SU(6) violations due to one-gluon exchange

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The one-gluon-exchange corrections to the baryon magnetic moments and the weak semileptonic decays are shown to have the correct two-body operator in order to explain recent data. An explicit model calculation using a mode sum for the quark propagator is then performed. In this model calculation the two lowest states dominate the corrections. This value of SU(6) breaking explains the measured ratio $\Sigma^- \rightarrow ne\bar{\nu}/\Lambda \rightarrow pe\bar{\nu}$ as well as why $\mu_{\Xi^-} < \mu_\Lambda$ and it restores $\mu_p/\mu_n \simeq -\frac{3}{2}$ in chiral bag models.

I. INTRODUCTION

The pioneering articles on hadron spectroscopy of De Rújula, Georgi and Glashow[1] and DeGrand, Jaffe, Johnson, and Kiskis[2] showed that one-gluon exchange can be used to get a satisfactory description of most of the low-lying qqq and $q\bar{q}$ states. Since this time there have been attempts to calculate the effect of the same types of exchange on hadron properties different from the one on masses, such as magnetic moments μ and axial charges g_A (Refs. 3 and 4). Results have differed, however, and have not been widely recognized.

Recently Ushio and Konashi[5] and Ushio[6,7] have published three articles where they calculate the one-gluon corrections to μ and g_A in the same semiclassical approximation that was used in Refs. 1 and 2. The results are so nice and resolve so many difficulties in hadron physics through the specific breaking of SU(6) symmetry of baryon wave functions that they merit a closer scrutiny and discussion. Before doing this we shall, however, make some general arguments concerning the color-magnetic interaction and the correction to observables coming from one-gluon exchange. This will be presented in Sec. II. Kobzarev *et al.*[8] have independently calculated this correction to the magnetic moments and they obtained a result which is a factor $\frac{1}{4}$ compared to Ushio's calculation.[6] To resolve this discrepancy we have calculated in Sec. III the gluon radiative corrections to the magnetic moments μ and the axial-vector couplings g_A. This calculation is also warranted due to the precise measurements of μ and of g_A. We confirm Ushio's results for the magnetic correction $\delta\mu$ and also for δg_A. The corrections $\delta\mu$ and δg_A and their relative signs are exactly what is necessary in order to explain on the one hand that the magnetic moment for Ξ^- which is more negative than that of the Λ (Ref. 9), and, on the other hand, that the measured ratio $\Sigma^- \rightarrow ne\bar{\nu}/\Lambda \rightarrow pe\bar{\nu}$ is $\simeq 2$ (Ref. 10) and not 3 as predicted from using SU(6) baryon wave functions.

II. THE (COLOR-MAGNETIC) ONE-GLUON-EXCHANGE CORRECTION

Independently of the model one uses for bound quarks, relativistic or nonrelativistic, the coupling of a (magnetic) gluon to a quark i is in color-spin space given by an operator of the form $b(i)\boldsymbol{\sigma}(i)\lambda^a(i)$ when the spatial degrees of freedom are integrated out. The tree diagrams due to color-magnetic gluons between quark i and j will be of the form[1,2]

$$H' = \sum_{i<j} C_{ij}\lambda^a(i)\lambda^a(j)\boldsymbol{\sigma}(i)\cdot\boldsymbol{\sigma}(j) . \tag{1}$$

The constants C_{ij} are in general flavor dependent, and most importantly model dependent, since they are functions of the quark masses and also include the spatial integration of products of quark wave functions.

Suppose we want to find the color magnetic corrections to an operator corresponding to a physical observable which is a color singlet. The operator $O(j)$ acts on a quark j in spin-flavor space. Its matrix elements in a baryon built up of quarks is $\langle B' \mid \sum_j O(j) \mid B \rangle$ where $\mid B' \rangle$ and $\mid B \rangle$ denote the baryon flavor–spin-color states. Let us denote by $\mid B_0 \rangle$ and $\mid B_0' \rangle$ the three-quark baryon states where the color-magnetic interaction is *not* included. We examine the correction to these when color magnetism is included to order α_s only.

The correction must then be of the form (see Fig. 1)

$$\mid \delta B \rangle = \sum_n \mid n \rangle \left\langle n \left| \sum_{i,j} a_{ij}\lambda^a(i)\lambda^a(j) \times \boldsymbol{\sigma}(j)^{[q,1/2]}\cdot\boldsymbol{\sigma}(i)^{[p,1/2]} \right| B_0 \right\rangle . \tag{2}$$

In this expression the spin matrices $\boldsymbol{\sigma}(i)^{[p,1/2]}$ are transition matrices in spin space corresponding to the transition from a quark state with angular momentum $\frac{1}{2}$ to angular momentum p (Ref. 11), the transition being caused by the (magnetic) gluon absorbed or emitted by the quark i. The set $|n\rangle$ is a complete set of states and the coefficients a_{ij} contain integrals over all spatial degrees of freedom and the energies of the quarks in $|B\rangle$ and $|n\rangle$.

To first order in the magnetic gluon exchange the baryon wave function will be (in color-flavor spin space)

$$|B\rangle=\frac{1}{N}(|B_0\rangle+|\delta B\rangle)$$
$$=\left[|B_0\rangle+\sum_{n\neq B_0}\sum_{i,j}a_{ij}|n\rangle\langle n|\lambda^a(i)\lambda^a(j)\sigma(i)^{[q,1/2]}\sigma(j)^{[p,1/2]}|B_0\rangle\right]\frac{1}{N}\ , \tag{3}$$

where N is such that $\langle B|B\rangle=1$ and the correction to the observable O to first order in α_s will be

$$\delta O=\frac{1}{N}(\langle B_0|O|\delta B\rangle+\langle\delta B|O|B_0\rangle)\ . \tag{4}$$

Summing over all states $|n\rangle$ and the color operators one gets, in spin-flavor space,

$$\delta O=\left\langle B_0'\left|\sum_{i,j}b(i,j)\{O(i),\sigma(i)\cdot\sigma(j)\}_+\right|B_0\right\rangle\ . \tag{5}$$

Here the sum over i and j is over the valence quarks in baryons $|B_0\rangle$ and $|B_0'\rangle$ and the coefficients $b(i,j)$ contain all integrals relevant to quark excitation and (single) pairs created in the transition to the intermediate states $|n\rangle$.

The coefficients $b(i,j)$ are model dependent and depend furthermore in general on the flavor of quarks i and j. In the flavor-symmetric limit $b(i,j)=b_0$. When we discuss color-magnetic corrections we will mostly stay in the flavor-symmetric limit and we shall always restrict ourselves to tree diagrams ($i\neq j$). In this limit

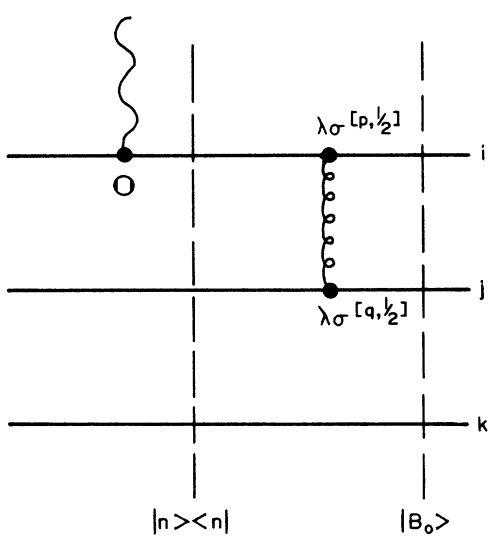

FIG. 1. The one-gluon-exchange correction diagram. The wavy line acts with the operator O on quark i which is correlated with quark j through a gluon exchange. The spin-transition operators take quark i from an angular momentum $J=\frac{1}{2}$ to a $J=p$ state and quark j from a $J=\frac{1}{2}$ to a $J=q$ state. See the text for details.

$$\langle O\rangle=\frac{1}{N}\left[\left\langle B_0'\left|\sum_i O(i)\right|B_0\right\rangle\right]$$
$$+b_0\left\langle B_0'\left|\sum_{i\neq j}\{O(i),\sigma(i)\cdot\sigma(j)\}\right|B_0\right\rangle \tag{6}$$

and it goes without saying that the value of the number b_0 dependents on which operator O we study.

In Sec. III we shall discuss a particular model (MIT) which gives very reasonable values for the corrections to μ and g_A. However, below we shall demonstrate in two examples how well the correction formula, Eq. (6), could explain problem cases of hadron structure. First we discuss the ratio of the proton and neutron magnetic moments and other magnetic moments. Second we discuss the axial charge ratios of the semileptonic decays $\Sigma^-\to ne\bar{\nu}$ and $\Lambda\to pe\bar{\nu}$. In both cases, it makes sense to use flavor-symmetry arguments in the correction term, Eq. (6).

Experimentally $\mu_p/\mu_n=-1.46$ and the closeness of this to -1.5 was one of the triumphs of SU(6) or the additive quark model with three valence quarks. However, recently experiments have found that $\mu(\Xi^-)<\mu(\Lambda)$ whereas the additive quark model predicts $\mu(\Lambda)<\mu(\Xi^-)$. We will discuss the proton-neutron ratio first.

The magnetic operator (here $O=\boldsymbol{\mu}$) in SU(6) is a sum of one-body quark operators:

$$\boldsymbol{\mu}=\sum_i q(i)\sigma(i)\mu(i)\ . \tag{7}$$

The ratio of the proton and neutron magnetic moments is then

$$\frac{\mu_p}{\mu_n}=-\frac{3}{2}\ . \tag{8}$$

In the constituent-quark model $\mu(i)=1/2m(i)$ with $m(i)$ being the constituent-quark mass. In bag models $\mu(i)$ is calculated from quark wave-function integrals and is a function of the bag radius. Both models preserve the $-\frac{3}{2}$ ratio.

The trouble comes when models are made more sophisticated by enforcing chiral symmetry which naturally adds pionic corrections to the magnetic moments (chiral bag models).[12,13] As these corrections are isovector in nature one gets ($\mu_q=u,d$ magnetic moment)

$$\mu_p=\mu_q+\delta\mu_\pi\ , \tag{9a}$$

$$\mu_n=-\frac{2}{3}\mu_q-\delta\mu_\pi\ , \tag{9b}$$

where the first contribution is from quarks and the second is the pion cloud contribution to the magnetic moment. Theoretically then

$$\mu_p/\mu_n = -\tfrac{3}{2}[(1+\delta\mu_\pi/\mu_q)/(1+3\delta\mu_\pi/2\mu_q)] \ . \quad (10)$$

If we use the experimental ratio in Eq. (10) we find

$$\frac{\delta\mu_\pi}{\mu_q} \simeq 0.06 \ ; \quad (11)$$

i.e., the pion contribution is very small contrary to the results of the chiral quark models.[12,13] One way out of this problem has been to introduce (*ad hoc*) isoscalar contributions to the magnetic moment operator.[14] However, with one-gluon corrections one has the additional correction term

$$\delta\mu = b_0\left\langle N \left| \sum_{i\neq j} \{q(i)\sigma(i)\mu(i), \sigma(i)\cdot\sigma(j)\}_+ \right| N \right\rangle \ . \quad (12)$$

For the proton then $\mu_p = \mu_q + \delta\mu_\pi$ as the correction $\delta\mu$ of Eq. (12) is zero. For the neutron $\mu_n = -\frac{2}{3}\mu_q - \delta\mu_\pi + \frac{2}{3}b_0\mu_q = -\frac{2}{3}\mu_q - \delta\mu_\pi + \frac{2}{3}C'$ where $C' = b_0\mu_q$. Clearly, if the constant C' is positive and $C' \simeq \frac{1}{2}\delta\mu_\pi$ one can accommodate a substantial pion contribution to the nucleon magnetic moment without spoiling agreement with observations. As we shall show the MIT model gives a positive C', see also Refs. 6 and 8. A specific example for all baryons is given in Table I where we also see that $\mu_{\Xi^-} < \mu_\Lambda$ because the one-gluon-exchange correction C' contributes in the correct manner to make μ_{Ξ^-} more negative and μ_Λ less negative (the pionic corrections to μ_{Ξ^-} are tiny[13,15]).

Next we will discuss the axial-vector coupling and concentrate on the ratio of the semileptonic decays:

$$[g_A/g_V(\Sigma^- \to ne\bar{\nu})]/[g_A/g_V(\Lambda \to pe\bar{\nu})] = X \ . \quad (13)$$

The axial operator here is $O = \mathbf{A}$. For zero momentum transfer $\mathbf{A}(0) = g_A$. This axial-vector coupling constant is in SU(6) a sum of one-body quark operators:

$$g_A = \sum_i \tau(i)\sigma_z(i)g_A(i) \ . \quad (14)$$

Here $\tau(i)$ is an operator which changes an s quark into a u quark ($\Delta S = 1$ decay) or a d quark into a u quark ($\Delta S = 0$ decay). In the constituent-quark model $g_A(i) = 1$ and in bag models $g_A(i)$ is calculated from quark wave-function integrals (see below). Computing the ratio, Eq. (13) with SU(6) spin-flavor wave functions one finds $X = -\frac{1}{3}$ where as experimentally the ratio $X \simeq -\frac{1}{2}$ (Ref. 10). Since the Λ and Σ^- have almost the same mass one would believe that the wave functions of the single strange quark in the two particles are very similar. Furthermore (in the isospin limit) all quarks in the final state should have the same spatial wave function whether it is a neutron or a proton as argued by Eeg, Høgaasen, and Lie-Svendsen.[16] This means that theoretically one finds the result $X \simeq -\frac{1}{3}$, which is also close to the result when the recoil corrections are properly included.[17] The correction due to the magnetic gluon exchange is, however, different in the two cases; it is zero for $\Lambda \to pe\bar{\nu}$ whereas the correction δg_A to $\Sigma^- \to ne\bar{\nu}$ is the largest among the weakly decaying baryon octet. The correction, using Eq. (6), becomes, in the flavor-symmetry limit,

$$\delta g_A = c'' \sum_{i\neq j} \langle B' \mid \tau(i)\sigma(j) \mid B \rangle \quad (15)$$

and for the ratio we find

$$X = -\tfrac{1}{3} + \tfrac{4}{3}c'' \ . \quad (16)$$

The constant c'' turns out to be negative in the MIT model as we will show, see also (Refs. 5 and 7), and just the right magnitude to reproduce the measured semileptonic decays which as discussed deviate from the SU(6) relations. We refer also to the discussion in Ref. 17; see also the attempt to explain this in the cloudy-bag model.[18]

TABLE I. Baryon magnetic moments in a chiral bag model where both pionic and one-gluon-exchange (OGE) corrections Eq. (12) are included. The quark magnetic moment (quark charges counted separately) are $\mu_q = \mu_u = \mu_d \simeq 2.2$ and $\mu_s = 1.9$. We here will use $C' = 0.20$, see Sec. III or Ref. 6 and $\delta\mu_\pi = 0.59$ which is not an unreasonable value in the chiral bag (Refs. 13 and 15); see also Ref. 24. All numbers are in units of μ_N.

Baryon	Quark	Pion	OGE	Magnetic moment	Experiment
μ_p	μ_q	$\delta\mu_\pi$	0	2.79	2.79
μ_n	$-\frac{2}{3}\mu_q$	$-\delta\mu_\pi$	$\frac{2}{3}C'$	−1.92	−1.91
μ_{Σ^+}	$\frac{8}{9}\mu_q + \frac{1}{9}\mu_s$	$\frac{1}{2}\delta\mu_\pi^*$	0	2.46[a]	2.38±0.02
μ_{Σ^-}	$-\frac{4}{9}\mu_q + \frac{1}{9}\mu_s$	$-\frac{1}{2}\delta\mu_\pi^*$	$-\frac{2}{3}C'$	−1.20[a]	−1.14±0.05
μ_{Ξ^0}	$-\frac{2}{9}\mu_q - \frac{4}{9}\mu_s$	$\simeq 0$	$\frac{2}{3}C'$	−1.20	−1.25±0.01
μ_{Ξ^-}	$\frac{1}{9}\mu_q - \frac{4}{9}\mu_s$	$\simeq 0$	$-\frac{2}{3}C'$	−0.73	−0.69±0.04
μ_Λ	$-\frac{1}{3}\mu_s$	0	$\frac{1}{3}C'$	−0.61[b]	−0.61

[a]These numbers should be reduced somewhat in a chiral bag model (Ref. 13).
[b]This includes −0.04 from Σ^0, Λ wave-function mixing (Ref. 22).

III. CALCULATION OF THE GLUONIC CORRECTIONS IN THE BAG MODEL

It is remarkable that the MIT bag gives, according to Ushio and Konashi,[5-7] the correct relative sign for $\delta\mu$ and δg_A and also approximately the absolute size to solve the SU(6) problems that we have discussed. However, as mentioned these gluon-radiative corrections from tree diagrams have not been widely recognized. Kobzarev *et al.*[8] have also calculated $\delta\mu$ and they seem to find a much smaller result so obviously Ushio's $\delta\mu$ calculation[6] has to be examined.

In what follows we give some details of our calculations. In Fig. 2 we display the Feynman diagrams we calculate using the usual semiclassical technique.[2,19] We calculate only the magnetic one-gluon-exchange corrections since the electric ones involve loop diagrams (namely, the "minimal" self-energy and vertex correction diagrams in order to satisfy the boundary condition for the color Coulomb field[2,5-7]). For the electric one-gluon-exchange terms there are cancellations between the different intermediate quark propagator modes for δg_A (Ref. 7). The parameters in the bag model are phenomenologically determined and we know the loop (self-energy) diagrams give large bag-radius-dependent mass corrections δm (Ref. 20). In addition one has partial cancellations among large-loop terms in radiative corrections.[4,7] Therefore we choose to ignore these contributions to δg_A (Ref. 7) (they only give tiny contributions to $\delta\mu$ according to Ushio[6]).

In calculating the diagrams in Fig. 2 we use the mode sum for the intermediate quark propagating from the vertex with coordinates $r(=\mathbf{r},t)$ to the vertex with coordinates r':

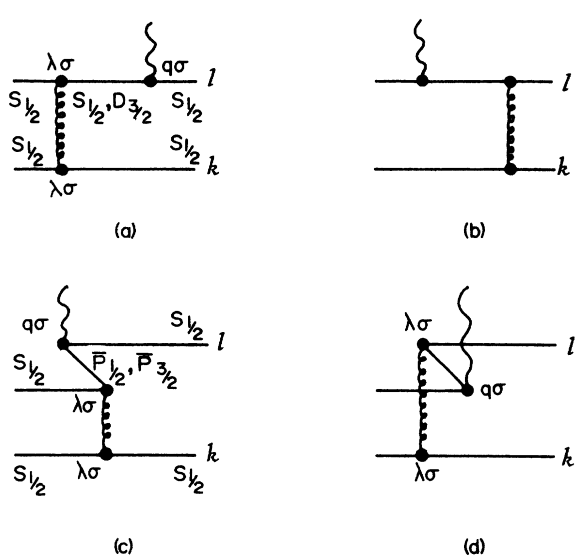

FIG. 2. Illustrations of the different Feynman tree diagrams which contribute to the magnetic moment corrections. Diagrams (a) and (b) contain intermediate three-quark states whereas (c) and (d) contain intermediate four-quark–one-antiquark states.

$$iS_F(r,r')=\sum_M [u_M(r)\bar{u}_M(r')e^{-iE_M(t-t')}\theta(t-t') - v_M(r)\bar{v}_M(r')e^{iE_M(t-t')}\theta(t'-t)] ,$$

where we sum over all intermediate quark (antiquark) states M (shorthand for $nljm$). The time integrations (all time orderings) give the energy denominators $\Delta\omega$ of the different diagrams to be specified later.

The color-magnetic transition energy implied in Fig. 2 is given by

$$H_{\rm CM}(M)=-\sum_{k\neq l}\int_{\rm bag}\mathbf{A}_k^a\cdot\mathbf{j}_l^a(M)d^3r \; , \tag{17}$$

where the sum k,l includes all three quarks in the baryon. The color vector field $\mathbf{A}_k^a$ is generated by the $S_{1/2}$-state quark (lower vertex) with the wave function

$$\psi=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} iF(r) \\ G(r)\sigma\cdot\hat{\mathbf{r}}\end{bmatrix}\chi^{[1/2]} \tag{18}$$

and is

$$\mathbf{A}_k^a(\mathbf{r})=\frac{g}{4\pi}\frac{\lambda^a(k)}{2}[\sigma(k)\times\hat{\mathbf{r}}]f_k(r) \; , \tag{19}$$

where

$$f_k(r)=rM_k(r)+\mu_k(r)/r^2+r\mu_k(R)/2R^3 \; . \tag{20a}$$

In the MIT bag[2] the upper and lower Dirac components are $F(r)=-Nj_0(\omega_s r)$ and $G(r)=Nj_1(\omega_s r)$ where the quark energy $\omega_s=2.043/R$ (for massless quarks) and the normalization N is determined by

$$\int dr\, r^2(|F|^2+|G|^2)=1 \; . \tag{20b}$$

In Eq. (20a) $\mu(R)$ is the magnetic moment for an $S_{1/2}$ state quark in a bag of radius R and

$$\mu(r)=-\tfrac{1}{3}\int_0^r dr' r'^3[F^*(r')G(r')+F(r')G^*(r')]$$

and

$$M(r)=-\tfrac{1}{3}\int_r^R dr'[F^*(r')G(r')+F(r')G^*(r')] \; .$$

We first consider diagrams 2(a) and 2(b) with the intermediate quark $M=S'_{1/2}$ and $D_{3/2}$. The transition current of quark l, $j_l^a(M)$, from the $S_{1/2}$ to the M state at the upper gluon vertex in Figs. 2(a) and 2(b) is constructed using the definitions of Eeg and Wroldsen:[11]

$$\psi_M(\mathbf{r},t)=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} iF_M(r) \\ G_M(r)\sigma\cdot\hat{\mathbf{r}}\end{bmatrix}\phi_M e^{-i\omega_M t} \; , \tag{21}$$

where ϕ_M is short for

$$\phi(slj;m)=\sum_{m_l,m_s} C^{l\ s\ j}_{m_l\ m_s\ m}Y_l^{m_l}(\hat{r})\chi_{m_s}^{[s]} \; . \tag{22}$$

Here $C^{l\ s\ j}_{m_l\ m_s\ m}$ are the usual Clebsch-Gordan coefficients and, for example, $s=\frac{1}{2}$ gives $\chi_{+1/2}^{[1/2]}=\binom{1}{0}$ and $\chi_{-1/2}^{[1/2]}=\binom{0}{1}$ which are the usual Pauli spin matrices. We use here the multipole transition matrices (MTM) technique of Ref. 11 to evaluate Eq. (17) for the two possible intermediate angular momentum states $j=\frac{1}{2}$ and $\frac{3}{2}$. We find ($\alpha_s=g^2/4\pi$) for $M=S'_{1/2}$, the first excited S state:

$$H_{\rm CM}(S'_{1/2})=\frac{\alpha_s}{6}\sum_{l\neq k}\lambda^a(l)\lambda^a(k)\boldsymbol{\sigma}(l)\cdot\boldsymbol{\sigma}(k)R_\Delta(S')\ , \qquad (23)$$

where

$$R_\Delta(S')=\int_0^R dr\, r^2 f(r)[F^*_{S'}(\omega_{s'}r)G_s(\omega_s r)+G^*_{s'}(\omega_{s'}r)F_s(\omega_s r)]\ . \qquad (24)$$

Here $\omega_{s'}=5.40/R$ (massless quarks) is the energy of the first excited $S_{1/2}$ state in the bag.

For $M=D_{3/2}$ we find using the MTM techniques

$$H_{\rm CM}(D_{3/2})=\frac{\alpha_s}{4}\sum_{k\neq l}\lambda^a(k)\lambda^a(l)\chi^{[1/2]\dagger}\times\boldsymbol{\sigma}(k)\cdot\boldsymbol{\sigma}(l)^{[1/2,3/2]}\chi^{[3/2]}R_\Delta(D)\ , \qquad (25)$$

where $\sigma^{[1/2,3/2]}$ are the 2×4 dipole transition matrices which transform as vectors[11] and

$$R_\Delta(D)=\int_0^R dr\, r^2 f(r)[F_s(\omega_s r)G^*_D(\omega_D r)+G_s(\omega_s r)F_D(\omega_D r)]\ . \qquad (26)$$

Here $\omega_D=5.12/R$ (massless quarks) is the energy of the lowest $D_{3/2}$ state in the bag and for the MIT bag $F_D(\omega_D r)=N_D j_2(\omega_D r)$ and $G_D(\omega_D r)=N_D j_1(\omega_D r)$ where N_D is determined by Eq. (20b).

Next we have to calculate the transition from quark state M and the $S_{1/2}$ state for quark l which depends on the wavy line in Fig. 2 being a vector (a photon) for $\delta\mu$ or an axial vector (a W particle) for δg_A. We find

$$\delta\mu(S')=-\frac{\alpha_s}{18}\sum_{l\neq k}\lambda^a(l)\lambda^a(k)q(l)[\boldsymbol{\sigma}(k)\cdot\boldsymbol{\sigma}(l)]\sigma_z(l)\times\frac{R_\Delta(S')R_M(S')}{\Delta\omega(S')}\ , \qquad (27)$$

where $q(l)$ is the electric charge of quark l. Here

$$R_M(S')=\int_0^R dr\, r^3[F^*_s(\omega_s r)G_{s'}(\omega_{s'}r)+F_{s'}(\omega_{s'}r)G^*_s(\omega_s r)] \qquad (28)$$

and the energy denominator is

$$\Delta\omega(S')=-E_{S'_{1/2}}+E_{S_{1/2}}=(-5.40+2.04)/R$$

for a massless quark in the MIT bag. The correction to the axial-vector coupling for $M=S'_{1/2}$ is

$$\delta g_A(S')=\frac{\alpha_s}{6}\sum_{l\neq k}\lambda^a(l)\lambda^a(k)\tau(l)[\boldsymbol{\sigma}(k)\cdot\boldsymbol{\sigma}(l)]\sigma_z(l)\times\frac{R_\Delta(S')R_A(S')}{\Delta\omega(S')}\ , \qquad (29)$$

where

$$R_A(S')=\int_0^R dr\, r^2[F^*_s(\omega_s r)F_{s'}(\omega_{s'}r)-G^*_s(\omega_s r)G_{s'}(\omega_{s'}r)/3]\ . \qquad (30)$$

The flavor operator $\tau(l)$ is defined earlier, Eq. (14). Similarly we find, for $M=D_{3/2}$,

$$\delta\mu(D)=-\frac{\alpha_s}{24}\sum_{l\neq k}\lambda^a(l)\lambda^a(k)q(l)\times(\boldsymbol{\sigma}(k)\cdot\boldsymbol{\sigma}(l)^{[1/2,3/2]})\sigma_z(l)^{[3/2,1/2]}\times\frac{R_\Delta(D)R_M(D)}{\Delta\omega(D)}\ , \qquad (31)$$

where $\Delta\omega(D)=-E_{D_{3/2}}+E_{S_{1/2}}$ [$=(-5.12+2.04)/R$ for massless quark in an MIT bag] and

$$R_M(D)=\int_0^R dr\, r^3[F^*_s(\omega_s r)G_D(\omega_D r)+G^*_s(\omega_s r)F_D(\omega_D r)]\ , \qquad (32)$$

$$\delta g_A(D)=-\frac{\alpha_s}{6}\sum_{l\neq k}\lambda^a(l)\lambda^a(k)\tau(l)\times[\boldsymbol{\sigma}(k)\cdot\boldsymbol{\sigma}(l)^{[1/2,3/2]}]\sigma_z(l)^{[3/2,1/2]}\times R_\Delta(D)R_A(D)/\Delta\omega(D)\ , \qquad (33)$$

where

$$R_A(D)=\int_0^R dr\, r^2 G^*_s(\omega_s r)G_D(\omega_D r)\ .$$

Next we consider diagrams 2(c) and 2(d) where we again use Eq. (17) and then evaluate the relevant operators at the second vertex for quark l, namely, the vector or axial-vector probe. We calculate the diagrams directly, and independently check our calculation using the substitution rule of Close and Monaghan.[21] The intermediate antiquark can be in $P_{1/2}$ or $P_{3/2}$ states which gives the following correction to the magnetic moment.

For the $P_{1/2}$ state ($P\equiv P_{1/2}$) we have

$$\delta\mu(P)=-\frac{\alpha_s}{18}\sum_{k\neq l}\lambda^a(k)\lambda^a(l)q(l)[\boldsymbol{\sigma}(k)\cdot\boldsymbol{\sigma}(l)]\sigma_z(l)\times\frac{R_M(P)R_\Delta(P)}{\Delta\omega(P)}\ , \qquad (34)$$

where $\Delta\omega(P)=\omega_{P_{1/2}}+3\omega_{S_{1/2}}-2\omega_{S_{1/2}}$ [$=(3.81+2.04)/R$ for massless quarks in the MIT bag] and

$$R_M(P)=\int_0^R dr\, r^3[F^*_s(\omega_s r)F^*_p(\omega_p r)+G^*_s(\omega_s r)G^*_p(\omega_p r)]\ . \qquad (35)$$

Further

$$R_\Delta(P)=\int_0^R dr\, r^2 f(r)[F_s(\omega_s r)F_p(\omega_p r)+G_s(\omega_s r)G_p(\omega_p r)]\ , \qquad (36)$$

where $\omega_p=3.81/R$ for a massless quark in an MIT bag and F_p and G_p are the upper and lower Dirac component for the quark (*not* antiquark) $P_{1/2}$ wave function. For massless quarks in the MIT bag $F_p(\omega_p r)=N_p j_1(\omega_p r)$ and $G_p(\omega_p r)=N_p j_0(\omega_p r)$. For the $P_{3/2}$ state we have $(P'\equiv P_{3/2})$

$$\delta\mu(P')=-\frac{\alpha_s}{24}\sum_{k\neq l}\lambda^a(k)\lambda^a(l)q(l)\times[\sigma(k)\cdot\sigma(l)^{[1/2,3/2]}]\sigma_z(l)^{[3/2,1/2]}\times R_M(P')R_\Delta(P')/\Delta\omega(P')\ , \qquad (37)$$

where $\Delta\omega(P')=\omega_{P_{3/2}}+3\omega_{S_{1/2}}-2\omega_{S_{1/2}}$ with

$$R_M(P')=\int_0^R dr\, r^3[F_s^*(\omega_s r)F_{p'}^*(\omega_{p'} r)+G_s^*(\omega_s r)G_{p'}^*(\omega_{p'} R)] \qquad (38)$$

and

$$R_\Delta(P')=\int_0^R dr\, r^2 f(r)[G_{p'}(\omega_{p'} r)G_s(\omega_s r)+F_{p'}(\omega_{p'} r)F_s(\omega_s r)]\ , \qquad (39)$$

where $\omega_{p'}=\omega_{P_{3/2}}=3.20/R$ for a massless quark in the MIT bag, and again $F_{p'}$ and $G_{p'}$ are the upper and lower Dirac components of the quark (*not* antiquark) $P_{3/2}$ wave function. For massless quarks in the MIT bag $F_{p'}=-N_{p'}j_1(\omega_{p'} r)$ and $G_p=N_{p'}j_2(\omega_{p'} r)$.

The results for the axial-vector coupling are

$$\delta g_A(P)=-\frac{\alpha_s}{6}\sum_{k\neq l}\lambda^a(k)\lambda^a(l)\tau(l)[\sigma(k)\cdot\sigma(l)]\sigma_z(l)\times R_A(P)R_\Delta(P)/\Delta\omega(P)\ , \qquad (40)$$

where

$$R_A(P)=\int_0^R dr\, r^2[-F_s^*(\omega_s r)G_p^*(\omega_p r)+G_s^*(\omega_s r)F_p^*(\omega_p r)/3]\ , \qquad (41)$$

$$\delta g_A(P')=-\frac{\alpha_s}{6}\sum_{k\neq l}\lambda^a(k)\lambda^a(l)\tau(l)\times[\sigma(k)\cdot\sigma(l)^{[1/2,3/2]}]\sigma_z(l)^{[3/2,1/2]}\times R_A(P')R_\Delta(P')/\Delta\omega(P')\ , \qquad (42)$$

where

$$R_A(P')=\int_0^R dr\, r^2 G_s^*(\omega_s r)F_{p'}^*(\omega_{p'} r)\ . \qquad (43)$$

To calculate the operators we use the technique of Ref. 11 and find (for massless quarks), when we add all time orderings implied by Fig. 2,

$$\{\sigma(k)\cdot\sigma(l)^{[1/2,3/2]},\sigma_z(l)^{[3/2,1/2]}\}_+=\tfrac{4}{3}\sigma_z(k) \qquad (44)$$

and further we have

$$\{\sigma(k)\cdot\sigma(l),\sigma_z(l)\}_+=2\sigma_z(k)\ . \qquad (45)$$

This means we can summarize the results as follows [using $\lambda^a(k)\lambda^a(l)=-\frac{8}{3}$]:

$$\delta g_A(M)=\sum_{k\neq l}\sigma_z(k)\tau(l)\Delta g_A(M) \qquad (46)$$

and

$$\delta\mu(M)=\sum_{k\neq l}\sigma_z(k)q(l)\Delta\mu(M)\ . \qquad (47)$$

The results for $\Delta g_A(M)$ and $\Delta\mu(M)$ are given in Table II for the lowest and the next excited quark states (with massless quarks) in the MIT bag. The higher states are more suppressed than the tabulated ones which means the mode sum converges very rapidly.

IV. DISCUSSION

From Table II we see that the intermediate $q\bar{q}$ state with the antiquark in the $P_{1/2}$ and $P_{3/2}$ state determines

TABLE II. Contributions to the baryon magnetic moments and the axial-vector coupling from one-gluon-exchange processes illustrated in Fig. 2 from the different intermediate quark modes. Only two modes of each angular momentum state are shown to indicate the rapid convergences. The bag radius $R=5$ GeV^{-1}, $\alpha_s=2.2$ and the quarks are all massless (since this is a correction term). The bars over the P states indicate an intermediate antiquark ($\bar{q}$) state. The $\Delta\mu$ are in units of μ_N.

Intermediate quark state contributing M	Intermediate quark energy	$10^4\Delta\mu$	$10^4\Delta g_A$	Intermediate quark energy M	$10^4\Delta\mu$	$10^4\Delta g_A$
$S'_{1/2}$	$5.40/R$	22	32	$8.58/R$	1.0	2.2
$D_{3/2}$	$5.12/R$	8	12	$8.41/R$	0.4	0.8
$\bar{P}_{1/2}$	$3.81/R$	730	-275	$7.00/R$	-6.7	7.0
$\bar{P}_{3/2}$	$3.20/R$	1349	-332	$6.76/R$	-6.1	6.0
Sum		2109	-563		-11.4	16.0

$\Delta\mu$ and Δg_A and also gives $\Delta\mu > 0$ and $\Delta g_A < 0$ which is essential in order for the one-gluon-exchange correction to explain the measured SU(6) violations as discussed earlier. The values for $\Delta\mu$ and Δg_A change a little when flavor symmetry is broken through a nonvanishing mass for the strange quarks. We shall here neglect this kind of adjustments to the corrections $\Delta\mu$ and Δg_A.

As seen from Table I we restore the ratio $\mu_p/\mu_n \simeq -\frac{3}{2}$ in the chiral bag models as discussed. We also make $|\mu_\Lambda| < |\mu_{\Xi^-}|$ even when we include the isospin wave-function mixing between Σ^0 and Λ (Ref. 22). And finally we increase the value of $|\mu_{\Sigma^-}|$ due to the gluon-exchange corrections, $\delta\mu$. This brings the Σ^- magnetic moments from the chiral (cloudy) bag in close agreement with the newer experiments[10] without invoking the controversial recoil/c.m. corrections[12] which should be small.[23]

The one-gluon-exchange correction to g_A in the bag, δg_A, explains the measured semileptonic decays $\Sigma^- \to n e \bar{\nu}$ and $\Lambda \to p e \bar{\nu}$ as discussed. This correction also affects g_A/g_V of the weak semileptonic decay $\Xi^- \to \Lambda e \bar{\nu}$ (Ref. 7) but we find this is still consistent with the experimentally measured value.[10]

We confirm Ushio's results for $\delta\mu$ and for Δg_A we find values which are slightly larger than Ushio and Konashi's.[5,7] As stated before we are impressed by the ability of the bag model to give such nice quantitative results for the SU(6)-breaking corrections from the tree diagrams with one-gluon exchange. We would like, however, to end with a word of caution. Because the dominant contribution to the electromagnetic and weak form factors comes from intermediate states with an extra quark-antiquark pair, the precise quantitative results coming from the use of the MIT bag model must be regarded with some skepticism as we have no experimental data where we can directly check the model in this sector. It would therefore in our opinion be permissible to do as in Sec. II and treat the one-gluon-exchange corrections as QCD-inspired phenomenology using the two-body operator structure of the correction terms

$$\delta\mu = \left\langle B \left| \sum_{i,j} C(i,j) q(i) \sigma_Z(j) \right| B \right\rangle \tag{48}$$

and

$$\delta g_A = \left\langle B' \left| \sum_{i,j} g(i,j) \tau(i) \sigma_Z(j) \right| B \right\rangle \tag{49}$$

and treat the constants $C(i,j)$ and $g(i,j)$ as parameters. It is, however, a very respectable aspect of the MIT bag model that the signs and magnitudes for these constants solve many problems in quark theory that have been with us for several years.

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