THE K-MESIC ATOM

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Assuming that the $Y_0^*(1405)$ resonance dominates the $\bar{K}N$ system at threshold, we show in a three-body K^-d model that to get the correct sign of the real part of the kaon-nucleus optical potential, we must use an off shell $\bar{K}N$ amplitude and that higher $\bar{K}N$ multiple scattering including intermediate nuclear excitations are important.

The K atomic levels and level widths can be calculated by using a perturbing strong interaction complex potential in the Klein-Gordon equation with the Coulomb potential. By adjusting the real and imaginary parts of this strong potential, it is possible to fit experimental atomic levels and widths [1–3]. The problem then becomes to derive this K^--nucleus optical potential from elementary K-nucleon interactions, see the review talk by Ericson [4].

We will assume that the kaon-nucleus optical potential is known from experiments [1]. Further we will assume that the Y_0^* is strongly coupled to $\bar{K}N$ [4], and we will study a solvable model to find which effects are involved when we want to derive a kaon-nucleus optical potential from the kaon-nucleon scattering amplitude.

More specifically we will study the K^-d elastic scattering amplitude as a function of the binding energy of the deuteron. We will look upon K^-d as a three-body system like Hetherington and Schick [5], and display the contributions from the different terms in the $\bar{K}N$ multiple scattering series. The K^-d scattering amplitude is [5, 6]

$$\eta = \sum_{i,j\neq 2} T_{ij}, \tag{1}$$

$$T_{ij} = t_i \delta_{ij} + \sum_{k=1}^{3} t_i (1-\delta_{ik}) G_0 T_{kj}, \tag{2}$$

where particle 2 is the kaon and particles 1 and 3 are the nucleons. Further we have

$$t_i = V_i [1 - G_0 V_i]^{-1}, \tag{3}$$

where V_1 is the potential between particles 2 and 3, and

$$G_0 = [E + i\eta - \sum_{i=1}^{3} p_i^2/2m_i]^{-1} \tag{4}$$

is the free three-body propagator. If we iterate eq. (2), we get the Faddeév multiple scattering series. The first term on the right-hand side of eq. (2) is the single scattering approximation (SS). Our calculations are done in the three-body centre-of-mass system, i.e. $\boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{p}_3 = 0$. We take plane-wave matrix elements of eq. (2). For the SS term we get

$$\langle \boldsymbol{q}_1, \boldsymbol{k}_1 | t_1(E) | \boldsymbol{q}_1', \boldsymbol{k}_1' \rangle = (2\pi)^3 \delta^3(\boldsymbol{q}_1 - \boldsymbol{q}_1') \langle \boldsymbol{k}_1 | t_1(\epsilon_1) | \boldsymbol{k}_1' \rangle, \tag{5}$$

where

$$\epsilon_1 = E - \boldsymbol{p}_1^2/2m_1 - (\boldsymbol{p}_2 + \boldsymbol{p}_3)^2/2(m_2 + m_3). \tag{6}$$

Here ϵ_1 is the relative two-body energy available to the scattering of 2 off 3, $\boldsymbol{q}_1 = \boldsymbol{p}_2 + \boldsymbol{p}_3$ and $\boldsymbol{k}_1 = m_2 \boldsymbol{p}_3 - m_3 \boldsymbol{p}_2$. Generally the two-body scattering amplitude (5) is off the energy shell, i.e. $\boldsymbol{k}_1^2/2\mu_{23} \neq \epsilon_1 \neq \boldsymbol{k}_1'^2/2\mu_{23}$. It is off-energy-shell by $\Delta\epsilon_1 \sim \langle V_{NN} \rangle_D$ [7] since initially the two nucleons are bound in the deuteron and it is the free three-body propagator that enters the expression (3) for t_i. This energy expression will be the same to all orders in the multiple scattering series.

To solve the three-dimensional integral equation (2), we assume our two-body S-wave amplitudes to be separable in incoming and outgoing momenta. Further we assume that the $(\bar{K}N)_0$ amplitude is dominated by the S-wave resonance $Y_0^*(1405)$. The $\bar{K}N$

is coupled to $\pi\Sigma$ and we should have a 2 X 2 potential to describe this [8]. To simplify the problem we decouple $\bar{K}N$ and $\pi\Sigma$ by using a separable model in which

$$\langle k | t_\alpha(\epsilon) | k' \rangle = v(k)\tau_\alpha(\epsilon)v(k'), \tag{7}$$

where $v(k) = (\beta^2 + k^2)^{-1}$ and

$$\tau_\alpha(\epsilon) = \lambda \left\{ s\epsilon_\alpha - sER + \mathrm{i}\,\frac{\Gamma}{2}\,\frac{q_\alpha}{k_R}\left[\frac{v(q_\alpha)}{v(k_R)}\right]^2 \right\}^{-1}. \tag{8}$$

Here $sER = ER + m_\pi + m_\Sigma = 1405$ MeV, $k_R = (2\mu_{\pi\Sigma}ER)^{1/2}$, and

$$q_\alpha = [2\mu_\alpha(s\epsilon_\alpha - m_\pi - m_\Sigma)]^{1/2}. \tag{9}$$

The label α describes the $\bar{K}N \to \bar{K}N$ or the $\pi\Sigma \to \pi\Sigma$ channel; $s\epsilon_\alpha$ is the total energy and μ_α the reduced mass in channel α. The decoupling of KN from $\pi\Sigma$ is reflected in eq. (9). The expression for $\tau_\alpha(\epsilon)$ satisfies the analytic off-shell unitarity condition [9] in the $\pi\Sigma$ channel. The constant λ is determined by forcing the $\pi\Sigma$ amplitude to reach the unitarity limit for $s\epsilon_{\pi\Sigma} = sER$; $\Gamma = 20$ MeV and $\beta = 775$ MeV/c. In the $\bar{K}N$ channel $q_{KN} = [2\mu_{KN}(\epsilon + m_K + m_N - m_\pi - m_\Sigma)]^{1/2}$, where ϵ is the two-body energy given by eq. (6). A curious point about this $\bar{K}N$ amplitude is that if SU(3) were a good symmetry, the $\bar{K}N$ scattering length would be real.

This Breit-Wigner like amplitude (8) is (apart from mass-terms) proportional to $\epsilon_1 - ER \approx \epsilon_{KN}^{rel} + \langle V_{NN} \rangle_D - ER$ and ϵ_{KN}^{rel} is the relative $\bar{K}N$ energy. We see here that the effective resonance energy is $ER^{eff} = ER - \langle V_{NN} \rangle_D > ER$ [7]. Because the nucleon in a nucleus is bound, the resonance energy becomes larger than the free kaon nucleon resonance energy of -27 MeV.

The nucleon-nucleon S-wave potential is taken from Yamaguchi [10,5]. It gives the Hulthén deuteron wave function.

This three-body problem has been solved numerically by Hetherington and Schick [5]. We use their numerical technique, but our $\bar{K}N$ model is very different. For simplicity we set the isospin 1 $\bar{K}N$ amplitude equal to zero throughout this work. Our resulting K^-d amplitudes are displayed in fig. 1 and the parameters for the different Hulthén deuteron models are given in table 1. Only $l = 0$ partial wave contributes to the K^-d amplitude since the incoming momentum is zero.

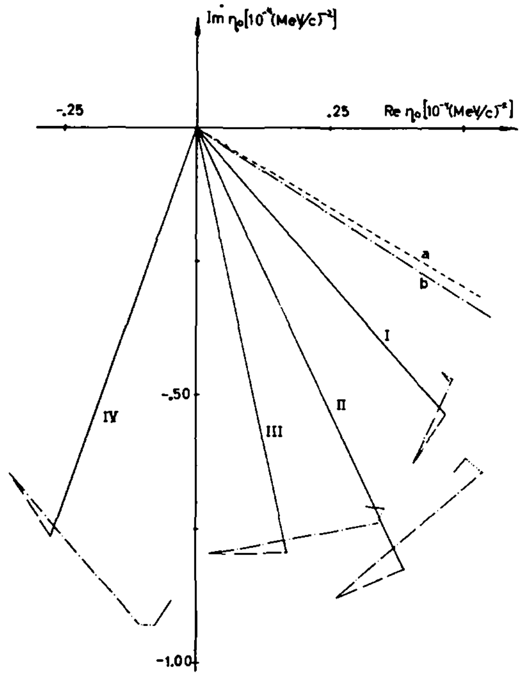

Fig. 1. Diagram of elastic kaon-deuteron scattering amplitude $f = -(\mu_{Kd}/2\pi)\,\eta_0$ at zero kaon momentum in its complex plane for four Hulthén deuteron models (Roman numerals, see table 1). For each deuteron model we have displayed the contributions to η_0 from different terms in the multiple scattering series. The composite broken line for each deuteron model terminates at the values for η_0, which are obtained by solving the integral equation (2). The lines a and b are the on-shell approximations discussed in the text.

The contributions from the different terms in the multiple scattering series in fig. 1 are: the solid lines starting at the origin are the single $\bar{K}N$ scattering contributions to the K^-d amplitude, the dashed lines are the double $\bar{K}N$ scattering contributions, the dashed-dotted lines the double $\bar{K}N$ scattering with intermediate N-N scattering (deuteron excitations), the dotted lines the triple $\bar{K}N$ contributions; the last solid lines end up at η_0 and represent the difference between η_0 obtained from the integral equation (2) and η_0 obtained from the four multiple scattering terms above. We have also included the single scattering on-shell amplitudes with (curve a) or without (curve b) the impulse approximation. On-shell means that we freeze

Table 1
Parameters for the different Hulthén-type deuteron models and corresponding λ_2, $\langle q^2 \rangle_D$ and $\langle V_{NN} \rangle_D$.

Deuteron Model	EB(MeV)	α_2(MeV/c)	β_2(MeV/c)	λ_2	$\langle V_{NN} \rangle_D$ (MeV)	$\langle q^2 \rangle_D$ (MeV/c)^2
I	− 2.225	45.706	285.89	$-\ 8.415 \times 10^5$	−16.15	13070.0
II	−10.0	96.90	134.86	$-\ 1.939 \times 10^5$	−23.92	13070.0
III	−10.0	96.90	285.89	-11.213×10^5	−39.45	27650.0
IV	−20	137.03	285.89	-13.688×10^5	−61.75	39190.0

the deuteron a la Glauber ($p_1 = p_3 = -p_{IN}/2 = 0$ in eq. (6)), and with impulse approximation means that we take $\epsilon_1 \sim p_{IN}^2 = 0$ in eq. (6).

As we make $\langle V_{NN} \rangle_D$ more negative, we pass through the $(\bar{K}N)_0$ resonance Y_0^* backwards. By starting with the strongest bound deuteron model IV in table 1, and increase p_{IN} for fixed deuteron model, the K^-d amplitude will go through a resonance-like behaviour. This means that the stronger the nucleon in the nucleus is bound (the larger the *potential* energy), the further below $\bar{K}N$ threshold we will be. The fermi motion, $\langle q^2 \rangle_D$ in table 1, is only an averaging effect and does not itself influence the energy dependence very much. That off-energy-shell effects are important has recently also been shown in ref. [11].

The multiple scattering terms with and without intermediate excitations are very important in determining the sign of the real part of kaon nucleus optical potential, as can be seen from fig. 1. The intermediate deuteron excitation here includes the whole excitation spectrum of our deuteron. In our calculations the multiple scattering effects work against the potential effect.

Another possible explanation for the sign of the real part of the kaon nucleus potential is the addition of $\bar{K}N$ potentials to make the K^- nucleus potential [12]. The philosophy behind this approach is very different from ours.

We have argued that Y_0^* is strongly coupled to the $\bar{K}N$ system, and that some of the same effects are at work in the K^--atoms as in the π-nucleus scattering in the $\Delta(1236)$ resonance region [7], although the relative importance of the effects are altered. One thing that gives this meson-nucleus energy behaviour is a meson-baryon amplitude of the Chew-Low form [7, 13]

$$t_{MB} \sim v(k)\,\tau(\epsilon)\,v(k').$$

It is demonstrated here that it is not enough to use an off-shell impulse approximation [7] as is done by Ericson and Hüfner for $\pi - {}^{12}C$ scattering [13], but higher terms in the multiple scattering series with intermediate nuclear excitations must be included.

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References

[1] G. Backenstoss et al., Phys. Lett. 38B (1972) 181.
[2] M. Krell, Phys. Rev. Lett. 26 (1971) 584;
R. Seki, Phys. Rev. C5 (1972) 1196.
[3] W.A. Bardeen and E. Wayne Torigoe, Phys. Lett. 38B (1972) 135.
[4] T.E.O. Ericson, 4th Int. Conf. on High-energy physics and nuclear structure, Dubna, 1971.
[5] J.H. Hetherington and L.H. Schick, Phys. Rev. 137 (1965) B935.
[6] L.D. Faddeév, Soviet Phys. JETP 12 (1961) 1014.
[7] F. Myhrer and D.S. Koltun, to be published;
F. Myhrer, thesis, Univ. of Rochester, unpublished.
[8] R.H. Dalitz and S.F. Tuan, Ann. of Phys. 10 (1960) 307;
S. Wycech, Nucl. Phys. B28 (1971) 541;
J. Revai, Phys. Letters 33B (1970) 587.
[9] C. Lovelace, Phys. Rev. 135 (1964) B1225.
[10] Y. Yamaguchi, Phys. Rev. 95 (1954) 1628.
[11] M. Alberg, E.M. Henley and L. Wilets, Phys. Rev. Lett. 30 (1973) 255.
[12] J.L. Uretsky, Phys. Rev. 147 (1966) 906;
F. Von Hippel and J.H. Douglas, Phys. Rev. 146 (1966) 1042;
J.H. Koch and M.M. Sternheim, Phys. Rev. Lett. 28 (1972) 1061.
[13] T.E.O. Ericson and J. Hüfner, Phys. Lett. 33B (1970) 601.