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The $\mu^- d$ capture rate in effective field theory

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Abstract

Muon capture on the deuteron is studied in a framework that essentially incorporates heavy baryon chiral perturbation theory (HBχPT). It is found that by far the dominant contribution to μd capture comes from a region of the final three-body phase-space in which the energy of the two neutrons is sufficiently small for HBχPT to be applicable. The single unknown low-energy constant having been fixed from the tritium beta decay rate, our calculation contains no free parameter. Our estimate of the μd capture rate is consistent with the existing data. The relation between μd capture and the νd reactions, which are important for the SNO experiments, is briefly discussed.

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1. Introduction

Electroweak processes in the two-nucleon systems invite detailed studies for multiple reasons. From the nuclear physics point of view, these processes offer a valuable testing ground of the basic inputs of nuclear physics. In astrophysics, the precise knowledge of the pp fusion cross section is of crucial importance for building a reliable model for stellar evolution [1]. Furthermore, experiments at the Sudbury Neutrino Observatory (SNO) [2,3] to observe solar neutrinos with a heavy-water Cerenkov counter have made it extremely important to estimate the νd reaction cross sections with high precision.

In this note we study μd capture: $\mu^- + d \to \nu_\mu + n + n$, in a formalism motivated by effective field theory (EFT). Our work is connected to the above-mentioned urgent need of accurate estimates of the νd cross sections, $\sigma_{\nu d}$. To expound this connection, we will first explain the standard nuclear physics approach (SNPA), see, e.g., Ref. [4]. This is a highly successful method for describing nuclear responses to electroweak probes. In this approach we consider one-body (1B) impulse approximation terms and two-body (2B) exchange-current terms acting on non-relativistic nuclear wave functions, with the exchange currents derived from a one-boson exchange model. The vertices in the relevant Feynman diagrams are obtained from a Lagrangian constructed to satisfy the low-energy theorems and current algebra [5], while the nuclear wave functions are generated by solving the A-body

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Schrödinger equation, $H|\Psi_A\rangle = E|\Psi_A\rangle$, where the Hamiltonian H contains realistic phenomenological nuclear potentials. The most elaborate study of μd capture based on SNPA was carried out by Tatara et al. (TKK) [6] and by Adam et al. [7].

Now, the best available estimation of $\sigma_{\nu d}$ based on SNPA is due to Nakamura et al. (NSGK) [8], while that based on EFT is due to Butler, Chen, and Kong (BCK) [9]. Since EFT is a general framework [10], it can give model-independent results, *provided* all the low-energy coefficients (LEC) in the effective Lagrangian, $\mathcal{L}_{\rm eff}$, are predetermined. $\mathcal{L}_{\rm eff}$ considered by BCK, however, does contain one unknown parameter (L_{1A}), which they adjusted to reproduce $\sigma_{\nu d}$ obtained by NSGK. After this adjustment, the results of BCK are found to be in perfect agreement with those of NSGK. The fact that an ab initio calculation (modulo one free parameter) based on EFT reproduces $\sigma_{\nu d}$ of NSGK extremely well offers strong support to the calculation based on SNPA. At the same time, it stresses the importance of carrying out an EFT calculation free from an adjustable parameter. An interesting possibility is to use μd capture data as input to control the unknown LEC. An immediate question, however, is whether this process is "gentle" enough to be amenable to EFT. The substantial energy transfer accompanying the disappearance of a muon can lead to a region of the final three-particle phase space in which the intrinsic state of the two neutrons receives such a large momentum that the applicability of EFT becomes a delicate issue. Let this unfavorable kinematical region be called the "dangerous" region. The problem of the dangerous region is reminiscent of the difficulty one encounters in applying EFT to threshold pion production in $N + N \to N + N + \pi$ [11–14]. It will turn out (see below), however, that, unlike the pion production case, μd capture receives only a tiny fraction of contribution from the dangerous region, and therefore the theoretical uncertainty caused by the dangerous region is practically negligible.

The EFT calculation in [9] used the power divergence subtraction scheme (PDS) [15]. We employ here a formalism in which the transition operators are derived from irreducible diagrams in heavy-baryon chiral perturbation theory (HBχPT), while the transition matrix elements are obtained with the use of the initial and final nuclear wave functions obtained in SNPA. For convenience, we refer to this approach as EFT*. The use of the SNPA wave functions causes some degree of deviation from genuine EFT but, as discussed in [16], EFT* can nevertheless reduce the model-dependence of SNPA drastically. According to [16], a next-to-next-to-next-to-leading order (N^{3}LO) calculation in EFT* contains one unknown LEC, denoted by $\hat{d}^R$. Like L_{1A} discussed by BCK, the parameter $\hat{d}^R$ controls the strength of a short-range exchange-current term and, once $\hat{d}^R$ is fixed from data, we can make a definite prediction for $\sigma_{\nu d}$. We, therefore, investigate here the relation between $\hat{d}^R$ and the μd capture rate, $\Gamma_{\mu d}$. Our study is essentially of exploratory nature, given the present limited accuracy (see below) of the experimental value of $\Gamma_{\mu d}$.

Another important point concerning $\hat{d}^R$ is that, as emphasized in [16], the strength of $\hat{d}^R$ can be reliably related to the tritium β-decay rate, Γ^t_β. Thus, using the experimental value of Γ^t_β, which is known with high precision, one can determine $\hat{d}^R$ and then proceed to make predictions on various two-nucleon weak-interaction processes, including the μd capture rate, pp fusion rate, and νd cross sections. We will present here the first estimate of the μd capture rate obtained in this approach; the pp fusion rate has already been discussed in [16], and the νd cross sections will be reported elsewhere [17].

2. The capture rate

Although μd capture can in principle occur from the two μd hyperfine states ($S_{\mu d} = 1/2$ and $S_{\mu d} = 3/2$), the capture is known to take place practically uniquely from the hyperfine doublet state. Therefore, concentrating on this dominant capture, we refer to hyperfine-doublet μd capture simply as μd capture and denote the hyperfine-doublet μd capture rate by $\Gamma_{\mu d}$. The measured value of $\Gamma_{\mu d}$ is $\Gamma^{\rm exp}_{\mu d} = 409 \pm 40\ {\rm s}^{-1}$ [18] and $\Gamma^{\rm exp}_{\mu d} = 470 \pm 29\ {\rm s}^{-1}$ [19]. We remark that a high-precision measurement of $\Gamma_{\mu d}$ is being contemplated at PSI [20].

Denoting by L the orbital angular momentum of the two-neutron relative motion in the final state, we can write

$$\Gamma_{\mu d} = \sum_{L=0,1,2\ldots} \Gamma^{L}_{\mu d}, \tag{1}$$

where $\Gamma^{L}_{\mu d}$ is the rate of μd capture leading to the L state. Here we shall be primarily concerned with $\Gamma^{L=0}_{\mu d}$ since it is this quantity that contains information about $\hat{d}^R$. The contributions of $\Gamma^{L}_{\mu d}$ ($L \geqslant 1$) are significant,[1] but their calculation is not expected to involve any major EFT-related issues. In general, due to the centrifugal force, the $L \geqslant 1$ contributions cannot be too sensitive to short-range physics, which implies that chiral expansion for them should converge rapidly. Specifically, the $L = 1$ contributions are dominated by the axial-charge (AC) and E1 transitions, whose one-body operators (which are NLO in chiral counting) are well known. The lowest order meson-exchange corrections (MEC) to the one-body operators come from soft one-pion-exchange (OPE), which is N^{2}LO in chiral counting. These soft-OPE terms, dictated by chiral symmetry, are well known, and they are model-independent. For $L \geqslant 2$ states, within the accuracy of our evaluation only one-body contributions [6] have to be included. In our exploratory study, therefore, we concentrate on a detailed evaluation of $\Gamma^{L=0}_{\mu d}$, and for $\Gamma^{L}_{\mu d}$ ($L \geqslant 1$) we simply use the results obtained by Ref. [6].

Muon capture by the deuteron is effectively described by the current–current Hamiltonian of weak interactions

$$H_W = \frac{G_V}{\sqrt{2}} \int d^3x\, L_\alpha(\vec{x}\,) J^\alpha(\vec{x}\,) + \text{h.c.}, \tag{2}$$

where the leptonic and the hadronic charged currents are

$$L_\alpha(\vec{x}\,) = \bar{\psi}_\nu(\vec{x}\,)\gamma_\alpha(1-\gamma_5)\psi_\mu(\vec{x}\,) \quad \text{and} \quad J^\alpha(\vec{x}\,) = \left(V^\alpha - A^\alpha\right)^{a=1}(\vec{x}\,) - i\left(V^\alpha - A^\alpha\right)^{a=2}(\vec{x}\,), \tag{3}$$

respectively, and $G_V = 1.14939 \times 10^{-5}$ GeV^{-2} [21]; α (a) is the Lorentz (isospin) index. In the center-of-mass system of the initial $\mu^- d$ atom from which capture occurs, we can safely assume $\vec{p}_\mu = \vec{p}_d = \vec{0}$. Consequently, the four-momentum transfer to the leptonic system, $q^\alpha \equiv (p_\nu - p_\mu)^\alpha$, reads $(q^0, \vec{q}\,) = (E_\nu - m_\mu, \vec{p}_\nu)$. The μd capture amplitude is then given by

$$\langle f|H_W|i\rangle = \frac{G_V}{\sqrt{2}} \Psi_{\mu-d}(\vec{0}\,) l_\alpha \langle \Psi_{nn}(-\vec{q}, \vec{p}; s_1 s_2)| j^\alpha(\vec{q}\,)|\Psi_d(s_d)\rangle, \tag{4}$$

where the Fourier-transformed currents are

$$j^\alpha(\vec{q}\,) \equiv \int d^3\vec{x}\, \mathrm{e}^{-i\vec{q}\cdot\vec{x}} J^\alpha(\vec{x}\,), \tag{5}$$

$$l_\alpha \equiv \mathrm{e}^{i\vec{x}\cdot\vec{q}} \langle \nu(p_\nu, s_\nu)|L_\alpha(\vec{x}\,)|\mu^-(p_\mu, s_\mu)\rangle = \bar{u}_\nu(\vec{p}_\nu, s_\nu)\gamma_\alpha(1-\gamma_5)u_\mu(\vec{p}_\mu, s_\mu). \tag{6}$$

In Eq. (4), $\Psi_{\mu-d}(\vec{0}\,) = 1/\sqrt{\pi} a_0^{3/2}$ is the $1S$ wave function of the $\mu^- d$ atom, where $a_0 \equiv (m_\mu + m_d)/m_\mu m_d \alpha$, with $\alpha \simeq 1/137.036$ the fine structure constant.[2] $\Psi_d(s_d)$ in Eq. (4) represents the deuteron wave function with the z-component of its spin s_d; $\Psi_{nn}(-\vec{q}, \vec{p}; s_1 s_2)$ represents the final nn wave function, with total nn momentum $-\vec{q}$, relative nn momentum $\vec{p} = (\vec{p}_1 - \vec{p}_2)/2$, and the z-components of the neutron spins, s_1 and s_2. It is easy to obtain

$$\Gamma_{\mu d} = \frac{|G_V \Psi_\mu(\vec{0}\,)|^2}{4(2J_{\mu d}+1)} \int \frac{d^3\vec{p}}{(2\pi)^3} \int \frac{d^3\vec{p}_\nu}{(2\pi)^3} 2\pi\delta(\Delta E)$$
$$\times \sum_{S_{\mu_d}=-J_{\mu d}}^{J_{\mu d}} \sum_{s_1 s_2 s_\mu s_d} \left|\langle \Psi_{nn}|j^\alpha(\vec{q}\,)|\Psi_d\rangle l_\alpha \left\langle \tfrac{1}{2}, s_\mu; 1, s_d \middle| J_{\mu d}, S_{\mu d}\right\rangle\right|^2, \tag{7}$$

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[1] For example, $\Gamma^{L=1}_{\mu d} \approx \frac{1}{3}\Gamma_{\mu d}$ according to Ref. [6].

[2] The small correction due to the finite size of the deuteron is taken into account in the actual calculation.

where $J_{\mu d} = 1/2$, ΔE is the energy difference between the final and initial states, $\Psi_d \equiv \Psi_d(s_d)$ and $\Psi_{nn} \equiv \Psi_{nn}(-\vec{q}, \vec{p}; s_1 s_2)$.

3. HBχPT Lagrangian and the hadronic currents

To derive the transition operators, we adopt Weinberg's counting rule. In HBχPT the leading order (LO) Lagrangian is given by

$$\mathcal{L}_0 = \overline{B}[iv\cdot D + 2ig_A S\cdot\Delta]B - \frac{1}{2}\sum_A C_A\big(\overline{B}\Gamma_A B\big)^2 + f_\pi^2\,\mathrm{Tr}\big(i\Delta^\mu i\Delta_\mu\big) + \frac{f_\pi^2}{4}\mathrm{Tr}(\chi_+) \tag{8}$$

with

$$D_\mu \equiv \partial_\mu + \frac{1}{2}\big[\xi^\dagger, \partial_\mu\xi\big] - \frac{i}{2}\xi^\dagger\mathcal{R}_\mu\xi - \frac{i}{2}\xi\mathcal{L}_\mu\xi^\dagger, \qquad \Delta_\mu = \frac{1}{2}\big\{\xi^\dagger, \partial_\mu\xi\big\} + \frac{i}{2}\xi^\dagger\mathcal{R}_\mu\xi - \frac{i}{2}\xi\mathcal{L}_\mu\xi^\dagger$$

and $\chi_+ = \xi^\dagger\chi\xi^\dagger + \xi\chi^\dagger\xi$, where $\mathcal{R}_\mu = \frac{\tau^a}{2}(\mathcal{V}_\mu^a + \mathcal{A}_\nu^a)$ and $\mathcal{L}_\mu = \frac{\tau^a}{2}(\mathcal{V}_\mu^a - \mathcal{A}_\nu^a)$ denote the external gauge fields. In the absence of the external scalar and pseudo-scalar fields $\chi = m_\pi^2$, and we define the pion field as $\xi = \exp\big(i\frac{\vec{\tau}\cdot\vec{\pi}}{2f_\pi}\big)$. It is convenient to choose the four-velocity v^μ and the spin operator S^μ as $v^\mu = (1, \vec{0})$ and $S^\mu = (0, \vec{\sigma}/2)$.

The next-to-leading-order (NLO) Lagrangian (including the "$1/m_N$" terms) in the one-nucleon sector is given in [22] while that in the two-nucleon sector is given in [12].[3] Combining them, we can write the NLO Lagrangian relevant to our case as

$$\begin{aligned}\mathcal{L}_1 = \overline{B}\bigg(&\frac{v^\mu v^\nu - g^{\mu\nu}}{2m_N}D_\mu D_\nu + c_1\,\mathrm{Tr}\,\chi_+ + \bigg(4c_2 - \frac{g_A^2}{2m_N}\bigg)(v\cdot i\Delta)^2 + 4c_3 i\Delta\cdot i\Delta \\ &+ \bigg(2c_4 + \frac{1}{2m_N}\bigg)\big[S^\mu, S^\nu\big][i\Delta_\mu, i\Delta_\nu] - i\frac{1+c_6}{m_N}\big[S^\mu, S^\nu\big]f_{\mu\nu}^+\bigg)B \\ &- 4id_1\overline{B}S\cdot\Delta B\overline{B}B + 2id_2\epsilon^{abc}\epsilon_{\mu\nu\lambda\delta}v^\mu\Delta^{\nu,a}\overline{B}S^\lambda\tau^b B\overline{B}S^\delta\tau^c B + \cdots,\end{aligned} \tag{9}$$

where

$$\epsilon_{0123} = 1, \qquad \Delta_\mu = \frac{\tau^a}{2}\Delta_\mu^a,$$
$$f_{\mu\nu}^+ = \xi\big(\partial_\mu\mathcal{L}_\nu - \partial_\nu\mathcal{L}_\mu - i[\mathcal{L}_\mu, \mathcal{L}_\nu]\big)\xi^\dagger + \xi^\dagger\big(\partial_\mu\mathcal{R}_\nu - \partial_\nu\mathcal{R}_\mu - i[\mathcal{R}_\mu, \mathcal{R}_\nu]\big)\xi.$$

We find it convenient to use the dimensionless low-energy constants $\hat{c}$'s and $\hat{d}$'s defined by

$$c_{1,2,3,4} = \frac{1}{m_N}\hat{c}_{1,2,3,4}, \qquad d_{1,2} = \frac{g_A}{m_N f_\pi^2}\hat{d}_{1,2}. \tag{10}$$

The values of these low energy constants, $\hat{c}_{1,2,3,4}$, are taken from Ref. [22]:

$$\hat{c}_1 = -0.60 \pm 0.13, \qquad \hat{c}_2 = 1.67 \pm 0.09, \qquad \hat{c}_3 = -3.66 \pm 0.08, \qquad \hat{c}_4 = 2.11 \pm 0.08 \tag{11}$$

and $c_6 = \kappa_V = 3.70$. These values were determined at tree level (or NLO) in the one-nucleon sector, which correspond to N^{3}LO in our two-nucleon calculation.

[3] Our definition of the pion field differs from that used in Ref. [12] by a minus sign.

3.1. The one-body currents

The one-body currents can be obtained either by an explicit HBχPT calculation or by the Foldy–Wouthuysen (FW) reduction of the well-known relativistic expressions. The former method requires one-loop diagrams which consist of vertices from $\mathcal{L}_0$ (for N^{2}LO contributions) and those which contain one vertex from $\mathcal{L}_1$ (for N^{3}LO contributions); also needed are the corresponding counter-terms from $\mathcal{L}_2$ and $\mathcal{L}_3$. We adopt here the FW reduction method for convenience. Since the range of $t \equiv q^2 = m_\mu(m_\mu - 2E_\nu)$ for μd capture is small ($-m_\mu^2 \lesssim t \leqslant m_\mu^2$), the t-dependences in the standard form factors, $F_{1,2}^V(t)$ and $G_A(t)$, give only less than 2% effects, which can be reliably taken into account by expansion in t,

$$F_1^V(t) = 1 + \frac{t}{6}r_V^2 + \mathcal{O}(t^2), \qquad F_2^V(t) = \kappa_V + \mathcal{O}(t), \qquad G_A(t) = g_A\left(1 + \frac{t}{6}r_A^2 + \mathcal{O}(t^2)\right).$$

Keeping the terms linear in t is consistent with HBχPT to the order we calculate. Some caution is required for the $G_P(t)$ term, which contains the pion-pole contributions,

$$\frac{G_P(t)}{2m_N} \equiv \beta(t)\frac{2m_N G_A(t)}{m_\pi^2 - t}, \tag{12}$$

where $\beta(t)$ is a slowly varying function. Comparison with the explicit HBχPT calculation up to N^{2}LO [23] leads to

$$\beta(t) = \frac{f_\pi g_{\pi NN}}{g_A m_N} - \frac{1}{6}r_A^2 m_\pi^2 + \mathcal{O}\left(\frac{Q^3}{\Lambda_\chi^3}\right) = 1 + \big[(-2.0 \sim 1.5) \pm 0.3\big]\ \%. \tag{13}$$

The authors of Ref. [6] found that $\Gamma_{\mu d}$ is reduced only by $\sim 2\%$ when β increases by 10%. Thus, limiting ourselves to the $\beta = 1$ case entails at most 0.4% error.[4]

The resulting one-body vector (1B) current components are

$$\begin{aligned}
V_{1\mathrm{B}}^{0,-}(\vec{q}\,) &= \sum_i \tau_i^- \mathrm{e}^{-i\vec{q}\cdot\vec{r}_i}\left[1 + \frac{t}{6}r_V^2 - \frac{\vec{q}^{\,2}}{8m_N^2} + (1+2\kappa_V)\frac{i\vec{q}\cdot\vec{\sigma}_i\times\bar{\vec{p}}_i}{4m_N^2} - \kappa_V\frac{\vec{q}^{\,2}}{4m_N^2}\right],\\
\vec{V}_{1\mathrm{B}}^{-}(\vec{q}\,) &= \sum_i \tau_i^- \mathrm{e}^{-i\vec{q}\cdot\vec{r}_i}\left[\frac{\bar{\vec{p}}_i + \frac{i}{2}(1+\kappa_V)\vec{q}\times\vec{\sigma}_i}{m_N} + (1+2\kappa_V)\left(i\vec{\sigma}_i\times\bar{\vec{p}}_i - \frac{1}{2}\vec{q}\right)\frac{\omega}{4m_N^2}\right],
\end{aligned} \tag{14}$$

where $\tau_i^- \equiv (\tau_i^x - i\tau_i^y)/2$ and $\bar{\vec{p}}_i = (\vec{p}_i^{\,\prime} + \vec{p}_i)/2$. The one-body axial-vector current components can be written for convenience as

$$A_{1\mathrm{B}}^\alpha = \hat{A}_{1\mathrm{B}}^\alpha + \frac{q^\alpha}{m_\pi^2 - t}q_\beta \hat{A}_{1\mathrm{B}}^\beta, \tag{15}$$

which defines $\hat{A}_{1\mathrm{B}}$, where

$$\begin{aligned}
\hat{A}_{1\mathrm{B}}^{0,-}(\vec{q}\,) &= \sum_i g_A\tau_i^- \mathrm{e}^{-i\vec{q}\cdot\vec{r}_i}\left[\frac{\vec{\sigma}_i\cdot\bar{\vec{p}}_i}{m_N} - \omega\frac{\vec{\sigma}_i\cdot\vec{q}}{8m_N^2}\right],\\
\hat{\vec{A}}_{1\mathrm{B}}^{-}(\vec{q}\,) &= \sum_i g_A\tau_i^- \mathrm{e}^{-i\vec{q}\cdot\vec{r}_i}\left[\vec{\sigma}_i\left(1 + \frac{t}{6}r_A^2\right) + \frac{2\big(\bar{\vec{p}}_i\vec{\sigma}_i\cdot\bar{\vec{p}}_i - \vec{\sigma}_i\bar{\vec{p}}_i^{\,2}\big) - \frac{1}{2}\vec{q}\vec{\sigma}_i\cdot\vec{q} + i\vec{q}\times\bar{\vec{p}}_i}{4m_N^2}\right].
\end{aligned} \tag{16}$$

[4] If the deviation of β from 1 were important, we would have to include the one-body pseudoscalar term, $\widehat{P}_{1\mathrm{B}}$ in Eq. (15), as is necessary for the two-body $\widehat{P}_{2\mathrm{B}} = \widehat{P}$ in Eq. (18).

The above equations correspond to N^{2}LO in HBχPT [24]. Apart from the mentioned t-dependence of the form factors, the N^{2}LO contribution is found to be negligible, $\sim 0.1\%$, indicating a rapid convergence. We therefore limit ourselves to N^{2}LO for the one-body currents although, to be completely consistent, we should in principle include N^{3}LO.

3.2. Two-body exchange currents

Since the evaluation of the two-body exchange current is the focus of this work we discuss the various parts of this exchange current in detail. We write the two-body vector and axial-vector currents as $V^\mu_{2\mathrm{B}}(\vec k_1,\vec k_2)$ and $A^\mu_{2\mathrm{B}}(\vec k_1,\vec k_2)$, where $\vec k_i=\vec p_i\,'-\vec p_i$ is the momentum transferred to the ith nucleon. For the two-body vector-charge current we know $V^0_{2\mathrm{B}}(\vec k_1,\vec k_2)=\mathcal{O}(Q^4)$ [5] [25], which therefore can be ignored. The spatial components of the vector current have a one-pion-exchange contribution of order Q^2 [26]:

$$\vec V_{2\mathrm{B}}(\vec k_1,\vec k_2)=-i(\tau_1\times\tau_2)^-\frac{g_A^2}{4f_\pi^2}\left[\frac{\vec\sigma_1(\vec\sigma_2\cdot\vec k_2)}{m_\pi^2-k_2^2}-\frac{\vec\sigma_2(\vec\sigma_1\cdot\vec k_1)}{m_\pi^2-k_1^2}+\frac{\vec\sigma_1\cdot\vec k_1}{m_\pi^2-k_1^2}\frac{\vec\sigma_2\cdot\vec k_2}{m_\pi^2-k_2^2}(\vec k_2-\vec k_1)\right]+\mathcal{O}(Q^4),\tag{17}$$

where $f_\pi\simeq 93$ MeV is the pion decay constant, and we make use of the notation $(\tau_1\times\tau_2)^-\equiv(\tau_1\times\tau_2)^x-i(\tau_1\times\tau_2)^y$. Analogously to Eq. (15), we write the axial-vector current as [16,27],

$$A^\alpha_{2\mathrm{B}}=\hat A^\alpha_{2\mathrm{B}}+\frac{q^\alpha}{m_\pi^2-t}\left(q_\beta\hat A^\beta_{2\mathrm{B}}+\widehat P\right),\tag{18}$$

with

$$\hat A^0_{2\mathrm{B}}(\vec k_1,\vec k_2)=i(\vec\tau_1\times\vec\tau_2)^-\frac{g_A}{4f_\pi^2}\frac{\vec\sigma_1\cdot\vec k_1}{m_\pi^2-k_1^2}-\frac{2g_A}{m_Nf_\pi^2}\left(\hat c_2+\hat c_3-\frac{g_A^2}{8}\right)\tau_1^-\frac{k_1^0\vec\sigma_1\cdot\vec k_1}{m_\pi^2-k_1^2}+(1\leftrightarrow 2),\tag{19}$$

$$\begin{aligned}\hat{\vec A}_{2\mathrm{B}}(\vec k_1,\vec k_2)=-\frac{g_A}{2m_Nf_\pi^2}\Bigg\{&\left[\frac{i}{2}(\vec\tau_1\times\vec\tau_2)^-\vec{\bar p}_1+4\hat c_3\tau_2^-\vec k_2+\left(\hat c_4+\frac14\right)(\vec\tau_1\times\vec\tau_2)^-(\vec\sigma_1\times\vec k_2)\right.\\&\left.+\frac{1+c_6}{4}(\vec\tau_1\times\vec\tau_2)^-(\vec\sigma_1\times\vec q\,)\right]\frac{\vec\sigma_2\cdot\vec k_2}{m_\pi^2-k_2^2}\\&+\left[2\hat d_1(\tau_1^-\vec\sigma_1+\tau_2^-\vec\sigma_2)+\hat d_2(\vec\tau_1\times\vec\tau_2)^-(\vec\sigma_1\times\vec\sigma_2)\right]+(1\leftrightarrow 2)\Bigg\},\end{aligned}\tag{20}$$

$$\widehat P(\vec k_1,\vec k_2)=-\frac{g_Am_\pi^2}{2m_Nf_\pi^2}\left\{8\hat c_1\vec\tau_2^-\frac{\vec\sigma_2\cdot\vec k_2}{m_\pi^2-k_2^2}+(1\leftrightarrow 2)\right\}.\tag{21}$$

Only one combination of the LEC, $\hat d_1$ and $\hat d_2$, is relevant for the μd capture process,

$$\hat d^R\equiv\hat d_1+2\hat d_2+\frac13\hat c_3+\frac23\hat c_4+\frac16.\tag{22}$$

Exactly the same combination of LEC's appears in triton β-decay, pp-fusion and the solar *hep* process [16]. Adopting the same strategy as in Ref. [16], we fix $\hat d^R$ from Γ^t_β(exp), the experimental value of the tritium β-decay rate.

[5] The terms of $\mathcal{O}(Q^\nu)$ correspond to those of N$^\nu$LO.

To facilitate the calculations, we perform a Fourier transformation (FT) of the above two-body currents. To control short-range physics in performing FT, we introduce a Gaussian cut-off regulator

$$S_\Lambda(\vec{k}^2) = \exp\left(-\frac{\vec{k}^2}{2\Lambda^2}\right). \tag{23}$$

where Λ is a cut-off parameter. It is to be emphasized that, although our calculation without regularization involves no infinities, we still need a regulator since EFT, by definition, is valid only up to a certain momentum scale. The regulated delta and Yukawa functions read

$$\begin{aligned} \delta_\Lambda^{(3)}(\vec{r}\,) &\equiv \int \frac{d^3\vec{q}}{(2\pi)^3} S_\Lambda^2(\vec{q}\,^2) e^{i\vec{q}\cdot\vec{r}} = \frac{\Lambda^3}{(4\pi)^{3/2}} \exp\left(-\frac{\Lambda^2 r^2}{4}\right), \\ y_{0\Lambda}(m, r) &\equiv \int \frac{d^3\vec{q}}{(2\pi)^3} S_\Lambda^2(\vec{q}\,^2) e^{i\vec{q}\cdot\vec{r}} \frac{1}{\vec{q}\,^2 + m^2}. \end{aligned} \tag{24}$$

We remark that this is exactly the same regularization method as used in Ref. [16].

In performing FT, we need to specify the time components of the momentum transferred to the nucleons. Energy conservation imposes the constraint: $k_1^0 + k_2^0 = -q^0 = m_\mu - E_\nu$. In our calculation we will adopt the so-called fixed-kinematics assumption (FKA) [11], where the energy transfer is assumed to be shared equally between the two nucleons, i.e., $k_1^0 = k_2^0 = (m_\mu - E_\nu)/2$, which naturally brings in the quantity $\widetilde{m}_\pi \equiv \sqrt{m_\pi^2 - (m_\mu - E_\nu)^2/4}$. The uncertainty related to FKA becomes large as $|q^0|$ grows. The contribution from the large $|q^0|$ region, however, will turn out to be so tiny that the assumptions related to k_i^0 cause little uncertainty in our calculation.

4. The capture rate for the transition to the $^1S_0$ *nn* state

The deuteron and the $^1S_0$ wave function may be written as

$$\psi_d(\vec{r}; s_d) = \frac{1}{\sqrt{4\pi} r}\left[u_d(r) + \frac{S_{12}(\hat{r})}{\sqrt{8}} w_d(r)\right]\chi_{1,s_d}\xi_{0,0}, \qquad \psi_0(r) = \frac{1}{\sqrt{4\pi} r} u_0(r)\chi_{0,0}\xi_{1,-1} \tag{25}$$

with

$$\int_0^\infty dr\left[u_d^2(r) + w_d^2(r)\right] = 1 \quad \text{and} \quad \lim_{r\to\infty} u_0(r) = \frac{\sin\delta_0}{p}[\cos pr + \cot\delta_0 \sin pr].$$

Here $S_{12}(\hat{r}) = 3\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2$, χ (ξ) is the Pauli spinor (isospinor), and δ_0 is the *nn* $^1S_0$ phase shift. To facilitate numerical work, we approximate ΔE as

$$\Delta E = E_\nu + 2\sqrt{m^2 + \vec{p}\,^2 + \frac{E_\nu^2}{4}} - M_{\mu d} + \mathcal{O}\left(\frac{(\vec{p}_\nu\cdot\vec{p}\,)^2}{4m^3}\right), \tag{26}$$

where $m \equiv m_n = 939.566$ MeV is the neutron mass, $M_{\mu d} \equiv m_\mu + m_d = 1981.272$ MeV. In our calculation we will neglect the $\mathcal{O}\big(\frac{(\vec{p}_\nu\cdot\vec{p})^2}{4m^3}\big)$ term since, as we shall show, the major contributions comes from the low $p \equiv |\vec{p}|$ region. Choosing the z-axis along $\vec{p}_\nu$, we write $\vec{q} = \vec{p}_\nu = E_\nu\hat{z}$. This simplifies the structure of the transition amplitudes as

$$\langle\psi_0|j^0(\vec{q}\,)|\Psi_d(s_d)\rangle = \delta_{s_d,0}\mathcal{M}_t, \qquad \langle\psi_0|\hat{e}_\lambda^*\cdot\vec{j}(\vec{q}\,)|\Psi_d(s_d)\rangle = \delta_{s_d,\lambda}\mathcal{M}_\lambda, \tag{27}$$

where $\hat{e}_\pm = \mp(\hat{x} \pm i\hat{y})/\sqrt{2}$, $\hat{e}_0 = \hat{z}$, and $\lambda = \pm 1, 0$. We decompose the matrix elements into vector and axial vector current contributions, $\mathcal{M}_{t,\lambda} = \mathcal{M}_{t,\lambda}[V] - \mathcal{M}_{t,\lambda}[A]$, and arrive at

$$\Gamma_{\mu d}^{L=0} = \frac{|G_V \Psi_\mu(\vec{0})|^2}{2\pi^2} \int_0^{p^{\max}} dp\, 2p^2 E_\nu^2 \left(1 - \frac{E_\nu}{M_{\mu d}}\right) \frac{2}{3} \left|2\mathcal{M}_{-1} + \mathcal{M}_0 - \mathcal{M}_t\right|^2. \tag{28}$$

Note that $\mathcal{M}_{-1} = -(\mathcal{M}_{+1}[V] + \mathcal{M}_{+1}[A])$, $\mathcal{M}_0 = -\mathcal{M}_0[A]$ and, to the order under consideration, $\mathcal{M}_t = -\mathcal{M}_t[A]$. The matrix elements of the vector current are

$$\begin{aligned}
\mathcal{M}_\lambda[V] = {} & \lambda\sqrt{2} \int_0^\infty dr \left\{ q u_0 \left(u_d j_0 - \frac{j_2 w_d}{\sqrt{2}}\right) \frac{\mu_V}{2m_N} - \omega u_0^{(1)} \left(u_d + \frac{w_d}{\sqrt{2}}\right) j_1 \frac{2\mu_V - 1}{4m_N^2} \right\} \\
& + \lambda(4\sqrt{2}) \left(-\frac{g_A^2}{8 f_\pi^2}\right) q \int_0^\infty dr\, u_0 \int_{-1/2}^{1/2} dx \left[\left(j_0^x u_d - \frac{j_2^x w_d}{\sqrt{2}}\right) \left(y_0^L - \frac{2}{3} y_1^L\right) - x q r j_1^x \left(u_d + \frac{w_d}{\sqrt{2}}\right) y_0^L \right. \\
& \left. + \frac{1}{3} \left(j_2^x u_d - \left(\sqrt{2}\, j_0^x + \frac{j_2^x}{\sqrt{2}}\right) w_d \right) y_1^L \right],
\end{aligned} \tag{29}$$

where $j_n^x \equiv j_n(qrx)$ are the spherical Bessel functions,

$$y_n^L \equiv y_{n\Lambda}\left(\sqrt{m_\pi^2 + \tfrac{1-4x^2}{4}\vec{q}^{\,2}}, r\right),$$

$$y_{1\Lambda}(m,r) \equiv -r \frac{\partial}{\partial r} y_{0\Lambda}(m,r), \qquad y_{2\Lambda}(m,r) \equiv \frac{1}{m^2} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} y_{0\Lambda}(m,r).$$

Using Eq. (18), we obtain for the axial current

$$\left\{ \begin{matrix} \mathcal{M}_t[A] \\ \mathcal{M}_\lambda[A] \end{matrix} \right\} = \left\{ \begin{matrix} \mathcal{M}_t[\hat{A}] \\ \mathcal{M}_\lambda[\hat{A}] \end{matrix} \right\} + \frac{1}{m_\pi^2 - t} \left\{ \begin{matrix} \omega \\ \delta_{\lambda,0}|\vec{q}\,| \end{matrix} \right\} \left(\omega\mathcal{M}_t[\hat{A}] - |\vec{q}\,|\mathcal{M}_0[\hat{A}] + \mathcal{M}[\widehat{P}]\right), \tag{30}$$

$$\begin{aligned}
\mathcal{M}_t[\hat{A}] = {} & \sqrt{2} g_A \int_0^\infty dr \left\{ \frac{1}{m_N} u_0^{(1)} \left(u_d - \sqrt{2}\, w_d\right) j_1 - \frac{q\omega}{8m_N^2} u_0 \left(u_d j_0 + \sqrt{2}\, w_d j_2\right) \right\} \\
& - \frac{\sqrt{2} g_A}{f_\pi^2} \left[1 - \left(\hat{c}_2 + \hat{c}_3 - \frac{g_A^2}{8}\right) \frac{m_\mu - E_\nu}{m_N}\right] \int_0^\infty dr\, u_0 \left(u_d - \sqrt{2}\, w_d\right) j_1 \frac{y_{1\Lambda}}{r},
\end{aligned} \tag{31}$$

$$\begin{aligned}
\mathcal{M}_\lambda[\hat{A}] = {} & \sqrt{2} g_A \int_0^\infty dr \left\{ \left[1 + \frac{t}{6} r_A^2 - \frac{\vec{\bar{p}}^{\,2}}{3m_N^2} - \delta_{\lambda,0} \frac{\vec{q}^{\,2}}{8m_N^2}\right] u_0 \left(u_d j_0 - \frac{w_d}{\sqrt{2}} j_2^\lambda\right) \right. \\
& \left. - \frac{1}{6m_N^2} u_0^{(2)} \left[\left(u_d - \frac{w_d}{\sqrt{2}}\right) j_2^\lambda - \sqrt{2}\, w_d j_0 \right] \right\} \\
& - \left(4\sqrt{2}\right) \frac{g_A}{2m_N f_\pi^2} \int_0^\infty dr \left[\frac{y_1}{r} \left(\mathcal{O}^{\rm kin} - \frac{1 + c_6}{4} (1 - \delta_{\lambda,0}) |\vec{q}\,| u_0 \left(u_d + \frac{w_d}{\sqrt{2}}\right) j_1 \right) \right. \\
& \left. - \frac{\widetilde{m}_\pi^2}{3} y_{0\Lambda} \left(\hat{c}_3 + 2\hat{c}_4 + \frac{1}{2}\right) u_0 \left(u_d j_0 - \frac{w_d}{\sqrt{2}} j_2^\lambda\right) \right.
\end{aligned}$$

$$+\frac{\widetilde{m}_\pi^2}{3} y_{2\Lambda}\left(\hat{c}_3-\hat{c}_4-\frac{1}{4}\right)u_0\left(\sqrt{2}\,w_d j_0-\left(u_d-\frac{w_d}{\sqrt{2}}\right)j_2^\lambda\right)+\hat{d}^R\delta_\Lambda(\vec{r})u_0u_d\Bigg], \tag{32}$$

$$\mathcal{M}\big[\widehat{P}\big]=\big(4\sqrt{2}\big)\frac{g_A}{2m_N f_\pi^2}\int_0^\infty dr\left[2\hat{c}_1 m_\pi^2\frac{y_{1\Lambda}}{r}u_0\big(u_d-\sqrt{2}\,w_d\big)j_1\right], \tag{33}$$

where

$$\mathcal{O}^{\text{kin}}=-\delta_{\lambda,0}\frac{|\vec{q}\,|}{8}u_0\big(u_d-\sqrt{2}\,w_d\big)j_1+\frac{1}{12}\big(j_0+j_2^\lambda\big)\big[u_0\big(u_d'-\sqrt{2}\,w_d'\big)-u_0'\big(u_d-\sqrt{2}\,w_d\big)\big]$$
$$-\frac{1}{4\sqrt{2}}\big(2j_0-j_2^\lambda\big)u_0w_d, \tag{34}$$

$$j_2^\lambda\equiv(1-3\delta_{\lambda,0})j_2,\qquad j_n=j_n\left(\frac{1}{2}qr\right),$$
$$u_0^{(1)}(r)=u_0'(r)-\frac{u_0(r)}{r},\qquad u^{(2)}(r)=u_0''(r)-3\frac{u_0'(r)}{r}+3\frac{u_0(r)}{r^2}.$$

In the above expressions the curly brackets denote 1B contributions, and for clarity we have suppressed the dependence on r in some equations.

5. Results

Table 1 shows $\Gamma_{\mu d}^{L=0}$ as a function of the cut-off parameter, Λ. As discussed, the short-range exchange current contribution depends on the single low-energy constant $\hat{d}^R$, see Eqs. (32), (22), and $\hat{d}^R$ determined from Γ_β^t(exp) is a function of Λ (see Ref. [16]). We observe that the variation of $\Gamma_{\mu d}^{L=0}$ over the range of Λ under consideration is less than 0.7 s^{-1}. The $\hat{d}^R$-dependence in the table indicates the importance of the contribution of the short-distance exchange current. Without the $\hat{d}^R$ term, $\Gamma_{\mu d}^{L=0}$ would change as much as 16 s^{-1} for $\Lambda = 500$–800 MeV. Thus, renormalizing the $\hat{d}^R$-term using Γ_β^t(exp) reduces the variation of $\Gamma_{\mu d}$ with respect to Λ by a factor ≈ 20, leading to the practically Λ-independent behavior of $\Gamma_{\mu d}$. Considering this stability we will hereafter only discuss the case corresponding to $\Lambda = 600$ MeV and $\hat{d}^R = 1.78$.

The capture rate contains several interference terms, which are listed in Table 2 in a cumulative manner. We note that the axial charge (AC) plays only a minor role; its destructive interference with GT decreases the capture rate by ~ 1 s^{-1}. Meanwhile, the M1 contribution interferes constructively with GT, increasing $\Gamma_{\mu d}$ by ~ 59 s^{-1}. Furthermore, the two-body MEC in the $L = 0$ channel increases the capture rate by ~ 13 s^{-1}.

Table 1
$L = 0$ capture rate (in s^{-1}) calculated as a function of the cutoff Λ. Also listed are the corresponding values of $\hat{d}^R$ determined from Γ_β^t(exp) [16]

Λ (MeV)	$\hat{d}^R$	$\Gamma_{\mu d}^{L=0}$ [s^{-1}]
500	1.00 ± 0.07	$254.7 - 9.85\hat{d}^R + 0.159(\hat{d}^R)^2 = 245.0 \pm 0.7$
600	1.78 ± 0.08	$261.1 - 9.09\hat{d}^R + 0.132(\hat{d}^R)^2 = 245.3 \pm 0.7$
800	3.90 ± 0.10	$271.0 - 6.76\hat{d}^R + 0.070(\hat{d}^R)^2 = 245.7 \pm 0.6$

Table 2
Cumulative contributions to $\Gamma_{\mu d}$ (calculated for $\Lambda = 600$ MeV and $\hat{d}^R = 1.78$). The row labeled "1B" corresponds to the case that contains one-body contributions only, while the row labeled "1B + 2B" to the case that includes both the one-body and MEC contributions. The three columns labeled "$L = 0$" show contributions from the $L = 0$ channel, with the contributions of the different transition operators displayed in a cumulative manner. The fifth column gives contribution from the $L \geqslant 1$ channels, as evaluated in TKK, Ref. [6], and the last column shows the sum of the $L = 0$ and $L \geqslant 1$ contributions

$\Gamma_{\mu d}$ [s^{-1}]	$L=0$			$L \geqslant 1$	Total
	$\lvert\mathrm{GT}\rvert^2$	$\lvert\mathrm{GT}+\mathrm{AC}\rvert^2$	$\lvert\mathrm{GT}+\mathrm{AC}+\mathrm{M1}\rvert^2$		
$\lvert 1\mathrm{B}\rvert^2$	178	177	232	138	370
$\lvert 1\mathrm{B}+2\mathrm{B}\rvert^2$	187	186	245	141	386

Table 3
Matrix elements calculated for representative values of E_{nn} (MeV) and the cumulative $L = 0$ capture rate for the case: $\Lambda = 600$ MeV and $\hat{d}^R = 1.78$. In each entry for the matrix element, the first number (preceding a "+" or "−" sign) gives the one-body contribution, while the second number gives the two-body contribution

E_{nn}	$\mathcal{M}_{+1}[A]$	$\mathcal{M}_{+1}[V]$	$\mathcal{M}_0[A]$	$\mathcal{M}_t[A]$	$\Gamma_{\mu d}^{L=0}$ [s^{-1}]
0.0	73.09 + 1.24	14.68 + 0.53	50.22 + 0.81	0.79 − 0.23	0
1.0	20.88 + 0.38	4.15 + 0.16	14.26 + 0.25	0.18 − 0.07	91
10.0	2.59 + 0.12	0.47 + 0.04	1.82 + 0.08	0.06 − 0.01	231
30.0	0.49 + 0.05	0.07 + 0.01	0.39 + 0.04	0.04 − 0.00	244
$E_{nn}^{\max}$	0.056 − 0.003	0	0.056 − 0.003	0	245

Our final result for $\Gamma_{\mu d}^{L=0} = 245\ \mathrm{s}^{-1}$ in Table 2 should be compared with TKK's result, $\Gamma_{\mu d}^{L=0}(\mathrm{TKK}) = 259\ \mathrm{s}^{-1}$.[6] By adding the $1 \leqslant L \leqslant 5$ contribution, $\Gamma_{\mu d}^{L\geqslant 1} = 141\ \mathrm{s}^{-1}$, calculated by TKK, we arrive at the total capture rate

$$\Gamma_{\mu d} = 386\ \mathrm{s}^{-1}, \tag{35}$$

to be compared with TKK's result $\Gamma_{\mu d}(\mathrm{TKK}) = (397 \sim 400)\ \mathrm{s}^{-1}$.

As mentioned earlier, a primary question is whether μd capture process is "gentle enough" for applying HBχPT with reasonable confidence. As noted the "dangerous" region for HBχPT occurs when the two neutrons carry most of the final energy. To address this issue, it is useful to consider the differential capture rate, $d\Gamma_{\mu d}/dE_{nn}$, where $E_{nn} \equiv 2\big(\sqrt{m_n^2 + \vec{p}^{\,2}} - m_n\big)$ is the energy of the final two-neutron relative motion. An equally informative quantity is the "cumulative" capture rate

$$\Gamma_{\mu d}(E_{nn}) \equiv \int_0^{E_{nn}} \frac{d\Gamma_{\mu d}}{dE'_{nn}}\, dE'_{nn}. \tag{36}$$

From these quantities we can assess to what extent μd capture is free from the "dangerous" kinematic region. We show in Table 3 the matrix elements, $\mathcal{M}_{+1}[A]$, $\mathcal{M}_{+1}[V]$, $\mathcal{M}_0[A]$ and $\mathcal{M}_t[A]$, calculated for representative values of E_{nn}, and for $\Lambda = 600$ MeV and $\hat{d}^R = 1.78$. Table 3 also gives $\Gamma_{\mu d}^{L=0}(E_{nn})$. The graphical representation of $\Gamma_{\mu d}(E_{nn})$ can be found in Fig. 1. We learn from Table 3 that the matrix elements decrease quite fast as E_{nn} increases, a feature that can be easily understood as follows. The ${}^1S_0$ nn radial wave function is proportional to $(\sin\delta_0)/p = \pm[(p\cot\delta_0)^2 + p^2]^{-1/2}$. Since the nn scattering length is very large, $p\cot\delta_0$ diminishes rapidly when the nn relative momentum p gets small. The examination of Table 3 also reveals that the one-body amplitudes decrease more quickly than the two-body amplitudes. This is a consequence of the softness of the deuteron wave

[6] We have re-run the code of TKK using $g_A = 1.267$. TKK's original result corresponding to $g_A = 1.262$ was $\Gamma_{\mu d}^{L=0}(\mathrm{TKK}) = 257\ \mathrm{s}^{-1}$.

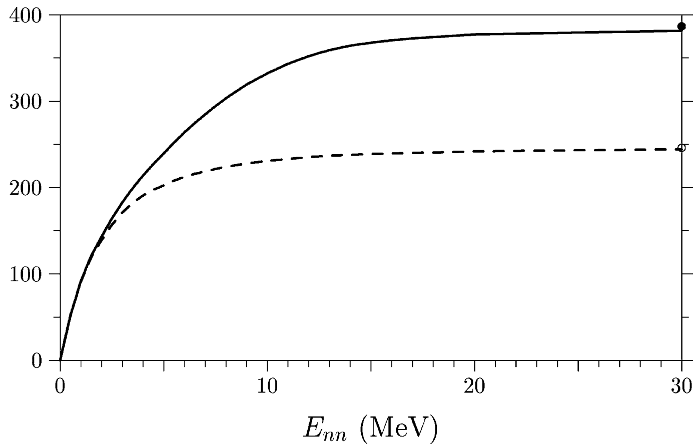

Fig. 1. Cumulative $\mu^- d$ capture rate (in s^{-1}) calculated for $\Lambda = 600$ MeV and $\hat{d}^R = 1.78$. The dashed line gives the $L = 0$ contribution, $\Gamma_{\mu d}^{L=0}(E_{nn})$, while the solid line shows the total contribution, $\Gamma_{\mu d}(E_{nn}) \equiv \Gamma_{\mu d}^{L=0}(E_{nn}) + \Gamma_{\mu d}^{L\geqslant 1}(E_{nn})$. The empty and solid circles for the values at $E_{nn} = E_{nn}^{\max} \simeq 102$ MeV, for $L = 0$ and $L \geqslant 0$, respectively.

function, which cannot supply high momentum transfers needed for producing large values of p. As a result, the contributions from high E_{nn}—where the applicability of EFT is questionable—is negligible. For instance, the contribution to $\Gamma_{\mu d}^{L=0}$ from $E_{nn} > 30$ MeV is just 1.1 s^{-1}, and that from $E_{nn} > 50$ MeV is less than 0.1 s^{-1}.

We now can make a rough estimate of the theoretical error associated with this calculation. Uncertainty related to the G_P term, Eq. (12) (or β) is ~ 1 s^{-1}, while uncertainty reflecting the Λ-dependence is less than 1 s^{-1}; uncertainty in Γ_β^t(exp) (or that in $\hat{d}^R$ for a given Λ) can affect $\Gamma_{\mu d}$ at the level of 1 s^{-1}. Furthermore, owing to the above-discussed “gentleness” of the μd capture kinematics, the higher-order corrections to the 2B MEC should converge rapidly in powers of $m_\mu/m_N \sim 0.1$; the uncertainly due to the higher-order contributions in expected to be ~ 1 s^{-1}. If we assign a rather conservative error, 2 s^{-1}, to the $L \geqslant 1$ contributions obtained in Ref. [6], the overall uncertainty in our estimate becomes 5 s^{-1} or $\sim 1\%$ in the total capture rate.

As mentioned, there is a serious disagreement between the two measured values of $\Gamma_{\mu d}$. Our theoretical result is consistent with $\Gamma_{\mu d}$(exp) in Ref. [18]. In the present exploratory study we have not considered radiative corrections [28], which are expected to be smaller than the existing uncertainty in $\Gamma_{\mu d}$(exp). When the planned precision measurement of the $\Gamma_{\mu d}$ at PSI [20] is realized, the issue of radiative corrections should certainly be addressed. The EFT* approach as described here will provide a useful tool for this purpose as well. Once the accuracy in $\Gamma_{\mu d}$(exp) is significantly improved, we will be able to use μd capture to determine the low energy constant $\hat{d}^R$, a quantity critically important for the accurate evaluation of the νd cross sections used in the analysis of the SNO experiments. At present the tritium β-decay is a much more accurate source of information on $\hat{d}^R$ than μd capture, but it is hoped that in the near future $\Gamma_{\mu d}$ will provide an independent constraint on $\hat{d}^R$. We consider this redundancy extremely important.

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References

[1] J.N. Bahcall, R. May, Astrophys. J. 155 (1969) 501.
[2] The SNO Collaboration, Phys. Lett. B 194 (1987) 321, and references therein.
[3] Q.R. Ahmad et al., Phys. Rev. Lett. 87 (2001) 071301.
[4] J. Carlson, R. Schiavilla, Rev. Mod. Phys. 70 (1998) 743.
[5] M. Chemtob, M. Rho, Nucl. Phys. A 163 (1971) 1;
E. Ivanov, E. Truhlík, Nucl. Phys. A 316 (1979) 451;
E. Ivanov, E. Truhlík, Nucl. Phys. A 316 (1979) 437.
[6] N. Tatara, Y. Kohyama, K. Kubodera, Phys. Rev. C 42 (1990) 1694.
[7] J. Adam, E. Truhlík, S. Ciechanowicz, K.-M. Schmitt, Nucl. Phys. A 507 (1990) 675.
[8] S. Nakamura, T. Sato, V. Gudkov, K. Kubodera, Phys. Rev. C 63 (2001) 034617.
[9] M. Butler, J.-W. Chen, X. Kong, Phys. Rev. C 63 (2001) 035501.
[10] For reviews, see: U. van Kolck, Prog. Part. Nucl. Phys. 43 (1999) 409;
S.R. Beane et al., in: M. Shifman (Ed.), At the Frontier of Particle Physics—Handbook of QCD, Vol. 1, World Scientific, Singapore, 2001, p. 133;
G.E. Brown, M. Rho, hep-ph/0103102, Phys. Rep., accepted.
[11] B.-Y. Park, F. Myhrer, J.R. Morones, T. Meissner, K. Kubodera, Phys. Rev. C 53 (1996) 2661.
[12] T.D. Cohen, J.L. Friar, G.A. Miller, U. van Kolck, Phys. Rev. C 53 (1996) 2661.
[13] T. Sato, T.-S.H. Lee, F. Myhrer, K. Kubodera, Phys. Rev. C 56 (1997) 1246.
[14] S. Ando, T.-S. Park, D.-P. Min, Phys. Lett. B 509 (2001) 253.
[15] D.B. Kaplan, M.J. Savage, M.B. Wise, Phys. Lett. B 424 (1998) 390;
D.B. Kaplan, M.J. Savage, M.B. Wise, Nucl. Phys. B 534 (1998) 329.
[16] T.-S. Park, L.E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky, S. Rosati, K. Kubodera, D.-P. Min, M. Rho, nucl-th/0106026;
T.-S. Park, L.E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky, S. Rosati, K. Kubodera, D.-P. Min, M. Rho, nucl-th/0107012.
[17] S. Ando et al., in progress.
[18] M. Cargelli et al., in: M. Morita, H. Ejiri, H. Ohtsubo, T. Sato (Eds.), Proceedings of the XXIII Yamada Conference on Nuclear Weak Processes and Nuclear Structure, Osaka, 1989, World Scientific, Singapore, 1989, p. 115.
[19] G. Bardin et al., Nucl. Phys. A 453 (1986) 591.
[20] P. Kammel et al., nucl-ex/0202011, to appear in Hyperfine Interactions;
P. Kammel, private communication.
[21] J.C. Hardy, I.S. Towner, V.T. Koslowsky, E. Hagberg, H. Schmeing, Nucl. Phys. A 509 (1990) 429.
[22] V. Bernard, N. Kaiser, U.-G. Meißner, Nucl. Phys. B 457 (1995) 147.
[23] V. Bernard, H.W. Fearing, T.R. Hemmert, U.-G. Meißner, Nucl. Phys. A 635 (1998) 121.
[24] S. Ando, F. Myhrer, K. Kubodera, Phys. Rev. C 63 (2001) 015203.
[25] T.-S. Park, K. Kubodera, D.-P. Min, M. Rho, Phys. Lett. B 472 (2000) 232.
[26] T.-S. Park, D.-P. Min, M. Rho, Phys. Rev. Lett. 74 (1995) 4153;
T.-S. Park, D.-P. Min, M. Rho, Nucl. Phys. A 596 (1996) 515.
[27] T.-S. Park, D.-P. Min, M. Rho, Phys. Rep. 233 (1993) 341;
T.-S. Park, I.S. Towner, K. Kubodera, Nucl. Phys. A 579 (1994) 381;
T.-S. Park, H. Jung, D.-P. Min, Phys. Lett. B 409 (1997) 26;
T.-S. Park, K. Kubodera, D.-P. Min, M. Rho, Astrophys. J. 507 (1998) 443.
[28] J.F. Beacom, S.J. Parke, Phys. Rev. D 64 (2001) 091302.