PION-DEUTERON SCATTERING IN THE Δ(1236) ENERGY REGION AS A THREE-BODY PROBLEM*

F. MYHRER** and D.S. KOLTUN

Department of Physics and Astronomy, University of Rochester, Rochester, N.Y. 14627, USA

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We study pion-deuteron scattering in the first π-N resonance energy region, in a three-body model based on Faddeev's equations. We discuss the effects of multiple scattering, nuclear binding, and virtual excitation of the target, on the energy behavior of the cross section.

There is considerable interest in how a resonance in pion-nucleon (π-N) scattering is modified when the nucleon is an interacting part of a nucleus. With this problem in mind, we have calculated pion-deuteron (π-d) elastic scattering for energies in the region of the π-N (3/2, 3/2) resonance (Δ1236) using a three-body model of the reaction [1]. This allows us to examine the effects of dynamical excitation of the target, as well as of multiple scattering and nucleon motion.

Our method is similar to that used by Hetherington and Schick [2] for K-d scattering, based on Faddeev's equations. We assume a separable S-wave N-N potential and a resonating P-wave π-N scattering amplitude, which is separable in momentum and energy. We reduce Faddeev's equations for each π-d partial wave amplitude, generalizing the method of ref. [1] to include interactions with relative $L \neq 0$, as required by the P-wave π-N interaction. We have neglected the spin dependence of the π-N amplitude in order to keep the size of the calculation moderate. It is important to keep the P-wave feature of the resonance, however, as we discuss below.

Our π-N P-wave amplitude is

$$\langle k|t_{\mathrm{P}}(\epsilon)|k'\rangle = 3\boldsymbol{k}\cdot\boldsymbol{k}'\,\tau(\epsilon)\,v(k^2)\,v(k')^2 \qquad (1)$$

where $v(k^2) = (\beta^2+k^2)^{-1}$ and

$$\tau(\epsilon) = \lambda\left[\epsilon - E_{\mathrm{R}} + \tfrac{1}{2}\mathrm{i}\Gamma\frac{(2\mu\epsilon)^{3/2}}{k_{\mathrm{R}}^3}\left(\frac{v(2\mu\epsilon)}{v(k_{\mathrm{R}}^2)}\right)^2\right]^{-1} \qquad (2)$$

The π momenta (in π-N c.m.) are k' and k before and after scattering; μ denotes the π-N reduced mass. The energy ϵ available to the π-N system depends on the total three-body energy $E(\pi$-d c.m.) and the momentum p of the "spectator" nucleon, through

$$\epsilon = E - \frac{p^2}{2M} - \frac{p^2}{2(m+M)} \qquad (3)$$

where m, M are the pion and nculeon masses, respectively. The resonance energy is E_{R} and $k_{\mathrm{R}}^2 = 2\mu E_{\mathrm{R}}$. (We use non-relativistic expressions for energies.) We fix the parameters in (1) and (2) so that the on-shell amplitude (i.e., with $k^2 = 2\mu\epsilon = k'^2$) resembles the experimental π-N amplitude as a function of energy ϵ (except for the neglect of spin). We take E_{R} = 156.5 MeV, Γ = 120 MeV, β = 336.13 MeV/c; and λ is chosen so that the amplitude (1) reaches the unitary limit for $\epsilon = E_{\mathrm{R}}$.

The separable amplitude (1) is similar in form to that derived by Chew and Low [3], with a cut-off function $v(k^2)$. The form is not derived from a separable π-N potential; however, one could find such a potential with a more general choice of $v(k^2)$, as has been shown by Landau and Tabakin [4]. The ϵ-dependence of $\tau(\epsilon)$ in (2) has been chosen to satisfy the off-shell unitarity condition [5].

For the purpose of examining some details of the theory, we have also constructed a fictitious S-wave resonant π-N amplitude of the form

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** Present address: Fysisk Institutt, Universitetet i Trondheim-NLHT, N-7000 Trondheim, Norway.

$$\langle k|t_S(\epsilon)|k'\rangle = 2\mu\epsilon\tau(\epsilon)\,\upsilon(k^2)\,\upsilon(k'^2) \quad (4)$$

with $\tau(\epsilon)$ also given by (2). The amplitudes (1) and (4) are simply related in the forward direction, on shell:

$$\langle k|t_P(k^2/2\mu)|k\rangle = 3\,\langle k|t_S(k^2/2\mu)|k\rangle\,. \quad (5)$$

We use Yamaguchi's form [6] for the separable N-N potential; this gives a Hulthèn wave function for the deuteron ground state.

With these choices of π-N and N-N interactions, the three-body equation may be reduced to coupled integral equations [1, 2]. We solve these to obtain the partial wave amplitudes η_l which contribute to the π-d elastic scattering amplitude

$$f(\theta) = -\left(\frac{\mu_d}{2\pi}\right)\sum_l (2l+1)\,P_l(\cos\theta)\eta_l \quad (6)$$

where μ_d is the π-d reduced mass. We may also write each amplitude η_l as a sum of multiple scattering amplitudes, corresponding to a solution of the integral equations by iteration:

$$\eta_l = \eta_l(\mathrm{SS}) + \eta_l(\mathrm{DS}) + \eta_l(\mathrm{TS}) + \eta_l(\text{“TS”}) + \ldots\,. \quad (7)$$

The first three terms denote single, double, and triple scattering of the pion, without interactions of the nucleons, and represent the begining of a multiple scattering series which contains no dynamical effect of the target. The term labelled “TS” represents two π-N collisions with an intervening N-N interaction; this is the first term which contains the excitation spectrum of the target (deuteron), which is missing from most multiple scattering approximations.

Numerical results for the forward scattering amplitude $f(0)$ are listed in table 1, where the P and S label the P- and S-wave π-N resonance models. Although the amplitude $f(\vartheta)$ is forward peaked, this is largely due to the $\eta_l(\mathrm{SS})$ term. Excluding this term the scattering is dominated by $l = 1$ for the P-wave model, and by $l = 0$ for the S-wave model. The separate contributions of the multiple scattering terms of (7) to η_1 are illustrated in fig. 1, for the P-wave resonance model.

One interesting result is the pion momentum (in π-d c.m.) at which $\mathrm{Re}\,f(0) = 0$; this may be considered a definition of the “resonance” for π-d scattering. This momentum is slightly below 215 MeV/c in our calculations. For a free nucleon at rest (in d c.m.) the resonant momentum would be 205 MeV/c (in π-d c.m.). As can be seen from table 1 and fig. 1, this upward shift is a detailed results of several different effects:

a) The potential that binds the nucleon to the target increases the π energy at which the π-N amplitude resonates. This is apparent from the SS term in $f(0)$, which resonates slightly above 215 MeV/c. This is an effect of keeping the full three-body kinematics of the problem, which forces the amplitude (1) off-energy-shell ($k^2 \neq 2\mu\epsilon$). In contrast to keeping

Table 1
Forward amplitudes (in fm.) for π-d elastic scattering, for the three π-N resonance models: P, S, and “S”, at various c.m. momenta. The contributions of the first few multiple scattering terms (7) and the total values are included.

	term				
Momentum and model	SS	DS	“TS”	TS	Total
170 MeV/c					
P	3.693 + 2.108i	0.065 + 0.726i	−0.229 − 0.247i	−0.145 + 0.490i	2.957 + 2.683i
S	1.066 + 0.650i	−0.120 + 0.065i	−0.020 − 0.039i	0.000 − 0.033i	0.951 + 0.652i
“S”	3.199 + 1.949i	−1.083 + 0.588i	−0.180 − 0.349i	0.011 − 0.881i	2.160 + 1.971i
215 MeV/c					
P	0.290 + 6.159i	−1.066 − 0.252i	0.440 − 0.102i	−0.134 − 0.912i	−0.048 + 5.348i
S	0.041 + 1.909i	−0.033 − 0.260i	0.088 − 0.009i	0.028 + 0.098i	0.065 + 1.733i
“S”	0.122 + 5.728i	−0.294 − 2.343i	0.789 − 0.084i	0.760 + 2.633i	0.343 + 4.102i
230 MeV/c					
P	−1.419 + 5.342i	−0.605 − 0.654i	0.336 + 0.135i	0.483 − 0.570i	−1.324 + 4.691i
S	−0.490 + 1.657i	0.097 − 0.198i	0.065 + 0.036i	−0.047 + 0.073i	−0.402 + 1.532i
“S”	−1.470 + 4.971i	0.870 − 1.781i	0.588 + 0.323i	−1.267 + 1.983i	−0.681 + 3.847i

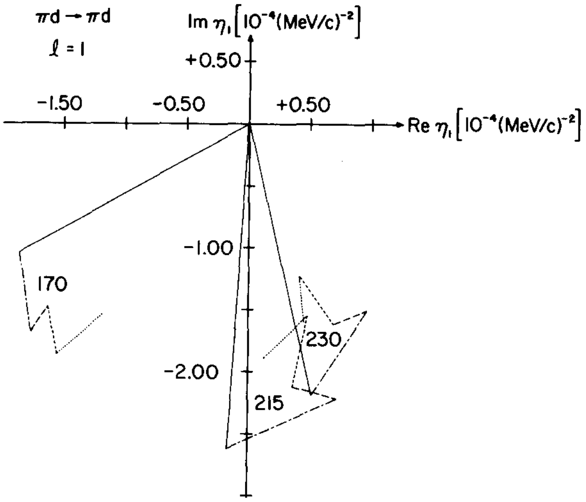

Fig. 1. Diagram of the $l = 1$ partial wave amplitude η_1 for π-d elastic scattering in the P-wave π-N resonance model, for the c.m. momenta 170, 215 and 230 MeV/c. The multiple scattering contributions are: SS-solid line, DS-dash-dot, "TS"-long dash, TS-short dash; end of dotted line gives total η_1.

only the effect of the bound nucleon kinetic energy on the on-energy-shell amplitude [7], this contributes an upward shift to the resonant momentum.

b) The multiple scattering terms (DS+TS+...) contribute a downward shift to the resonant momentum, for the P-wave model. This is in agreement with other models of multiple scattering effects on a P-wave resonance [8]. The "TS" term partly cancels this shift.

c) The effect of a) is larger than b), giving a net upward shift of the resonance in our model for π-d elastic scattering. However, for a target with more nucleons than the deuteron, one might expect the multiple scattering effects of b) to dominate, as in ref. [8]. This would be consistent with the downward shift of the resonant π momentum (for which Re $f(0) = 0$) seen experimentally [9, 10].

d) In contrast, for the S-wave resonance model, the multiple scattering contributes an upward shift to the resonant momentum, enhancing the potential effect a). This behavior is similar to that of $K^- - d$ scattering, in which the $K^- - p$ system has an S-wave resonance below threshold, which is moved up both by the binding potential a) and the multiple scattering effects [11].

We have introduced an "S" model, for which the π-N scattering amplitude is taken to be $t_{\text{"S"}} \equiv 3t_S$ where t_S is defined in (4). From (5) we see that this choice sets the forward, on-energy-shell amplitude for this "S" model to be identical to that for the P-wave model as a function of $\epsilon(= k^2/2m)$. However, the amplitudes will differ as a function of angle. This provides a useful test of the assumption of small angle scattering often employed in multipole scattering theory (e.g. by Glauber [12]), for which these two models should give similar results for π-d scattering. We see from the numerical results listed in table 1, that the "S" and P models do not give the same π-d scattering, even in the forward direction. These results are in contrast to the apparently successful application of this approximation to π-^{12}C elastic scattering in the same energy region [13].

We have also used our three-body model of π-d scattering to calculate the amplitude for the absorption reaction $\pi^+ + d \to p + p$ for the same energies. We find that the excited deuteron states (in the three-body channel) make considerable contribution to this process [1].

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