusters where only the quark-quark forces are given. These techniques have been used (Wheeler, 1937; Hill and Wheeler, 1953; Wildermuth and Kanellopoulos, 1958, 1959; Shimodaya, Tamagaki, and Tanaka, 1962; Kaminura, 1977; Wildermuth and Tang, 1977) in nuclear physics to understand, for example, the S-wave phase shift of $\alpha\alpha$ scattering. There one calculates the elastic scattering of two four-nucleon clusters where input is a phenomenological nucleon-nucleon potential (Tamagaki and Tanaka, 1965; Okai and Park, 1966). The calculated $\alpha\alpha$ scattering phase shifts describe the "measured" phase shifts very well. The results are not very sensitive to which reasonable NN potentials are used in the calculation (Spitz, Klar, and Schmid, 1985). It was earlier speculated (Tamagaki, 1967) that similar effects occur in nucleon-nucleon low partial-wave scattering.

The first attempts to understand the NN repulsion from an underlying quark model using the adiabatic approximation were by Liberman (1977), DeTar (1978, 1979), Harvey (1981b), and others (Toki, 1980; Babutsidze *et al.*, 1981). Neudatchin and collaborators also examined the question of short-range NN repulsion in bag models (Neudatchin, Obukhovsky, Kukulin, and Golonova, 1975; Neudatchin, Smirnov, and Tamagaki, 1977; Obukhovsky, Neudatchin, Smirnov, and Tchuvil'sky, 1979). Later Harvey and co-workers (Harvey, 1981c; Harvey and LeTourneux, 1984; Harvey, LeTourneux, and Lorazo, 1984), Faessler and co-workers (Faessler, Fernandez, Lübeck, and Shimizu, 1982, 1983; Faessler and Fernandez, 1983; Bräuer, Faessler, Fernandez, and Shimizu, 1985; Zhang, Bräuer, Faessler, and Shimizu, 1985a, 1985b), Oka and Yazaki (1980, 1984), Ohta, Oka, Arima, and Yazaki (1982), and others (Warke and Shanker, 1979, 1980, Cvetič, Golli, Mankoc-Borstnik, and Rosina, 1980, 1981, 1983; Ribero, 1980; Storm and Watt, 1983; Suzuki and Hecht, 1983; Burger and Hofmann, 1984; Wakamatsu, Yamamoto, and Yamauchi, 1984; Suzuki, 1985; Yamauchi, Yamamoto, and Wakamatsu, 1985) refined these early calculations. [These calculations were also applied to the deuteron (Williams *et al.*, 1982; Takeuchi and Yazaki, 1985; Takeuchi, Shimizu, and Yazaki, 1986).] The NN phase shifts calculated by these groups strongly indicate that a large part of the observed NN S-wave repulsion originates from the quark substructure of the nucleons. A similar mechanism appears in the $\alpha\alpha$ repulsion, which originates from the nucleon substructure of the α particle. One dominant effect in both cases is the requirement of antisymmetric wave functions, for the six quarks in the NN case (see the "flip-flop" model discussion of Horowitz, Moniz, and Negele, 1985, and Lenz *et al.*, 1986) and for the eight nucleons in the $\alpha\alpha$ case. Here we shall give some simple physics arguments why the technically complex resonating group or generator coordinate calculations are generating the observed NN repulsion. In the following we draw heavily on the concepts discussed in Sec. III.

B. Initial assumptions

To calculate NN phase shifts we first have to construct a six-quark wave function that is totally antisymmetric. We assume that this wave function can be written as a product of two three-quark cluster wave functions and a relative wave function:

$$\Psi_6=\mathcal{A}\left[\sum_{B,B'}\Psi_B(1,2,3)\Psi_{B'}(4,5,6)\chi_{BB'}(r)\right], \qquad (5.1)$$

where Ψ_B ($\psi_{B'}$) is a well-defined wave function of a three-quark baryon B (B'), and $\chi_{BB'}(r)$ is the relative BB' wave function where the B and B' are separated by a distance r. The operator $\mathcal{A}$ ensures that the total six-quark wave function is antisymmetric under interchange of any two quarks. This six-quark wave function satisfies the Schrödinger equation,

$$(H_6-E)\Psi_6=0\ , \qquad (5.2)$$

where H_6 is a six-quark Hamiltonian as given in Eq. (3.4). In the six-quark calculations $V_{ij}^{\rm HYP}$ is the Fermi-Breit Hamiltonian of the one-gluon exchange, as written down by De Rújula, Georgi, and Glashow [1975; see Eqs. (3.4) and (3.6)]. For $V_{ij}^{\rm CONF}$ we have used in the six-quark calculations the harmonic-oscillator potential [$n=2$ in Eq. (3.5)] as well as other two-body confinement potentials (Harvey, LeTourneux, and Lorazo, 1984). Two-body confining potentials do have problems with long-range color van der Waals forces, as will be discussed. The so-called "flip-flop" models have been introduced (Yazaki, 1984; Oka, 1985; Koike, Morimatsu, and Yazaki, 1986; Lenz *et al.*, 1986) to avoid this problem.

The final NN phase shifts calculated with the "flip-flop" model are very similar to the ones calculated with the two-body potential models in which the color octet parts of the six-quark wave function responsible for the color van der Waals forces have been "projected" out (Oka and Horowitz, 1985; Koike, 1986). To solve the above Schrödinger equation (5.2), one integrates out the relative quark coordinates in the known baryon cluster ψ_B (assumed relative S states) and arrives at coupled integral equations for $\chi_{BB'}(r)$:

$$\sum_{BB'}\int[H_{BB'}(r,r')-EN_{BB'}(r,r')]\chi_{BB'}(r')d^3r'=0\ , \qquad (5.3)$$

where $H_{BB'}$ is the resonant group method Hamiltonian and $N_{BB'}$ the resonant group normalization kernels (see Oka and Yazaki, 1984). These equations are solved to find the scattering phase shifts for NN scattering. We shall not go through the details of such a calculation (see the review by Oka and Yazaki, 1984), but rather present the results and a few physics arguments to make the results of these six-quark resonating group method calculations understandable. One should keep in mind that in the calculation one truncates the sum over baryon clusters (BB') in Eq. (5.3). The accuracy of this approximation has been tested (Harvey, LeTourneux, and Lorazo, 1984). A second approximation is to truncate the quark Hilbert space to include only S- and P-state quarks. The effects of expanding this space have also been tested and discussed by Oka and Yazaki (1984; see also recent discussions by Silvestre-Brac *et al.*, 1986, and by Stancu and Wilets, 1987).

In these six-quark calculations, with two-body confinement potentials of the type of Eq. (3.5), one has color van der Waals forces between nucleons (Greenberg and Lipkin, 1981) which are unphysical (longest-range *NN* force is OPE). This problem has been discussed carefully by several authors, e.g., Liu (1983), Robson (1984), and Lenz *et al.* (1986). In six-quark calculations the problem has been discussed by Maltman and Isgur (1984), who say that the attraction in their model due to color degrees of freedom replaces the attraction due to two-pion exchange in the normal *NN* models, but they have strong color van der Waals forces in their calculation, as remarked by Harvey, LeTourneux, and Lorazo (1984). One open question is whether the "flip-flop" model will, depending on its parameters, give some attraction due to the excited hidden color states (Koike and Yazaki, 1986; Sato, 1986), i.e., can part of the two-pion exchange be understood in simple terms on the quark level? Others avoid the color van der Waals problem by truncation of the available wave-function components (see Oka and Yazaki, 1984). Since this problem does not strongly affect the main results of the resonant group method calculations with a truncated set of channels [see Pfenninger, Faessler, and Bräuer, 1987, who use Robson's "model" (1984)], we shall not dwell on it, but refer the reader to the literature (Greenberg and Lipkin, 1981; Wong, 1982, Robson, 1984).

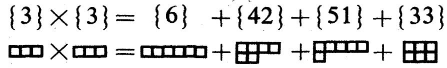
C. Six-quark and two-nucleon wave functions

First let us construct the total six-quark wave functions. A nucleon has a symmetric spin-flavor wave function

$\{3\}=$ □□□

[see discussion after Eq. (3.3)]. This means that the two-nucleon (six-quark) wave function has the following possible spin-flavor symmetries [here we shall use curly brackets to denote spin-flavor (spin-isospin) symmetries]:

$$\{3\}\times\{3\}=\ \{6\}\ +\{42\}+\{51\}+\{33\}$$



(5.4)

In Eq. (5.4) we use two different notations for a Young tableau. The lower line is the "picture" introduced in Sec. III. The upper line, commonly used in six-quark calculations, denotes the number of boxes in each row in the corresponding Young tableau. As before, a single row of boxes means a completely symmetric wave function. In Eq. (5.4) the nucleon SU(4) spin-isospin wave function Ψ^{FS} is denoted by $\{3\}$. This wave function is symmetric, and when it is multiplied by another symmetric wave function, Ψ^{FS}, we get the four possible flavor-spin wave functions of the six quarks [see Eq. (5.4)]. One is completely symmetric under the interchange of any two quarks $\Psi^{FS}_{\{6\}}$.[2] The other three have mixed symmetry. The spatial quark symmetry of a nucleon is also [3] (or □□□), since the three quarks are all in *S* states. This means that six-quark spatial wave functions will have the same symmetries as are given in Eq. (5.4).

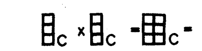
We have assumed here that the space and the spin-wave functions can be separated in a product wave function (see discussion in Sec. III.A). Furthermore, we know that all baryons have to be color singlets. In SU(3) the column of three boxes is a complete antisymmetric wave function and is an SU(3) singlet. This baryon wave function can consequently be written as

$$\Psi_3^{\rm color}\propto \varepsilon_{ijk}q_iq_jq_k\ \ (i,j,k=1,2,3)$$
$$=(rwb+brw+wbr-wrb-rbw-bwr)\ ,$$

where ε_{ijk} is the Levi-Civita symbol of Eq. (2.7) and q_i denotes the quark color wave function (here $q_1\equiv r\equiv\Psi_r$ where r means the color r quark wave function; $q_2=w$ and $q_3=b$). The product of two SU(3) singlets give only one possible SU(3) six-quark wave function with 36 terms:

$$\Psi_6^{\rm color}\propto \varepsilon_{ijk}\varepsilon_{lmn}q_iq_jq_kq_lq_mq_n\ ,$$

where $(i,j,k,l,m,n=1,2,3)$. Therefore, in $SU_c(3)$ color space, the only two-baryon state possible is

 $\Psi_6^{\rm color}$. (5.5)

Since this six-quark color wave function on the right-hand side of Eq. (5.5) is not completely antisymmetric under the interchange of any two quarks (it has mixed symmetry), and we note that the spatial wave functions [6] and [42] are symmetric under interchange of two nucleons (two three-quark clusters), we can for two nucleons in relative *S* state have only the following six-quark wave function (which has to be made antisymmetric under interchange of *any* two quarks):

[2]As an illustration of the wave function $\Psi^{FS}_{\{6\}}$ above, consider only one part of the proton wave function with spin z component $S_z=+\frac{1}{2}$: $u\uparrow u\uparrow d\downarrow$. The arrows denote spin up or spin down for each quark. The product of two such wave functions will give one part of the state $\psi^{FS}_{\{6\}}$. To ease the reading we denote $u\uparrow$ by a and $d\downarrow$ by b. The symmetric part of this product is

$$\text{symmetric }\{aab\times aab\}=aaaabb+aaabba+aabbaa$$
$$+abbaaa+bbaaaa+aaabab$$
$$+aababa+ababaa+babaaa$$
$$+aabaab+abaaba+baabaa$$
$$+abaaab+baaaba+baaaab\ .$$

The mixed-symmetry wave functions in Eq. (5.4) are not so easy to construct, and it depends on conventions used. We refer the reader to a textbook, e.g., Kaplan (1975).

$$\Psi_6(r)=\Psi_6^{\text{color}}\times(a[6]\{33\}+b[42]\{51\}+c[42]\{33\})\ . \qquad (5.6)$$

Here Ψ_6^{color} is the six-quark color singlet wave function above and we have used square brackets to denote the symmetries of the spatial wave functions and where a, b, and c are "Clebsch-Gordan coefficients" such that this wave function is antisymmetric under interchange of any two quarks. Again Eq. (5.6) is a compact notation for the total six-quark wave function,

$$\Psi_6(r)\doteq\Psi_6^{\text{color}}(a\Psi_{[6]}^{\text{space}}\Psi_{\{33\}}^{FS}+b\Psi_{[42]}^{\text{space}}\Psi_{\{51\}}^{FS}+c\Psi_{[42]}^{\text{space}}\Psi_{\{33\}}^{FS})\ .$$

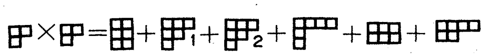
As shown by Harvey (1981a), when $r\to\infty$ we have two well-separated free nucleons (three-quark clusters) in relative S state, and the coefficients a, b, and c in Eq. (5.6) are then determined to be

$$\Psi_6(r\to\infty)=\Psi_6^{\text{color}}\times(\tfrac{1}{3}[6]\{33\}+\tfrac{2}{3}[42]\{33\}-\tfrac{2}{3}[42]\{51\})\ . \qquad (5.7)$$

Since the two nucleons are in relative S state, the only two spin-S and isospin-T combinations allowed are $S=0,T=1$ and $S=1,T=0$. The three terms above are the three symmetry basis states. However, we can also write a six-quark wave function in a baryon-baryon basis state, as was done in Eq. (5.1). For example, the six-quark wave function with deuteron quantum numbers $(S=1,T=0)$ in which all six quarks are in lowest S state and therefore have a symmetric spatial wave function [6]—see Eq. (5.7)—is (Matveev and Sorba, 1977a, 1977b)

$$\Psi_6=\tfrac{1}{3}\,|NN\rangle+(\tfrac{4}{45})^{1/2}\,|\Delta\Delta\rangle+(\tfrac{4}{5})^{1/2}\,|B_8B_8\rangle\ , \qquad (5.8)$$

where $|NN\rangle$ ($|\Delta\Delta\rangle$) is the two-nucleon (two-delta) state, and $|B_8B_8\rangle$ are the two-baryon colored-octet states. In color space the $|NN\rangle$ and $|\Delta\Delta\rangle$ are just Eq. (5.5), giving an overall color singlet six-quark wave function, whereas in color space $|B_8B_8\rangle$ is given by the six-quark singlet in the product



$$8\times 8=1+8_1+8_2+10+\overline{10}+27\ , \qquad (5.9)$$

i.e., two color **8** states couple to a color singlet, Eq. (5.5) [the notation in Eq. (5.9) is the same as in Eq. (3.1)]. The two $SU_c(3)$ octet states B_8 cannot be physical baryons. Harvey (1981a) has written the orthogonal transformation between the two basis states for $T=1;S=0$ or $T=0;S=1$ (see Table IV). In short, we can, following Eq. (5.1), write the six-quark wave functions as

$$\Psi_6=\sum_\beta|\beta\rangle\chi_\beta(r)\ , \qquad (5.10)$$

where $\beta=NN$, $\Delta\Delta$, or B_8B_8 are the three baryon-baryon channels. In the resonant group method calculations Oka and Yazaki (1980) and Faessler *et al.* (1982, 1983) let the variational principle

TABLE IV. Transformation coefficients between the six-quark basis and the baryon-baryon basis for $T,S=0,1$ or $1,0$; see Harvey (1981a).

	[6]{33}	[42]{33}	[42]{51}
$\|NN\rangle$	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$
$\|\Delta\Delta\rangle$	$\frac{2}{\sqrt{45}}$	$\frac{4}{\sqrt{45}}$	$\frac{5}{\sqrt{45}}$
$\|B_8B_8\rangle$	$\frac{2}{\sqrt{5}}$	$-\frac{1}{\sqrt{5}}$	0

$$\langle\delta\Psi_{6q}\,|H-E|\,\Psi_{6q}\rangle=0 \qquad (5.11)$$

choose the relative amplitudes of the different spatial symmetries [6] and [42] in Table IV as the distance between the two baryon clusters r is changing, arriving at NN having spatial symmetric states only for relative distance $r=0$. As shown by Harvey, LeTourneux, and Lorazo (1984; see their Fig. 5.1), this configuration path chosen by Oka and Yazaki (1980, 1981) is the optimal choice in order for a one-channel ($\beta=NN$) calculation to give NN S-wave phase shifts very close to the NN phase shifts of the complete three-channel coupled equations. As stressed by Harvey *et al.* (1984), only the complete channel ($\beta=NN,\Delta\Delta,B_8B_8$) calculations (in resonant group method or in generator coordinate method) give NN phase shifts that are reliable. The resultant $^3S_1$ and $^1S_0$ phase shifts [Fig. 15(b)] found in these calculations are very similar to phase shifts from a hard-core scattering, $\delta_0=r_0k$, where k is the nucleon momentum and r_0 the hard-core radius (r_0 is found to be almost energy independent), and where, as discussed by Tamagaki (1967), this radius is associated with the position of the outermost energy-independent node in the relative wave function $\chi(r)$ of Eq. (5.1). These phase shifts come from the asymptotic wave functions in resonant group or generator coordinate calculations, which both give the same results (Harvey, LeTourneux, and Lorazo, 1984). Asymptotically (and mathematically) the three physical channels, $|NN\rangle$, $|\Delta\Delta\rangle$, and $|B_8B_8\rangle$, are orthogonal and well defined (the color octet cannot exist asymptotically due to confinement). However, when r is small (the two three-quark clusters overlap), the only easily defined orthogonal states are the symmetry states, Eq. (5.6). This means that the relative wave function χ_{NN}, for example, is not a well-defined function for small r. The question now is why do the resonant group calculations find repulsive NN phase shifts? What is the physics behind this? And why do they not get strong repulsion when $\alpha_s=0$, i.e., when there are no color hyperfine interactions, Eq. (3.6), included [Harvey, LeTourneux, and Lorazo, 1984; see Fig. 15(a)]? The hard-core-like S-wave NN phase shifts are not quite the same whether one uses two-body confinement forces, in which the parts giving long-range color van der Waals forces have been projected out, or the quark "flip-flop" confinement model (Oka and

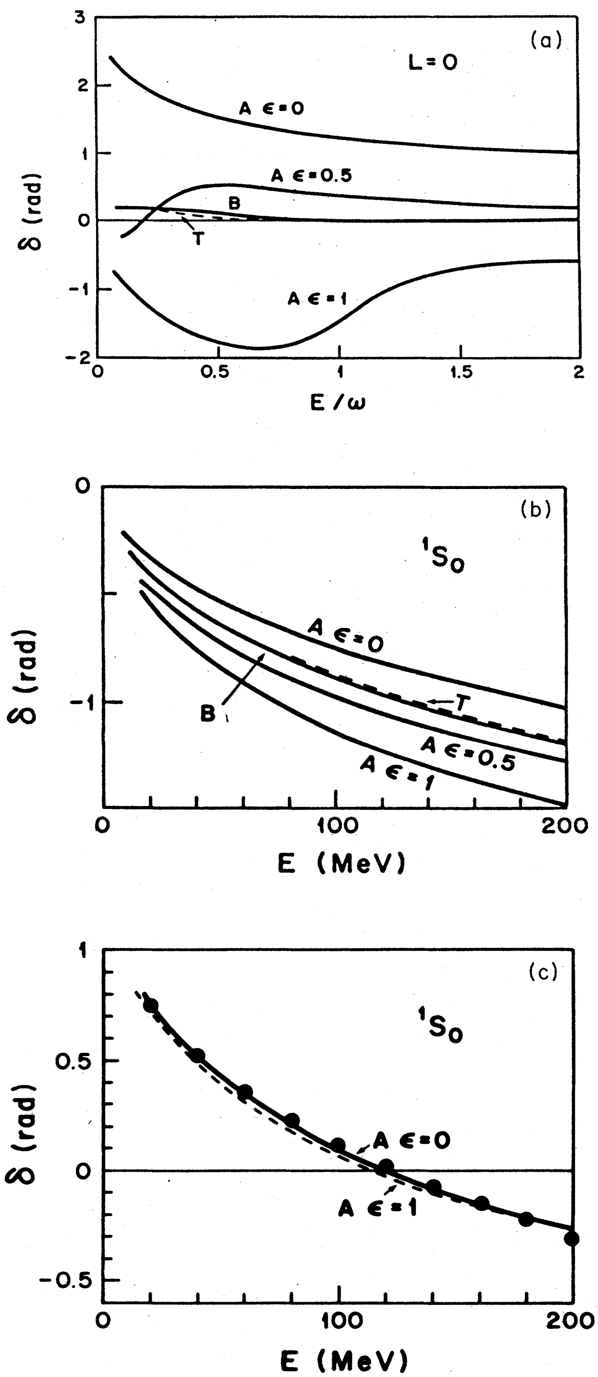


FIG. 15. The $^1S_0$ nucleon-nucleon phase shift as a function of energy, as calculated in the resonant group method six-quark calculations of Oka and Horowitz (1985) from whom the figure is taken. (a) Only quark confinement potentials are included ($\alpha_s=0$). Here T denotes a two-body potential like Eq. (3.5), B and A with different values of ε are different models of string ("flip-flop") confining forces [see Oka and Horowitz (1985)]. (b) The same as (a) except now $\alpha_s=1.77$ so as to give the N-Δ mass difference (Oka and Horowitz, 1985). (c) Here $\alpha_s=1.77$ and the long-range effective meson exchange potentials are included as well. The solid circles are the data points, and as can be seen there is hardly any dependence on the confinement models used.

Horowitz, 1985; Koike, 1986), in which one does not have the color van der Waals problem [see Fig. 15(b)]. To answer these questions let us discuss some details of these resonant group calculations.

D. Why six-quark calculations give *NN* repulsions

As a basis we choose the quark Gaussian wave functions because the overall center-of-mass coordinate can easily be separated, something which is very hard if not impossible in baglike models. The oscillator basis states we choose here are the ground state $0S(0\hbar\omega)$ and the excited $0P$ states $(1\hbar\omega)$. Oka and Yazaki (1984) have discussed the inclusion of $2\hbar\omega$ states $1S$ and $0D$, the latter coupled to $0S$ through the q-q tensor forces of Eq. (3.6), which are shown to be weak. Since Faessler *et al.* (1982) and Oka and Yazaki (1984) find the same NN phase shifts in their calculations, but the latter calculate also in a larger oscillator basis, we assume, as do Oka and Yazaki (1984), that the higher oscillator states cannot influence the results very strongly. We shall in the following keep only the lowest S- and P-quark states. The three-quark color singlet "nucleons" in this basis are

$$\begin{aligned} & \qquad C \times FS \;\times \text{space} \\ |N\rangle &= \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} \times \square\square\square \times \square\square\square \;, \\ |N^*\rangle &= \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} \times {}^{S}\!\left(\begin{smallmatrix}\square\square\\\square\;\;\end{smallmatrix} \times \begin{smallmatrix}\square\square\\\square\;\;\end{smallmatrix}\right) . \end{aligned} \qquad (5.12)$$

In the latter the symmetric combination of space and flavor-spin $SU_{FS}(4)$ wave functions has to be taken in order to have a total antisymmetric three-quark wave function. The latter state is the negative-parity N^* state [the symmetric spatial wave function $\square\square\square$ is only the Lorentz boosted $|N\rangle$ wave function, as discussed, for example, by DeGrand (1976), Fiebig and Schwesinger (1983), and Myhrer and Wroldsen (1984a)]. We have not included the $|N^*\rangle$ Roper resonance, which is a combination of two quarks in $0S$ state and one in $1S$ state (discussed by Silvestre-Brac *et al.*, 1986) and two in $0P$ and one in $0S$, since this state also has strong collective components (deformed chiral bags; see, for example, Brown, Durso, and Johnson, 1983, and Murthy, Dey, Dey, and Bhaduri, 1984) and cannot be described properly in a pure harmonic-oscillator basis unless a perturbing anharmonic potential is used (Isgur and Karl, 1979a).

In order to given an explanation of the resonant group method results, we shall concentrate on the color magnetic interaction [see Eq. (3.6) or Eq. (3.23)] of H_6, Eq. (5.2), since one finds that the NN phase shifts depend critically on the value of α_s, and for $\alpha_s=0$ one does not have hard-core-like NN phase shifts (Harvey, LeTourneux, and Lorazo, 1984; Oka and Horowitz, 1985). We know that this interaction gives the N-Δ mass difference of $\Delta M \simeq 300$ MeV for three quarks in the oscillator potential (α_s fitted to do just that; see Close, 1979). But this same color magnetic interaction also affects the NN

states. The six-quark states with S-wave quarks which means a spatial symmetric wave function [6], increase their energy when we include the color magnetic interaction (DeGrand, Jaffe, Johnson, and Kiskis, 1975). The [42] spatial state with four quarks in S state and two quarks in P state has initially higher energy, but the color magnetic interaction is strongly attractive (Obukhovsky, Neudatchin, Smirnov, and Tchuvil'sky, 1979; Harvey, 1981b; Myhrer and Wroldsen, 1986). As shown in Eq. (5.7), the Ψ_6 two-nucleon function has two major spatial components:

$$\Psi[42]=\sqrt{1/2}[42](\{33\}-\{51\}) \tag{5.13}$$

and

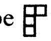
$$\Psi[6]=[6]\{33\} . \tag{5.14}$$

If the Ψ_6 of Eq. (5.7) were a pure [42] state, we would have a hard-core repulsion. The argument for this can be traced back to the results for $\alpha\alpha$ S-wave scattering, where in each α the four nucleons are in $0S$ states (Tamagaki and Tanaka, 1965; Okai and Park, 1966; Tamagaki, 1967). When we scatter two α's, the completely symmetric eight-nucleon spatial wave function [8] (= ) is ruled out by the Pauli exclusion principle, so for $\alpha\alpha$ scattering we can only have a mixed-spatial-symmetry state [44], in which four nucleons are in $0S$ states and the other four are in P states, which is orthogonal to [8]. This results in a hard-core-like S-wave $\alpha\alpha$ phase shift, but the $\alpha\alpha$ potential itself is highly nonlocal [neither the potential nor the relative wave function between the α clusters is well defined in resonant group method calculation when the two clusters overlap (Spitz and Schmid, 1986; Fiebig and Harvey, 1987); only the asymptotic scattering phase shifts are well defined]. In short, the relative S-wave $\alpha\alpha$ wave function must be an S-wave function that has two nodes, and this is what effectively causes the strong short-range repulsion in $\alpha\alpha$ S-wave scattering. This is consistent with the findings of Okai and Park (1966), who find the hard-core radius in the phase shift to be associated with the outermost node in the relative S-wave $\alpha\alpha$ wave function. This spatial wave function has to have energy-independent nodes, since it has to be orthogonal to a completely symmetric S-wave $\alpha\alpha$ wave function allowed by potential theory but forbidden as a physical state by the Pauli principle. However, this is just an argument since, as stated above, the relative wave functions of the physical channels are not well defined (see Orlowski and Kim, 1985, Spitz and Schmid, 1986, and discussions in Harvey, LeTourneux, and Lorazo, 1984, and Fiebig and Harvey, 1987). What is certain is that, if a mixed-symmetry spatial state dominates (as in $\alpha\alpha$ scattering), then we find hard-core-like S-wave phase shifts, as confirmed by the two other examples below.

(1) If we consider the six-quark states with flavor-spin $\{51\}$ symmetry, we know from Eq. (5.13) that this flavor-spin wave function Ψ^{FS} can only couple to the spatial [42] symmetry state, in order to have a completely antisymmetric six-quark wave function, provided the two three-quark clusters are in a relative S wave [see also Eq. (5.7)]. In the review of Oka and Yazaki (1984; their Table 4.2 and their Fig. 4.4) it is shown that $\Delta\Delta(S=3; T=2)$, $\Delta\Delta(2;3)$, and $N\Delta(2;2)$ scattering, which are all in a $\{51\}$ $SU_{SF}(4)$ state and thereby in a spatial mixed-symmetry [42] state, all give hard-core-like phase shifts.

(2) Consider K^+N elastic scattering in which the strange K meson consists of the valence quarks $\bar{s}u$ in an S wave (Bender, Dosch, Pirner, and Kruse, 1984). Here only the u quark will have to be antisymmetrized with respect to the u quarks in the nucleon. So we have to couple a single quark in flavor-spin $\{1\}$ with the symmetric (three-quark) nucleon flavor-spin wave function $\{3\}$, giving

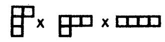
$$\{1\}\times\{3\}=\{4\}+\{31\} . \tag{5.15}$$

The SU(3) color symmetry of the four light u and d quarks has to be  since the antiquark has color symmetry ; these couple together to give a color singlet . Since we do K^+N S-wave scattering, and the total K^+N light-quark wave function has to be antisymmetric, we have two possibilities:

$C\times FS\times$space

 (5.16)

or

 . (5.17)

The first wave function (5.16) must involve a spatially excited [31] four-quark state. Bender, Dosch, Pirner, and Kruse (1984) find that K^+N scattering in the spatially mixed-symmetry [31] state gives a hard-core-like repulsive phase shift, whereas Eq. (5.17) gives attraction. Again this is consistent with the argument that the [31] S-wave spatial state of Eq. (5.16) has to be orthogonal to the spatial symmetric state [4] of Eq. (5.17). Therefore the [31] wave function has an energy-independent node, and the position of this node gives us roughly the "hard-core radius" in the phase shift.

In short, when one has to have a spatially excited, mixed-symmetry quark state due to total antisymmetry among quarks, one finds a repulsive hard-core phase shift.

For NN ($S=1, T=0$ or $S=0, T=1$) scattering the situation is, as stated, more complex. Here one has, due to the many degrees of freedom, both a spatial symmetric [6] wave function ($\frac{1}{9}=11\%$) and a spatially excited [42] symmetry wave function ($\frac{8}{9}=89\%$), where the numbers in parentheses refer back to Eq. (5.7). However, these numbers are only correct if the [6] and [42] spatial states are energy degenerate. As shown in a bag model estimate (Myhrer and Wroldsen, 1986), the [42] spatial symmetry component of Ψ_6 of Eq. (5.7) is about 700 MeV higher in energy than the [6] spatial symmetry component for the NN channel when only quark kinetic energies are considered. However, the expectation values of the color magnetic interaction $\Delta E_{\rm CM} = -\sum_{i<j} C\lambda_i^a \sigma_i \lambda_j^a \sigma_j$ [see also Eq. (3.24)] calculated using the six-quark states, Eqs. (5.13) and (5.14), showed that this color magnetic operator has a very strong nondiagonal attractive contribution in the SU(4) (isospin$\times$spin) {33} and {51} states of Eq. (5.13). This strong color magnetic attraction, together with the color magnetic repulsion in the spatial [6] state, almost cancels the increased quark excitation energy in the spatial [42] states involving *P*-state quarks (Myhrer and Wroldsen, 1986). In other words, the same α_s that generates different Δ and N masses ($\sim$300 MeV) also makes the spatial six-quark [6] and [42] states almost degenerate in energy [earlier calculations (Obukhovsky *et al.*, 1979; Harvey, 1981b) gave indications that this might happen in the *NN* system]. The crucial point is that if these states, Eqs. (5.13) and (5.14), are energy degenerate then they will mix by our group-theoretical arguments with coefficients given in Table IV and Eq. (5.7), and the [42] state will dominate *NN S*-wave scattering.

For the $\Delta\Delta$ channel, similar calculations found the color magnetic interaction of the [42] spatial state to be repulsive for $T=0, S=1$ and $T=1, S=0$ (Myhrer and Wroldsen, 1986). As a consequence the [42] spatial state is not very important in these $\Delta\Delta$ channels due to the large energy gap between the [6] and the [42] states. In addition, when we calculate the *NN* scattering far below the $\Delta\Delta$ threshold, it is not surprising that the $\Delta\Delta$ channel does not contribute strongly to the hard-core-like *NN* phase shifts, as several groups have noted (Faessler, Fernandez, Lübeck, and Shimizu, 1982, 1983; Storm and Watt, 1983; Burger and Hofmann, 1984; Harvey, LeTourneux, and Lorazo, 1984; Oka and Yazaki, 1984; Suzuki, 1985). In the chiral bag models, part of N-Δ mass difference is due to gluon exchange with $\alpha_s = 1.55$ and part is due to pionic contributions (Myhrer, Brown, and Xu, 1981). The energy degeneracy of the two states [6] and [42] also occurs in the chiral models (Myhrer and Wroldsen, 1986) where the quark-quark interaction energies due to pions in a chiral bag model will contribute very little (<10 MeV) to the energy difference discussed above.

When $\alpha_s = 0$, the phase shifts of resonant group method calculations give no hard-core-like behavior, and the phase shift's behavior as a function of energy is erratic; see Fig. 15(a). Also shown are the phase shifts when $\alpha_s \neq 0$, Fig. 15(b), where the dependence on different confinement models is evident (Oka and Horowitz, 1985). However, as calculated for $\alpha_s = 0$, the spatial wave functions [6] and [42] are not energy degenerate, and therefore the mixed-symmetry state [42] will not dominate due to energy suppression factors of this state *not* included in Eq. (5.7). We know that α_s gives an N-Δ mass difference of only 300 MeV in the three-quark system (in chiral bags only part of this comes from gluonic interactions as stated above). However, as has been discussed above, this is also of the order of the energy difference between the two six-quark spatial states, six quarks in *S* states, or four quarks in *S* and two quarks in *P* state, before including the color magnetic interaction. To summarize, it is not the energy of the color magnetic interaction itself that matters, but that this energy is comparable to the lowest quark excitation energy. For *NN* scattering this creates two (almost) energy-degenerate spatial symmetry states [6] and [42], and for the *NN* channel the latter will dominate (see Table IV) and result in a hard-core-like repulsion.

With the quark degrees of freedom only, we *cannot* reproduce the measured *S*-wave *NN* phase shifts. The reason is simple enough. Our picture of the nucleon is that it has a quark core (bag) surrounded by a pionic cloud (due to chiral symmetry). In this section we have so far considered only scattering of the two quark cores of the nucleons. This means, due to confinement, that we have *no NN* interaction or scattering when the two quark cores do not overlap. In the Introduction we stated that the rms radius of the quark core is typically 0.5–0.6 fm (Myhrer, 1984; Weise, 1984). This means that two nucleons which are 2 fm apart will not interact unless we include the mesonic cloud around the quark cores. Then the two nucleons 2 fm apart will interact via meson exchange forces. Thus we should add to the quark forces the long-range meson exchange potentials (and only use an α_s which gives part of the N-Δ mass splitting, since part of the difference is due to the pionic cloud). When long-range OPE and TPE forces are included in resonant group method calculations (Faessler and Fernandez, 1983; Oka and Yazaki, 1984; Wakamatsu, Yamamoto, and Yamauchi, 1984; Bräuer, Faessler, Fernandez, and Shimizu, 1985a, 1985b; Yamauchi, Yamamoto, and Wakamatsu, 1985; Zhang, Bräuer, Faessler, and Shimizu, 1985), the calculated and the "measured" *NN* $^1S_0$ and $^3S_1$ phase shifts are in remarkably good agreement with each other; see Fig. 15(c). Comparing Figs. 15(b) and 15(c), we can also see that there is hardly any dependence on the confinement models when OPE and TPE potentials are included (see Oka and Yazaki, 1984, for further discussion). The strengths of the effective long-range meson exchange in these calculations have been adjusted to reproduce the scattering lengths (connected with deuteron binding and the threshold $^1S_0$ *NN* "resonance"), which are given mainly by the long-range meson exchange forces. Figure 15 shows that, regardless of the quark confinement model used, one has a universal energy dependence of the *NN* phase shifts. The explanation for this energy behavior of the *S*-wave phase shifts in *NN* is the same as for the $\alpha\alpha$ *S*-wave phase shifts—the constituents in N and α, respectively, have to be antisymmetrized (Tamagaki, 1967).

Similarly the *NN* $^1P_1$ phase shift's energy dependence has been calculated and found to be close to the experimental phase shifts (Burger and Hofmann, 1984; Oka and Yazaki, 1984). As Faessler (1985) also discusses, his calculations can easily accommodate a repulsive ω exchange with a ωNN coupling $g_\omega^2/4\pi \simeq 4.5$, which is close to the SU(3) value also used by Vinh Mau *et al.* (1973) in their parameter-free description of NN phase shifts with $l \geq 2$. It should be pointed out here that the six-quark calculations give only small NN D-wave phase shifts (Burger and Hofmann, 1984; Harvey, LeTourneux, and Lorazo, 1984; Oka and Yazaki, 1984). We should stress that in these calculations there were only two free parameters in the long-range "scalar-meson" exchange attraction. All the others were determined by reproducing static baryon properties and by reproducing the long-range OPE potential. These two free parameters were adjusted as stated to describe the NN scattering length and effective range, and the same two parameters give reasonable ${}^1S_0$, ${}^1P_1$, and ${}^1D_2$ phase shifts (Oka and Yazaki, 1984; see also Oka and Horowitz, 1985, and Koike, 1986). No meson-nucleon form factors have been used in the calculations mentioned immediately above, although the use of such form factors has been examined (Strobel, Bräuer, Faessler, and Fernandez, 1985). In conclusion, we seem to be on the right track toward a deeper model understanding of the nuclear forces, but there are open questions and problems, one of which will be discussed next.

E. The noncentral short-range *NN* potentials

For NN S-wave scattering the $\mathbf{L}\cdot\mathbf{S}$ force is zero and the tensor force does not play a dominant role. However, in ${}^3P$ waves one should see effects of the spin-orbit terms, which should have been included in the Fermi-Breit forces [Eq. (3.6)]. For P-wave NN scattering the six-quark spatial symmetry wave functions that will contribute are the [51] and [33] (Harvey, 1981a; Oka and Yazaki, 1984). Several researchers have tried to calculate the short-range spin-orbit force between baryons based on quark models (e.g., Pirner, 1979; Brown, 1982b; Holinde, 1985; Wang and Wong, 1985; He, Wang, and Wong, 1986). One basic problem has already been pointed out by Isgur and Karl (1977). There does not seem to be any room for $\mathbf{L}\cdot\mathbf{S}$ quark forces in excited nucleon N^* or Δ^* states. Therefore Isgur and Karl (1977) simply neglected the Fermi-Breit $\mathbf{L}\cdot\mathbf{S}$ forces. The detailed bag model calculations by Fiebig and Schwesinger (1983) and Myhrer and Wroldsen (1984a, 1984b) show that the scalar confinement $\mathbf{L}\cdot\mathbf{S}$ splitting (in bag models) is partly canceled by the Fermi-Breit quark-quark interaction $\mathbf{L}\cdot\mathbf{S}$ forces for the three-quark N^* and Δ^* negative-parity states. In other words, the resultant calculated N^* and Δ^* spectra show little evidence for large effective $\mathbf{L}\cdot\mathbf{S}$ quark forces, although the two canceling $\mathbf{L}\cdot\mathbf{S}$ terms are large. For example, in the MIT bag, the $P_{3/2}$ quark has an energy $=3.20/R \simeq 640$ MeV (for $R=1$ fm), whereas the $P_{1/2}$ quark has an energy $=3.8/R \simeq 760$ MeV, indicating a spin-orbit force opposite in sign to the Fermi-Breit $\mathbf{L}\cdot\mathbf{S}$ terms of De Rújula, Georgi, and Glashow (1975).

Therefore one must be very careful in drawing any conclusion regarding the short-range $\mathbf{L}\cdot\mathbf{S}$ from V^{HYP} in six-quark calculations of the nuclear $\mathbf{L}\cdot\mathbf{S}$ force. It may well be that for the NN scattering (six quarks) the confinement mechanism that is so important in obtaining the correct N^* spectrum (Myhrer and Wroldsen, 1984a, 1984b) will also partly cancel the $\mathbf{L}\cdot\mathbf{S}$ force of the Fermi-Breit interaction used by Morimatsu, Ohta, Shimizu, and Yazaki (1984), Morimatsu, Yazaki, and Oka (1984), Suzuki and Hecht (1984), Holinde (1985), and Wang and Wong (1985). This question has been examined by Koike *et al.* (1986) in an SU(2) color model and by Koike (1986) for NN scattering. They constructed a confinement potential including an $\mathbf{L}\cdot\mathbf{S}$ term *à la* Dalitz (1982), with strength such that these $\mathbf{L}\cdot\mathbf{S}$ forces canceled the Fermi-Breit $\mathbf{L}\cdot\mathbf{S}$ for the negative-parity N^* and Δ^* states. Depending on their confinement model they got different results, so the question is still not settled. But with the "flip-flop" confinement model and an effective parametrization of the meson exchanges, Koike (1986) was able to describe the ${}^3P_{0,1,2}$ NN phase shifts. However, the results are, as stated, model dependent.

One further point, based on the results of Baym and Chin (1976), is that nonrelativistic and relativistic quarks have very different behavior; see, for example, the discussion by Brown (1982b). This raises immediately the question of whether one should use current quark masses ($\simeq 10$ MeV) or constituent quark masses ($\simeq 330$ MeV) in the calculations. Thus very little can be said about the $\mathbf{L}\cdot\mathbf{S}$ forces in NN scattering at the moment, and more detailed studies on this question are needed.

VI. CONCLUDING REMARKS

In this paper we have argued that the nuclear force can be understood from a model of the basic strong interactions among quarks when one imposes on the model the requirements of a chiral symmetry that guides the low-energy nuclear physics and particle physics phenomena. From the quark structure of the nucleon and the existence of the Goldstone pion, a collective $q\bar{q}$ 0^- state whose existence is required by the spontaneous breaking of chiral symmetry, we have a more fundamental understanding of the strength of the pion-nucleon coupling constant. In a quark model this is calculated using chiral symmetry requirements (see Sec. III.C.3). The picture that emerges of the nucleon is that the nucleon interior is a quark core, which is surrounded by a pionic cloud. A consequence is that large-impact-parameter NN scattering is dominated by meson exchanges, which means we can use an effective meson-nucleon theory for long-distance nuclear forces. An investigation of the two-pion exchange forces with the chiral quark model can help us to understand the details and perhaps improve the dispersion calculations of the two-pion exchange forces performed by the Stony Brook (Brown and Jackson, 1976) and Paris (Vinh Mau, 1979) groups (see Sec. IV). The fit to NN higher partial waves is still not understood completely, since form factors with free parameters are still needed (Machleidt, Holinde, and Elster, 1987). The values of the parameters in the form factors are not unreasonable, and approximate values can be given from meson-nucleon dispersion arguments. We are still lacking a more microscopic description based on quark models. Alternatively, one can expand on the idea of Fujiwara and Hecht (1986a, 1986b, 1986c, 1987) and include $q\bar{q}$, $qq\bar{q}\,\bar{q}$, $q\bar{q}g$, etc., excitations in the model space to explain the medium-range attraction. This medium-range attraction may also be due to possible hidden color excitations, but here an unknown excitation energy of color quark clusters causes problems (Koike and Yazaki, 1986; Sato, 1986). It is not clear that the above $q\bar{q}$ expansion converges.

The open questions in Sec. V concern the behavior of the nuclear potential at short distances where an effective meson-nucleon theory should not be used unless *derived* from an underlying quark model. Can one find a reasonable parametrization that can be used in nuclear calculations? The boundary model of Feshbach and Lomon (1964; Lomon, 1975) has been justified by the resonating group method calculations described here. The (almost) energy-independent boundary radius of Feshbach and Lomon can be associated with the (almost) energy-independent nodes in the relative NN wave functions due to the antisymmetry of the valence quarks (see again some critical comments by Fiebig and Harvey, 1987). However, we now also have a model that goes beyond the simple "hard-core" picture of the Reid hard-core potential or Feshbach and Lomon's model, which seems reasonable for low nuclear excitations and densities. The resonanting group method calculation does tell us that the short-range NN potential is *nonlocal* (Oka and Yazaki, 1984), so care should be taken in applying local NN repulsive short-range potentials. This quark model allows us to calculate the quark content of the deuteron, which, however, is not a well-defined quantity, as discussed by Yamauchi and Wakamatsu (1986a, 1986b). The reason is that the Ψ_6 contains an NN component (Sec. V.C). This quark model could also help to understand how stiff the nuclear matter equation of state is for increasing pressure and excitations (Crawford and Miller, 1987). The short-range spin-orbit potential is not satisfactorily understood; we know that both mesonic and quark degrees of freedom will contribute to this potential. However, this problem is a very difficult one due to the influence of the unknown scalar confinement requirement and its interplay with the spin-orbit structure of the gluon exchange forces. In addition the ω-meson exchange does give some repulsion (see the end of Sec. V.D) and a strong spin-orbit force. How to incorporate the ω exchange into a quark model has been considered by Vento (1981) and Su and Henley (1986). Both reproduce the standard ω-exchange potential, but they do not address the question of the total spin-orbit force.

Finally, if these six-quark calculations are correct, the nucleon-nucleon tensor force seems to be very weak at short distances. In Sec. V we have attempted to interpret some of the results of the complicated six-quark cluster calculations, but this topic is still debated. There are several uncertainties in these calculations. One open question is how properly to join the long-range meson exchange forces with quark degrees of freedom. One workable proposal has been suggested by Simonov (1981, 1984), and NN phase shifts have been calculated (Kalashnikova *et al.*, 1985). This model has recently been discussed by Fasano and Lee (1987) in the context of chiral models and the *P*-matrix approach (Jaffe and Low, 1979; Bakker, Grach, and Narodetskii, 1984). Again chiral symmetry has to be our guide, and much work has still to be done on this question.

One area where quark degrees of freedom enter the nuclear physics arena directly is in the interpretation of deep-inelastic scattering phenomena and the so-called EMC (European Muon Collaboration) effect, in which data indicate that the nucleon form factor is different in a heavy nucleus from the free nucleon form factor (for a review see Rith, 1984). In muon scattering from heavy nuclei at high momentum transfers ($q > 1$ GeV/c), data can be interpreted as if the nucleon constituents (the quarks) moved in a larger volume than the volume of the free nucleons. This might be due to a swelling of the individual nucleons in the nuclei, or perhaps to formation of six-quark (or bigger) clusters inside the nucleus. [That this change of scale could be due to nucleon binding is argued by Akulinichev, Kulagin, and Vagradov (1985) and by Dunne and Thomas (1985)]. If the EMC effect is a signal of six-quark cluster formation, as some speculate, then at even higher nuclear densities and excitation energies one might expect the formation of a quark-gluon plasma, which is another speculative and exciting subject in nuclear physics today. In other words, the EMC effect might be a signal for the transition from ordinary hadronic matter to quark-gluon matter, in which the quarks and gluons are no longer confined to the individual hadrons. Between nucleons in nuclei at normal densities we have an effective strong nonlocal repulsion at short distances, due to quark degrees of freedom (Sec. V). See here some recent developments on quark-clustering phenomena in nuclei (Koltun and Tosa, 1986; Ohta *et al.*, 1986; Takeuchi and Shimizu, 1986; Koltun, 1987; Robson, 1987) and hadronic properties from a quark model (Gardner and Moniz, 1987) and the possible saturation of nuclear matter due to quark degrees of freedom (Guichon, 1988). These degrees of freedom might be very important for high excitation energy, where one could have a quark-gluon plasma in which the asymptotic freedom of QCD would cause this plasma to resemble more and more a free Fermi-boson gas (for a review, see Satz, 1985).

Our principal aim has been to give an overview of our understanding of the nuclear forces at this time and to present a coherent modern view of the origin of the nuclear forces. These forces have been studied extensively for many years. The last few years have given us a new and more quantitative understanding, especially of short-distance phenomena, which 10 years ago were studied mainly on the basis of parametrizations and educated guesses. For higher partial waves the authors think the most economic model is an effective meson-nucleon theory. However, to understand the energy dependence of the measured S- and P-wave NN phase shifts, the antisymmetry requirement of the six quarks in NN scattering is essential (details of confinement forces seem not to matter). As stated, we are still in the middle of a new, active research effort inspired by the stimulating and seemingly fruitful introduction of quarks into nuclear phenomena, and we think we have a workable model to study short-distance nuclear forces.

ACKNOWLEDGMENTS

This research has been supported by an NSF Grant (F.M.) and by a NATO travel grant. The hospitality of the CERN theory group and SIN (Swiss Institute for Nuclear Research) during the summers of 1985 and 1986 are gratefully acknowledged (F.M.).

REFERENCES

Adler, S. L., and R. F. Dashen, 1986, *Current Algebra* (Benjamin, New York).

Akhiezer, A. I., and V. B. Berestetskii, 1965, *Quantum Electrodynamics* (Wiley, New York).

Akulinichev, S. V., S. A. Kulagin, and G. M. Vagradov, 1985, Phys. Lett. B **158**, 485.

Amati, D., E. Leader, and B. Vitale, 1960, Nuovo Cimento **17**, 68 (Part I); **18**, 409 (Part II); **18**, 458 (Part III).

Amati, D., E. Leader, and B. Vitale, 1963, Phys. Rev. **130**, 750.

Arnison, G., *et al.*, UA1 Collaboration (CERN), 1986, Phys. Lett. B **172**, 461.

Aubert, J. J., *et al.*, 1974, Phys. Rev. Lett. **33**, 1404.

Augustin, J. E., *et al.*, 1974, Phys. Rev. Lett. **33**, 1406.

Babutsidze, T. D., T. I. Kopaleishvili, I. Z. Machabeli, and M. Sh. Chachkhunashvili, 1981, Yad. Fiz. **33**, 1406 [Sov. J. Nucl. Phys. **33**, 754 (1981)].

Bakker, B. L. G., M. Bozoian, J. N. Maslow, and H. J. Weber, 1982, Phys. Rev. C **25**, 1134.

Bakker, B. L. G., I. L. Grach, and I. M. Narodetskii, 1984, Nucl. Phys. A **424**, 563.

Bardeen, W. A., M. A. Chanowitz, S. D. Drell, M. Weinstein, and T. M. Yan, 1975, Phys. Rev. D **11**, 1094.

Baym, G., and S. A. Chin, 1976, Nucl. Phys. A **262**, 527.

Bender, I., H. G. Dosch, H. J. Pirner, and H. G. Kruse, 1984, Nucl. Phys. A **414**, 359.

Bernheim, M., E. Jans, J. Mougey, D. Royer, D. Tarnowski, S. Turck-Chieze, I. Sick, G. P. Capitani, E. De Sanctis, and S. Frullani, 1981, Phys. Rev. Lett. **46**, 402.

Beyer, M., and H. J. Weber, 1984, Phys. Lett. B **146**, 383.

Beyer, M., and H. J. Weber, 1987, Phys. Rev. C **35**, 14.

Bjorken, J. D., 1967, in *Proceedings of the 1967 International Symposium on Electron and Photon Interactions at High Energies ... Stanford, California* (Clearinghouse for Federal Scientific and Technical Information, National Bureau of Standards, Springfield, VA), p. 109.

Bjorken, J. D., and E. A. Paschos, 1969, Phys. Rev. **185**, 1975.

Bogoliubov, P. N., 1968, Ann. Inst. Henri Poincaré **7A**, 163.

Borkowski, F., P. Peuser, G. G. Simon, V. H. Walther, and R. D. Wendling, 1974, Nucl. Phys. A **222**, 269.

Bräuer, K., A. Faessler, F. Fernandez, and K. Shimizu, 1985a, Nucl. Phys. A **320**, 609.

Bräuer, K., A. Faessler, F. Fernandez, and K. Shimizu, 1985b, Z. Phys. A **320**, 609.

Breit, G., 1960a, Proc. Natl. Acad. Sci. U.S.A. **46**, 746.

Breit, G., 1960b, Phys. Rev. **120**, 287.

Brodsky, S. J., 1982, in *Quarks and Nuclear Forces,* edited by D. C. Fries and B. Zeitnitz, Springer Tracts in Modern Physics No. 100 (Springer, Berlin), p. 81.

Brodsky, S. J., and G. Lepage, 1981, Phys. Scr. **23**, 945.

Brown, G. E., 1979, in *Mesons in Nuclei,* edited by M. Rho and D. Wilkinson (North-Holland, Amsterdam), p. 329.

Brown, G. E., 1982a, Nucl. Phys. A **374**, 63c.

Brown, G. E., 1982b, Prog. Part. Nucl. Phys. **8**, 147.

Brown, G. E., and J. W. Durso, 1971, Phys. Lett. B **35**, 120.

Brown, G. E., J. W. Durso, and M. B. Johnson, 1983, Nucl. Phys. A **397**, 447.

Brown, G. E., and A. D. Jackson, 1976, *Nucleon-Nucleon Interaction* (North-Holland, Amsterdam).

Brown, G. E., and M. Rho, 1979, Phys. Lett. B **82**, 177.

Brown, G. E., M. Rho, and V. Vento, 1979, Phys. Lett. B **84**, 383.

Büttgen, R., K. Holinde, and J. Speth, 1985, Phys. Lett. B **163**, 305.

Burger, J., and M. H. Hofmann, 1984, Phys. Lett. B **148**, 25.

Callan, C. G., R. F. Dashen, and D. H. Gross, 1979, Phys. Rev. D **19**, 1826.

Chemtob, M., J. W. Durso, and D. O. Riska, 1972, Nucl. Phys. B **38**, 141.

Chemtob, M., and D. O. Riska, 1971, Phys. Lett. B **35**, 115.

Chew, G. F., M. L. Goldberger, F. E. Low, and Y. Nambu, 1957, Phys. Rev. **106**, 1377.

Chodos, A., R. L. Jaffe, K. Johnson, C. B. Thorn, and V. Weisskopf, 1974a, Phys. Rev. D **9**, 3471.

Chodos, A., R. L. Jaffe, K. Johnson, and C. B. Thorn, 1974b, Phys. Rev. D **10**, 2599.

Chodos, A., and C. B. Thorn, 1975, Phys. Rev. D **12**, 2733.

Chodos, A., and C. B. Thorn, 1976, Nucl. Phys. B **104**, 21.

Close, F., 1979, *An Introduction to Quarks and Partons* (Academic, London).

Close. F., and R. R. Horgan, 1980, Nucl. Phys. B **164**, 413.

Cooper, E. D., and B. K. Jennings, 1986, Phys. Rev. D **33**, 1509.

Cottingham, W. N., M. Lacombe, B. Loiseau, J. M. Richard, and R. Vinh Mau, 1973, Phys. Rev. D **8**, 800.

Cottingham, W. N., and R. Vinh Mau, 1963, Phys. Rev. **130**, 735.

Crawford, G. A., and G. A. Miller, 1987, Phys. Rev. C **36**, 2569.

Cvetič, M., B. Golli, N. Mankoc-Borstnik, and M. Rosina, 1980, Phys. Lett. B **93**, 489.

Cvetič, M., B. Golli, N. Mankoc-Borstnik, and M. Rosina, 1981, Phys. Lett. B **99**, 486.

Cvetič, M., B. Golli, N. Mankoc-Borstnik, and M. Rosina, 1983, Nucl. Phys. A **395**, 349.

Dalitz, R. H., 1982, Prog. Part. Nucl. Phys. **8**, 7.

DeGrand, T. A., 1976, Ann. Phys. (N.Y.) **101**, 496.

DeGrand, T. A., and R. L. Jaffe, 1976, Ann. Phys. (N.Y.) **100**, 425.

DeGrand, T. A., R. L. Jaffe, K. Johnson, and J. Kiskis, 1975, Phys. Rev. D **12**, 2060.

De Rújula, A., H. Georgi, and S. L. Glashow, 1975, Phys. Rev. D **12**, 147.

DeTar, C. E., 1978, Phys. Rev. D **17**, 323; **19**, 1028(E) (1979).

DeTar, C. E., 1979, Phys. Rev. D **19**, 1451.

Dunne, G. V., and A. W. Thomas, 1985, Nucl. Phys. A **446**, 437c.
Durso, J., 1966, Phys. Rev. **149**, 1234.
Durso, J. W., A. D. Jackson, and B. J. VerWest, 1977, Nucl. Phys. A **282**, 404.
Durso, J., A. D. Jackson, and B. J. VerWest, 1980, Nucl. Phys. A **345**, 471.
Durso, J. W., M. Saarela, G. E. Brown, and A. D. Jackson, 1977, Nucl. Phys. A **278**, 445.
Eisenberg, J. M., A. Erell, and R. R. Silbar, 1986, Phys. Rev. C **33**, 1531.
Elster, Ch., and K. Holinde, 1984, Phys. Lett. B **136**, 135.
Ericson, T. E. O., and M. Rosa-Clot, 1985, Annu. Rev. Nucl. Part. Sci. **35**, 271.
Faessler, A., 1985, Prog. Part. Nucl. Phys. **13**, 253.
Faessler, A., and F. Fernandez, 1983, Phys. Lett. B **124**, 145.
Faessler, A., F. Fernandez, G. Lübeck, and K. Shimizu, 1982, Phys. Lett. B **112**, 201.
Faessler, A., F. Fernandez, G. Lübeck, and K. Shimizu, 1983, Nucl. Phys. A **402**, 555.
Fasano, C., and T.-S. H. Lee, 1987, Phys. Rev. C **36**, 1906.
Feshbach, H., and E. L. Lomon, 1964, Ann. Phys. (N.Y.) **29**, 19.
Feynman, R. P., 1969, Phys. Rev. Lett. **23**, 1415.
Feynman, R. P., M. Kislinger, and F. Ravndal, 1971, Phys. Rev. D **3**, 2706.
Fiebig, H. R., and M. Harvey, 1987, Nucl. Phys. A **469**, 557.
Fiebig, H. R., and B. Schwesinger, 1983, Nucl. Phys. A **393**, 349.
Frazer, W., and J. R. Fulco, 1960, Phys. Rev. **117**, 1603.
Fujiwara, Y., and K. T. Hecht, 1986a, Nucl. Phys. A **451**, 625.
Fujiwara, Y., and K. T. Hecht, 1986b, Nucl. Phys. A **456**, 669.
Fujiwara, Y., and K. T. Hecht, 1986c, Phys. Lett. B **171**, 17.
Fujiwara, Y., and K. T. Hecht, 1987, Nucl. Phys. A **462**, 621.
Gardner, S., and E. J. Moniz, 1987, Phys. Rev. C **36**, 2504.
Gell-Mann, M., 1964, Phys. Lett. **8**, 214.
Gell-Mann, M., R. J. Oakes, and B. Renner, 1968, Phys. Rev. **175**, 2195.
Green, A. M., and P. Haapakoski, 1974, Nucl. Phys. A **221**, 429.
Greenberg, O. W., and H J. Lipkin, 1981, Nucl. Phys. A **370**, 349.
Gross, F., and D. O. Riska, 1987, Phys. Rev. C **36**, 1928.
Guichon, P. A. M., 1988, Phys. Lett. B **200**, 235.
Guichon, P. A. M., and G. A. Miller, 1984, Phys. Lett. B **134**, 15.
Gursey, F., A. Pais, and L. A. Radicati, 1964, Phys. Rev. Lett. **13**, 299.
Gursey, F., and L. A. Radicati, 1964, Phys. Rev. Lett. **13**, 173.
Haapakoski, P., 1974, Phys. Lett. B **48**, 307.
Hamilton, J., and W. S. Woolcock, 1963, Rev. Mod. Phys. **35**, 737.
Hanson, G., G. S. Abrams, A. M. Boyarski, M. Breidenbach, F. Bulos, W. Chinowsky, G. J. Feldman, C. E. Friedberg, D. Fryberger, G. Goldhaber, D. L. Hartill, B. Jean-Marie, J. A. Kadyk, R. R. Larsen, A. M. Litke, D. Lüke, B. A. Lulu, V. Lüth, H. L. Lynch, C. C. Morehouse, J. M. Paterson, M. L. Perl, F. M. Pierre, T. P. Pun, P. R. Rapidis, B. Richter, B. Sadoulet, R. F. Schwitters, W. Tanenbaum, G. H. Trilling, F. Vannucci, J. S. Whitaker, F. C. Winkelmann, and J. E. Wiss, 1975, Phys. Rev. Lett. **35**, 1609.
Hansson, T. H., 1985, Nucl. Phys. B **249**, 742.
Harvey, M., 1981a, Nucl. Phys. A **352**, 301.
Harvey, M., 1981b, Nucl. Phys. A **352**, 326.
Harvey, M., 1981c, in *Proceedings International Symposium on Clustering Phenomena in Nuclei,* edited by P. Kramer and R. Schultheiss (Werkhefte der Universität Tübingen, Tübingen), p. 179.
Harvey, M., and J. LeTourneux, 1984, Nucl. Phys. A **424**, 419.
Harvey, M., J. LeTourneux, and B. Lorazo, 1984, Nucl. Phys. A **424**, 428.
Hasenfratz, P., and J. Kuti, 1978, Phys. Rep. **40**, 75.
Hasenfratz, P., J. Kuti, and A. S. Szalay, 1975, in *Charm, Color, and the ψ: Proceedings of the 10th Rencontre de Moriond,* Vol. II, edited by J. Tran Thanh Van (C.N.R.S., Paris), p. 209.
He, Y., F. Wang, and C. W. Wong, 1986, Nucl. Phys. A **451**, 653.
Henley, E. M., L. S. Kisslinger, and G. A. Miller, 1983, Phys. Rev. C **28**, 1277.
Hill, D. L., and J. A. Wheeler, 1953, Phys. Rev. **89**, 1102.
Høgaasen, H., and F. Myhrer, 1983a, Z. Phys. C **21**, 73.
Høgaasen, H., and F. Myhrer, 1983b, private communications.
Høgaasen, H., J. M. Richard, and P. Sorba, 1982, Phys. Lett. B **119**, 272.
Höhler, G., 1983, "Methods and Results of Phenomenological Analysis," Subvolume b, Part 2, of *Landolt-Börnstein: Numerical Data and Functional Relationships in Science and Technology,* New Series, edited by K.-H. Hellwege (Springer, Berlin), Vol. I/9b2, pp. 1–601.
Holinde, K., 1981, Phys. Rep. **68**, 121.
Holinde, K., 1985, Phys. Lett. B **157**, 123.
Horowitz, C. J., E. J. Moniz, and J. W. Negele, 1985, Phys. Rev. D **31**, 1689.
Hulthage, I., F. Myhrer, and Z. Xu, 1981, Nucl. Phys. A **364**, 322.
Inoue, T., and T. Maskawa, 1975, Prog. Theor. Phys. **54**, 1833.
Isgur, N., 1978, in *The New Aspects of Subnuclear Physics: Proceedings of the XVI International School of Subnuclear Physics, Erice, . . .,* edited by A. Zichichi (Plenum, New York), p. 107.
Isgur, N., and G. Karl, 1977, Phys. Lett. B **72**, 109.
Isgur, N., and G. Karl, 1978, Phys. Rev. D **18**, 4187.
Isgur, N., and G. Karl, 1979a, Phys. Rev. D **19**, 2643.
Isgur, N., and G. Karl, 1979b, Phys. Rev. D **20**, 1191.
Isgur, N., and G. Karl, 1980, Phys. Rev. D **21**, 3175.
Israilov, Z. Z., and M. M. Musakhanov, 1981, Phys. Lett. B **104**, 173.
Jackson, A., A. D. Jackson, and V. Pasquier, 1985, Nucl. Phys. A **432**, 567.
Jackson, A. D., D. O. Riska, and B. VerWest, 1975, Nucl. Phys. A **249**, 397.
Jaffe, R. L., 1982, in *Pointlike Structures Inside and Outside Hadrons,* 17th International School of Subnuclear Physics, Ettore Majorana, Erice, . . . 1979, edited by A. Zichichi (Plenum, New York), p. 99.
Jaffe, R. L., and F. E. Low, 1979, Phys. Rev. D **19**, 2105.
Jennings, B. K., and O. V. Maxwell, 1984, Nucl. Phys. A **422**, 589.
Jennings, B. K., E. A. Veit, and A. W. Thomas, 1984, Phys. Lett. B **148**, 28.
Kalashnikova, Yu. S., I. M. Narodetskii, Yu. A. Simonov, and A. I. Veselov, 1985, Phys. Lett. B **155**, 217.
Kälbermann, G., and J. M. Eisenberg, 1984, Phys. Rev. D **29**, 517.
Kaminura, M., 1977, Prog. Theor. Phys. Suppl. **62**, 236.
Kapadia, P. D., 1967, Nucl. Phys. B **3**, 291.
Kaplan, I. G., 1975, *Symmetry of Many Electron Systems,* (Academic, New York).
Kim, Y. E., and M. Orlowski, 1984a, Phys. Lett. B **140**, 275.

Kim, Y. E., and M. Orlowski, 1984b, Phys. Rev. C **29**, 2299.
Kisslinger, L. S., 1982, Phys. Lett. B **112**, 307.
Kogut, J. B., 1984, in *Recent Advances in Field Theory and Statistical Mechanics,* Les Houches Summer School 1982, session XXXIX, edited by J.-B. Zuber and R. Stora (North-Holland, Amsterdam), course 5, p. 319.
Koike, Y., 1986, Nucl. Phys. A **454**, 509.
Koike, Y., O. Morimatsu, and K. Yazaki, 1986, Nucl. Phys. A **449**, 635.
Koike, Y., and K. Yazaki, 1986, Phys. Lett. B **179**, 332.
Kokkedee, J. J. J., 1969, *The Quark Model* (Benjamin, New York).
Koltun, D. S., 1987, Phys. Rev. C **36**, 2047.
Koltun, D. S., and S. Tosa, 1986, Phys. Lett. B **172**, 267.
Kutschera, M., and C. J. Pethick, 1985, Nucl. Phys. A **440**, 670.
Lacomb, M., B. Loiseau, J.-M. Richard, and R. Vinh Mau, 1975, Phys. Rev. D **12**, 1495.
Lacomb, M., B. Loiseau, J.-M. Richard, R. Vinh Mau, J. Côté, P. Pirès, and R. deTourreil, 1980, Phys. Rev. C **21**, 861.
Lacomb, M., B. Loiseau, R. Vinh Mau, and W. N. Cottingham, 1985, Phys. Lett. B **161**, 31.
Lacomb, M., B. Loiseau, R. Vinh Mau, and W. N. Cottingham, 1986, Phys. Lett. B **169**, 121.
Lee, B. W., 1972, *Chiral Dynamics* (Gordon and Breach, New York).
Lenz, F., J. T. Londergan, E. J. Moniz, R. Rosenfelder, M. Stingl, and K. Yazaki, 1986, Ann. Phys. (N.Y.) **170**, 65.
Liberman, D. A., 1977, Phys. Rev. D **16**, 1542.
Liu, K. F., 1982, Phys. Lett. B **114**, 222.
Liu, K. F., 1983, Phys. Lett. B **131**, 195.
Llewelyn Smith, C. H., 1980, in *Quantum Flavordynamics, Quantum Chromodynamics, and Unified Theories,* edited by K. T. Mahanthappa and J. Randa (Plenum, New York/London), p. 59.
Locher, M. P., M. E. Sainio, and A. Švarc, 1986, Adv. Nucl. Phys. **17**, 47.
Lomon, E. L., 1975, Phys. Rev. D **12**, 3758.
Lomon, E. L., 1984, in *Few Body Problems in Physics, Proceedings of the 10th International Conference ... Karlsruhe ...,* edited by B. Zeitnitz (North-Holland, Amsterdam), Vol. II, p. 609.
Machleidt, R., 1984, in *Quarks and Nuclear Structure,* Lecture Notes in Physics No. 197, Proceedings of the 3rd Klaus Erkelenz Symposium, Bad Honnef, 1983 (Springer, Berlin), p. 352.
Machleidt, R., K. Holinde, and Ch. Elster, 1987, Phys. Rep. **149**, 1.
Maltman, K., and N. Isgur, 1984, Phys. Rev. D **29**, 952.
Marshak, R. E., Riazuddin, and C. P. Ryan, 1969, *Theory of Weak Interactions in Particle Physics* (Wiley, New York).
Matveev, V. A., and P. Sorba, 1977a, Nuovo Cimento **20**, 435.
Matveev, V. A., and P. Sorba, 1977b, Nuovo Cimento A **45**, 257.
Maxwell, O. V., and B. K. Jennings, 1985, Nucl. Phys. A **437**, 519.
McLeod, R. J., and D. J. Ernst, 1984, Phys. Rev. C **29**, 906.
Miller, G. A., 1984, in *Quarks and Nuclei,* International Review of Nuclear Physics Series Vol. 1, edited by W. Weise (World Scientific, Singapore), p. 189.
Miller, G. A., and P. Singer, 1983, Phys. Lett. B **130**, 98.
Miller, G. A., and P. Singer, 1985, Phys. Lett. B **154**, 75.
Morimatsu, O., S. Ohta, K. Shimizu, and K. Yazaki, 1984, Nucl. Phys. A **420**, 573.
Morimatsu, O., K. Yazaki, and M. Oka, 1984, Nucl. Phys. A **424**, 412.
Murthy, M. V. N., M. Dey, J. Dey, and R. K. Bhaduri, 1984, Phys. Rev. D **30**, 152.
Myhrer, F., 1984, in *Quarks and Nuclei,* International Review of Nuclear Physics Series Vol. I, edited by W. Weise (World Scientific, Singapore), p. 325.
Myhrer, F., G. E. Brown, and Z. Xu, 1981, Nucl. Phys. A **362**, 317.
Myhrer, F., and J. Wroldsen, 1984a, Z. Phys. C **25**, 281.
Myhrer, F., and J. Wroldsen, 1984b, Phys. Lett. B **139**, 81.
Myhrer, F., and J. Wroldsen, 1986, Phys. Lett. B **174**, 366.
Nadkarni, S., H. B. Nielsen, and I. Zahed, 1985, Nucl. Phys. B **253**, 308.
Nagels, M. M., T. A. Rijken, and J. J. deSwart, 1975, Phys. Rev. D **12**, 744.
Nambu, Y., 1957, Phys. Rev. **106**, 1366.
Neudatchin, V. G., I. T. Obukhovsky, V. I. Kukulin, and N. F. Golonova, 1975, Phys. Rev. C **11**, 128.
Neudatchin, V. G., Yu. F. Smirnov, and R. Tamagaki, 1977, Prog. Theor. Phys. **58**, 1072.
Nielsen, H., and G. C. Oades, 1972, Nucl. Phys. B **49**, 586.
Nyman, E. M., and D. O. Riska, 1986, Phys. Lett. B **175**, 392.
Obukhovsky, I. T., V. G. Neudatchin, Yu. F. Smirnov, and Yu. M. Tchuvil'sky, 1979, Phys. Lett. B **88**, 231.
Ohta, S., M. Fukugita, and A. Ukawa, 1986, Phys. Lett. B **173**, 15.
Ohta, S., M. Oka, A. Arima, and K. Yazaki, 1982, Phys. Lett. B **119**, 35; **123**, 477(E) (1983).
Oka, M., 1985, Phys. Rev. D **31**, 2274.
Oka, M., and C. J. Horowitz, 1985, Phys. Rev. D **31**, 2773.
Oka, M., and K. Yazaki, 1980, Phys. Lett. B **90**, 41.
Oka, M., and K. Yazaki, 1981, Prog. Theor. Phys. **66**, 556 and 572.
Oka, M., and K. Yazaki, 1984, in *Quarks and Nuclei,* International Review of Nuclear Physics Series, Vol. 1, edited by W. Weise (World Scientific, Singapore), p. 489.
Okai, S., and S. C. Park, 1966, Phys. Rev. **145**, 787.
Orlowski, M., and Y. E. Kim, 1985, "On-shell equivalent transformations for eliminating energy dependence from coupled channel equations with non-local linear energy-dependent potentials," Purdue University Preprint No. PNTG-85-15.
Oset, E., 1984, Nucl. Phys. A **430**, 713.
Pagels, H., 1975, Phys. Rep. **16**, 219.
Pais, A., 1964, Phys. Rev. Lett. **13**, 175.
Panofsky, W., 1968, in *Proceedings of the 14th International Conference on High Energy Physics,* Vienna, edited by J. Prentki and J. Steinberger (European Organization for Nuclear Research, CERN, Geneva), p. 23.
Partovi, M. H., and E. L. Lomon, 1970, Phys. Rev. D **2**, 1999.
Perry, R. J., and M. Rho, 1986, Phys. Rev. D **34**, 1169.
Pfenninger, Th., A. Faessler, and K. Bräuer, 1987, Nucl. Phys. A **466**, 495.
Pirner, H. J., 1979, Phys. Lett. B **85**, 190.
Quigg, C., 1983, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions* (Benjamin/Cummings, Reading, Massachusetts).
Rebbi, C., 1983, *Lattice Gauge Theories and Monte Carlo Simulations* (World Scientific, Singapore).
Rho, M., 1984, Annu. Rev. Nucl. Part. Sci. **34**, 531.
Ribero, J. E. T., 1980, Z. Phys. C **5**, 27.
Rinat, A. S., 1982, Nucl. Phys. A **377**, 341.
Riska, D. O., 1979, in *Mesons in Nuclei,* Vol. II, edited by M. Rho and D. Wilkinson (North-Holland, Amsterdam), p. 755.
Riska, D. O., 1985, Phys. Scr. **31**, 471.

Rith, K., 1984, in *Quarks and Nuclear Structure,* Lecture Notes in Physics No. 197, Proceedings of the 3rd Klaus Erkelenz Symposium, Bad Honnef, 1983 (Springer, Berlin), p. 214.
Robson, D., 1984, in *Hadron Substructure in Nuclear Physics,* edited by W.-Y. P. Hwang and M. H. Macfarlane (AIP Conference Proceedings No. 110 (AIP, New York), p. 100.
Robson, D., 1987, Phys. Rev. D **35**, 1019 (Part I) and 1029 (Part II).
Sakita, B., 1964, Phys. Rev. **136**, 1756B.
Sakurai, J. J., 1960, Phys. Rev. **119**, 1784.
Sato, T., 1986, "Quark-exchange and the medium range attraction in the nucleon-nucleon interaction," Swiss Institute for Nuclear Research, Preprint No. PR-86-06.
Satz, H., 1985, Annu. Rev. Nucl. Part. Sci. **35**, 245.
Schuh, A., H. J. Pirner, and L. Wilets, 1986, Phys. Lett. B **174**, 10.
Shimodaya, I., R. Tamagaki, and H. Tanaka, 1962, Prog. Theor. Phys. **27**, 793.
Silvestre-Brac, B., J. Carbonell, and C. Gignoux, 1986, Phys. Lett. B **179**, 9.
Simonov, Yu., 1981, Phys. Lett. B **107**, 1.
Simonov, Yu., 1984, Nucl. Phys. A **416**, 109c.
Spitz, G., H. Klar, and E. W. Schmid, 1985, Z. Phys. A **322**, 49.
Spitz, G., and E. W. Schmid, 1985, Few-Body Sys. **1**, 37.
Stancu, Fl., and L. Wilets, 1987, Phys. Rev. C **36**, 726.
Storm, M. H., and A. Watt, 1983, Nucl. Phys. A **408**, 397.
Strobel, G. L., K. Bräuer, A. Faessler, and F. Fernandez, 1985, Nucl. Phys. A **437**, 605.
Su, R.-K., and E. M. Henley, 1986, Nucl. Phys. A **452**, 633.
Suzuki, A., Y. Nogami, and N. Ohtsuka, 1983, Nucl. Phys. A **395**, 301.
Suzuki, Y., 1985, Nucl. Phys. A **444**, 637.
Suzuki, Y., and K. T. Hecht, 1983, Phys. Rev. C **27**, 299.
Suzuki, Y., and K. T. Hecht, 1984, Nucl. Phys. A **420**, 525.
Szymacha, A., and S. Tatur, 1981, Z. Phys. C **7**, 311.
Takeuchi, S., and K. Shimizu, 1986, Phys. Lett. B **179**, 197.
Takeuchi, S., K. Shimizu, and K. Yazaki, 1986, Nucl. Phys. A **449**, 617.
Takeuchi, S., and K. Yazaki, 1985, Nucl. Phys. A **438**, 605.
Tamagaki, R., 1967, Rev. Mod. Phys. **39**, 629.
Tamagaki, R., and H. Tanaka, 1965, Prog. Theor. Phys. **34**, 191.
Tasso Collaboration, Althoff, M., *et al.*, 1984, DESY-Report No. 84-001.
Théberge, S., A. W. Thomas, and G. A. Miller, 1980, Phys. Rev. D **22**, 2838; **23**, 2106(E) (1981).
Thirring, W., 1965, Acta Phys. Austriaca Suppl. **2**, 205.
Thomas, A. W., 1981, J. Phys. G **7**, L283.
Thomas, A. W., 1983, Adv. Nucl. Phys. **13**, 1.
Toki, H., 1980, Z. Phys. A **294**, 173.
Veit, E. A., B. K. Jennings, and A. W. Thomas, 1986, Phys. Rev. D **33**, 1859.
Vento, V., 1980, Ph.D. thesis (SUNY, Stony Brook).
Vento, V., 1981, Phys. Lett. B **107**, 5.
Vento, V., M. Rho, and G. E. Brown, 1981, Phys. Lett. B **103**, 285.
Vento, V., M. Rho, E. M. Nyman, J. H. Hun, and G. E. Brown, 1980, Nucl. Phys. A **345**, 413.
Vinh Mau, R., 1979, in *Mesons in Nuclei,* edited by M. Rho and D. H. Wilkinson (North-Holland, Amsterdam), p. 151.
Vinh Mau, R., 1980, Nucl. Phys. A **335**, 211.
Vinh Mau, R., J. M. Richard, B. Loiseau, M. Lacombe, and W. N. Cottingham, 1973, Phys. Lett. B **44**, 1.
Wakamatsu, M., R. Yamamoto, and Y. Yamauchi, 1984, Phys. Lett. B **146**, 148.
Wang, F., and C. W. Wong, 1985, Nucl. Phys. A **438**, 620.
Warke, C. S., and R. Shanker, 1979, Phys. Lett. B **89**, 17.
Warke, C. S., and R. Shanker, 1980, Phys. Rev. C **21**, 2643.
Weber, H. J., 1980, Z. Phys. A **297**, 271.
Weinberg, S., 1968, Phys. Rev. **166**, 1568.
Weise, W., 1984, in *Quarks and Nuclei,* International Review of Nuclear Physics Series Vol. 1, edited by W. Weise (World Scientific, Singapore), p. 57.
Wheeler, J. A., 1937a, Phys. Rev. **52**, 1083.
Wheeler, J. A., 1937b, Phys. Rev. **52**, 1107.
Wigner, E. P., 1937, Phys. Rev. **51**, 106.
Wildermuth, K., and T. Kanellopoulos, 1958, Nucl. Phys. **7**, 150.
Wildermuth, K., and T. Kanellopoulos, 1959, Nucl. Phys. **9**, 449.
Wildermuth, K., and Y. C. Tang, 1977, *A Unified Theory of the Nucleus* (Academic, New York).
Williams, S. A., F. J. Margetan, P. D. Morley, and D. L. Pursey, 1982, Phys. Rev. Lett. **49**, 771.
Wong, C. W., 1975, Phys. Rep. **15**, 283.
Wong, C. W., 1982, Prog. Part. Nucl. Phys. **8**, 223.
Wroldsen, J., and F. Myhrer, 1984, Z. Phys. C **25**, 59.
Wu, S. L., 1984, Phys. Rep. **107**, 59.
Yamauchi, Y., and M. Wakamatsu, 1986a, Nucl. Phys. A **457**, 621.
Yamauchi, Y., and M. Wakamatsu, 1986b, Phys. Lett. B **172**, 161.
Yamauchi, Y., R. Yamamoto, and M. Wakamatsu, 1985, Nucl. Phys. A **443**, 628.
Yazaki, K., 1984, Nucl. Phys. A **416**, 87.
Yukawa, H., 1935, Proc. Phys. Math. Soc. Jpn. **17**, 48.
Zahed, I., and G. E. Brown, 1986, Phys. Rep. **142**, 1.
Zhang, Zhong-Ye, K. Bräuer, A. Faessler, and K. Shimizu, 1985, Nucl. Phys. A **443**, 557.
Zweig, V., 1964, CERN Reports Nos. 8182/TH 401 and 8419/TH 412.