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Effective kaon mass in baryonic matter and kaon condensation ☆

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The effective kaon mass m_K^* in dense baryonic matter is calculated based on PCAC, current algebra and the Weinberg smoothness hypothesis. The off-shell behavior of the K–N scattering amplitudes is treated consistently with PCAC, and the effects of the subthreshold K–N resonances are also included. The m_K^* is found to depend crucially on the K–N sigma term, Σ_{KN}. Since the current estimates of Σ_{KN} are very uncertain, we discuss various scenarios treating Σ_{KN} as an input parameter; for certain values of Σ_{KN} a collective mode of a hyperon-particle–nucleon-hole state appears at high densities, possibly leading to kaon condensation.

Several years ago Kaplan and Nelson suggested the possibility of s-wave kaon condensation in baryonic matter using a chiral effective lagrangian [1]. This original suggestion was followed by many detailed studies on kaon condensation and its possible consequences for neutron stars [2–9]. In all of these works the condensation is driven by the *on-shell* contribution of the chiral-symmetry-breaking term, the major part of which can be identified with the K–N sigma term, $\Sigma_{KN}\equiv\frac{1}{2}(m_u+m_s)\langle N|\bar{u}u+\bar{s}s|N\rangle$. Theoretical estimates give a large positive value ($150\sim400$ MeV) for Σ_{KN}, a value which would give a large attractive contribution to the on-shell K–N interaction. The basic ansatz made in [1–9] is that this strong on-shell attraction is operative for the far off-shell kaons of the condensate as well.

Recently, Delorme, Ericson and Ericson (DEE) [10] have questioned the reliability of meson condensation driven by the on-shell sigma term. Their basic critique is that the meson–baryon scattering amplitudes given by Kaplan–Nelson's approach are devoid of some known off-shell structures. DEE's study of s-wave pion-condensation indicates that the pion effective mass m_π^* is almost density-independent, casting a strong doubt on boson condensation à la Kaplan–Nelson. In a previous paper [11], we have made a detailed study of s-wave pion condensation using current algebra, PCAC and the Weinberg expansion. Our results support the main point of DEE [10] and demonstrate the importance of the off-shell behavior of the π–N scattering amplitudes in determining m_π^*. In particular, we have found that the established positive value of the π–N sigma term ($\Sigma_{\pi N}=45\sim60$ MeV), which implies an attractive contribution for the on-shell pion, becomes repulsive for the off-shell pion near the soft-pion limit, precluding the possibility of s-wave pion condensation.

In the present paper we apply the same method [11] to the K–N system and discuss the behavior of the effective kaon mass m_K^* in baryonic matter. It will be demonstrated that the major results in [11] for m_π^* hold for $m_{K^+}^*$ as well. A notable difference in the $\bar{K}$–N channel comes from the existence of subthreshold resonances. These resonances give a new feature to the effective meson mass behavior; viz., there may appear hyperon-particle–nucleon-hole bound states, which are analogues of Migdal's π_s^+ in the π–N system [12]. We examine the energies of these bound states and their expected role in kaon condensation.

To evaluate the on- and off-shell K–N scattering amplitudes, we employ the Weinberg expansion method developed in [13] and applied later to the K–N sigma term estimation [14–16]. The scattering amplitude for the reaction: $K^\pm(k)+N(p)\to K^\pm(k')+N(p')$, $(N=p, n)$ can be parametrized as

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$$\langle \boldsymbol{k}', \boldsymbol{p}' | T | \boldsymbol{k}, \boldsymbol{p} \rangle = \mathrm{i}(2\pi)^4 \delta^4(p'+k'-p-k) \times \bar{u}(p')(A_N^{\pm} + B_N^{\pm} \Phi) u(p) , \tag{1}$$

where $\Phi = \frac{1}{2}(\not k + \not k')$, and $A_N^{\pm}$ and $B_N^{\pm}$ are functions of Lorentz invariants. The commonly used scalars are

$$s=(k+p)^2 , \quad t=(k-k')^2 , \quad u=(p-k')^2 ,$$
$$\nu = \frac{s-u}{4M_N} , \quad \nu_B = \frac{t-k^2-(k')^2}{4M_N} , \tag{2}$$

with M_N the nucleon mass. Since we constrain the nucleons to be on-shell, we need only four independent scalars. We shall use ν, t, k^2 and $(k')^2$ as independent variables. We define the Cheng–Dashen amplitudes $C_N^{\pm}(\nu, t, k^2, (k')^2)$ as

$$C_N^{\pm} = A_N^{\pm} + \nu B_N^{\pm} . \tag{3}$$

$C_N^{\pm}$ and the scattering amplitudes are equivalent for forward scattering. We define the crossing-even and -odd amplitudes $\hat{C}_N^{(\pm)}$

$$\hat{C}_N^{(\pm)} = \tfrac{1}{2}(C_N^{+} \pm C_{\bar{N}}^{-}) , \tag{4}$$

where $\hat{C}_N^{(+)}$ ($\hat{C}_N^{(-)}$) is an even (odd) function of ν [#1]. According to Weinberg's smoothness hypothesis [17], $\hat{C}_N^{(\pm)}$ can be decomposed into a smooth part $\hat{C}_{N,\mathrm{poly}}^{(\pm)}$ and a pole part $R_N^{(\pm)}$ including the Born terms

$$\hat{C}_N^{(\pm)} = \hat{C}_{N,\mathrm{poly}}^{(\pm)} + R_N^{(\pm)} . \tag{5}$$

For notational convenience we introduce $G^{(\pm)}$ defined by $G_N^{(+)} \equiv \hat{C}_N^{(+)}$ and $G_N^{(-)} \equiv (m_K/\nu)\hat{C}_N^{(-)}$. As is standard in the literature we parametrize the smooth part $G_{N,\mathrm{poly}}^{(\pm)}$ as a polynomial of second-order in the kaon energy-momentum:

$$G_{N,\mathrm{poly}}^{(\pm)} = A_N^{(\pm)} m_K^2 + B_N^{(\pm)} t + C_N^{(\pm)} (k^2 + k'^2) + D_N^{(\pm)} \nu^2 . \tag{6}$$

A major assumption is that this expansion is valid up to the K–N threshold region. One can then use the low-energy K–N scattering data to place constraints on the parameters in eq. (6). Although the K–N scattering data, which only provide on-shell information, cannot determine all the parameters, the off-shell structure of the K–N amplitudes is tightly controlled by current algebra and PCAC [18]. These off-shell constraints are:

(1) Adler's consistency condition for one soft-kaon,

$$\hat{C}_N^{(+)}(0, m_K^2, 0, m_K^2) = 0 , \tag{7}$$

where for the K–N case we have,

$$R_N^{(+)}(0, m_K^2, m_K^2, 0) = R_N^{(-)}(0, m_K^2, 0, m_K^2) = 0 . \tag{8}$$

(2) The sigma term and the Weinberg–Tomozawa term at the Weinberg point [18]

$$G_N^{(+)}(0,0,0,0) = -\frac{\Sigma_{KN}}{f_K^2} , \tag{9a}$$

$$G_p^{(-)}(0,0,0,0) = -\frac{1}{f_K^2} ,$$

$$G_n^{(-)}(0,0,0,0) = -\frac{1}{2f_K^2} , \tag{9b}$$

where Σ_{KN} is the K–N sigma term, and f_K the kaon decay constant. Eqs. (8), (7) and (9a), (9b) can be rewritten in terms of $A_N^{(\pm)} \sim D_N^{(\pm)}$ as

$$A_N^{(+)} + B_N^{(+)} + C_N^{(+)} = 0 , \tag{10a}$$

$$A_N^{(+)} = -\frac{\Sigma_{KN}}{f_K^2 m_K^2} ,$$

$$A_p^{(-)} = -\frac{1}{f_K^2 m_K} ,$$

$$A_n^{(-)} = -\frac{1}{2f_K^2 m_K} . \tag{10b}$$

Inserting eqs. (10a), (10b) into (6) leads to a crucial result for $G_{N,\mathrm{poly}}^{(+)}$:

$$G_{N,\mathrm{poly}}^{(+)} = \frac{k^2 + k'^2 - m_K^2}{f_K^2 m_K^2} \Sigma_{KN} + B_N^{(+)}(t - k^2 - k'^2) + D_N^{(+)} \nu^2 . \tag{11}$$

In this paper we will refer to the first term in (11) as the sigma term contribution.

As for the pole part $R_N^{(\pm)}$, we consider the five hyperons listed in table 1; the first four located below the K–N threshold are of particular importance in the present context. In calculating the Born terms, we use the following Yukawa couplings

[#1] K–N scattering has four independent channels with $I=0$, 1 and $S=\pm 1$. These four possibilities are represented here by the amplitudes $\hat{C}_{p,n}^{(\pm)}$.

Table 1
Resonances and their coupling constants g_Y as defined in eq. (12a)–(12c).

Resonance (Mass)	J^P	$g_Y^2/4\pi$	Ref.
$\Lambda(1116)$	$\frac{1}{2}^+$	13.9	A. Martin [19]
$\Sigma(1195)$	$\frac{1}{2}^+$	3.3	A. Martin [19]
$\Sigma^*(1385)$	$\frac{3}{2}^+$	2.3	O. Dumbrajs et al. [20]
$\Lambda^*(1405)$	$\frac{1}{2}^-$	1.56	O. Dumbrajs et al. [20]
$\Lambda^*(1520)$	$\frac{3}{2}^-$	1.56	from the decay width

$$\mathscr{L}=\frac{g_Y}{M_Y+M_N}\bar{Y}\gamma_\mu\gamma_5 N\partial^\mu K+\text{h.c.}$$

$$(Y=\Lambda,\Sigma)\,, \tag{12a}$$

$$\mathscr{L}=\frac{g_{\Lambda^*}}{M_{\Lambda^*}-M_N}\bar{\Lambda}^*\gamma_\mu N\partial^\mu K+\text{h.c.}\,,$$

$$\mathscr{L}=\frac{g_{\Sigma^*}}{M_N}\bar{\Sigma}^*_\mu N\partial^\mu K+\text{h.c.}\,, \tag{12b}$$

$$\mathscr{L}=\frac{g_{\Lambda^*(1520)}}{M_N}\bar{\Lambda}^*_\mu\gamma_5 N\partial^\mu K+\text{h.c.} \tag{12c}$$

To be consistent with (8), the derivative couplings are used here. For the propagator for the Rarita–Schwinger field we use the same form as in [14].

As stated, insofar as the soft-kaon expansion of eq. (6) is valid up to the K–N threshold, we can in principle determine $A_N^{(\pm)}\sim D_N^{(\pm)}$ from the experimental data combined with the PCAC constraints. Unfortunately, the low-energy K–N scattering data are not very accurate and their analysis is still unsettled. The variance among the existing analyses originates primarily from ambiguities in the $\bar{K}$–N subthreshold cut that appear in the dispersion relations used to determine g_Y [19]. These ambiguities are reflected in the published estimates of Σ_{KN}:$\Sigma_{Kp}=480^{+110}_{-600}$ MeV [14], -370 ± 110 MeV [21], 493 ± 716 MeV [16], 175 ± 890 MeV [22]. Evidently the value of Σ_{KN} is very uncertain, even attaining a negative value in some analysis, the significance of which we shall discuss. Here we treat Σ_{KN} as an unknown parameter ranging from -400 MeV to 600 MeV and fit the remaining coefficients in (6) so that the amplitude is consistent with the forward dispersion integral results given by Martin [19]. The coupling constants g_Y defined in (12a), (12c) take the values shown in table 1. We also adapt the values of g_Σ and g_Λ derived by Martin [19] to retain consistency with the forward dispersion integral data. With these inputs we can determine $A_N^{(\pm)}\sim D_N^{(\pm)}$ in (6); $A_N^{(\pm)}$ are fixed by (10a), (10b) and the other coefficients are given as

$$B_p^{(+)}=-\tfrac{1}{2}(A_p^{(+)}+1.43m_K^{-3})\,,$$
$$B_n^{(+)}=-\tfrac{1}{2}(A_n^{(+)}-0.465m_K^{-3})\,,$$
$$C_p^{(-)}=4.96m_K^{-3},\quad C_n^{(-)}=-5.57m_K^{-3}\,,$$
$$D_p^{(+)}=-20.9m_K^{-3},\quad D_p^{(-)}=5.92m_K^{-3}\,,$$
$$D_n^{(+)}=26.0m_K^{-3},\quad D_n^{(-)}=-9.34m_K^{-3}\,, \tag{13}$$

where we have used $f_K=113.6$ MeV. Although the forward scattering data cannot determine $B_N^{(-)}$, the $B_N^{(-)}$ terms have no contributions to the kaon effective mass.

Having determined the K–N scattering amplitude applicable to off-shell kinematics, we calculate the effective kaon mass m_K^* in baryonic matter. This is done by locating a pole in the in-medium kaon Green's function. The effects of the K–N interaction are included through the kaon self-energy $\Pi(\omega,\boldsymbol{k};\rho)$, leading to the in-medium dispersion relation

$$k^2-m_K^2\equiv\omega^2-\boldsymbol{k}^2-m_K^2=\Pi(\omega,\boldsymbol{k};\rho)\,, \tag{14}$$

where ρ is the matter density. From rotational symmetry we assume $\omega=f(m_K,\boldsymbol{k}^2;\rho)$. Solving the dispersion equation (14) for ω, we determine the effective mass m_K^* by

$$m_K^*(\rho)\equiv f(m_K,0;\rho)\,. \tag{15}$$

To calculate Π, we treat nuclear matter as a Fermi gas. Π is then given in the impulse approximation by

$$\Pi=-\int \mathrm{d}^3\bar{p}\,\rho_p(\boldsymbol{p})C_{Kp}(\boldsymbol{p},k)$$
$$-\int \mathrm{d}^3\bar{p}\,\rho_n(\boldsymbol{p})C_{Kn}(\boldsymbol{p},k)\,, \tag{16}$$

where $\rho_{p,n}(\boldsymbol{p})$ is the density distribution function of the proton or neutron with momentum $\boldsymbol{p}$, *and* $C(\boldsymbol{p},k)$ is the forward scattering amplitude or the Cheng–Dashen amplitude (3) at $t=0$ and $k^2=(k')^2$. For C we can use eqs. (5) and (6).

In fig. 1 we show the effective mass of K^+ in symmetric or neutron matter [#2]. The repulsive interac-

[#2] In symmetric matter, we encounter two independent K–N sigma terms but we assume here $\Sigma_{Kp}=\Sigma_{Kn}(\equiv\Sigma_{KN})$.

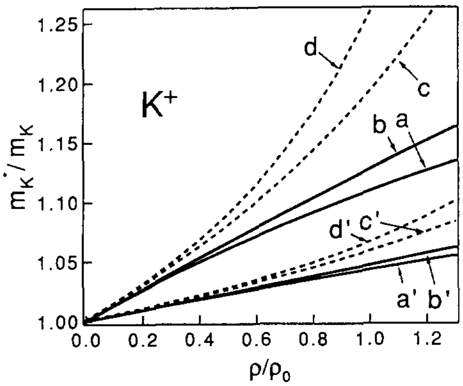

Fig. 1. The effective K^+ mass m_K^* in symmetric or neutron matter as a function of the density ρ (measured in units of the normal nuclear matter density ρ_0). The lines a–d correspond to symmetric matter and a′–d′ to neutron matter. The solid lines are for the positive sigma term: Σ_{KN}=400 MeV (200 MeV) for a, a′ (b, b′). The dashed lines are for the negative sigma term: $\Sigma_{KN}=-200$ MeV (-400 MeV) for c, c′ (d, d′). Here we take $\Sigma_{Kp}=\Sigma_{Kn}=\Sigma_{KN}$.

tions at threshold for both K^+–p and K^+–n channels [#3] explain the increase of $m_{K^+}^*$ with the density in both symmetric and neutron matters. The rapid increase of $m_{K^+}^*$ is caused by the $B_N^{(+)}$ and $D_N^{(+)}$ terms in eq. (11), but we probably should not take it too seriously, since the validity of the present treatment based on the low-energy expansion may deteriorate rapidly as a high-frequency regime becomes important due to the strong repulsion of the $B_N^{(+)}$ and $D_N^{(+)}$ terms. We also remark that the Σ_{KN}-dependence of $m_{K^+}^*$ is rather weak at low densities consistently with the sigma term contribution of (11) [#4].

The K^- case has more complicated features due to the existence of hyperon-particle–nucleon-hole states (to be denoted by Y^p–N^h). In the energy regime of our concern, the $\Lambda(1116)^p$–N^h and $\Lambda^*(1405)^p$–N^h states would contribute to symmetric matter, while the $\Sigma(1195)^p$–N^h and $\Sigma^*(1385)^p$–N^h states can contribute to both symmetric and neutron matters. These Y^p–N^h states form "continuum bands" as indicated by the shaded region in figs. 3 and 4. The upper and lower limits of these bands are given by

$$\omega_{\max}^Y = M_Y - M_N , \tag{17}$$

$$\omega_{\min}^Y = \sqrt{M_Y^2 + p_F^2} - \sqrt{M_N^2 + p_F^2} , \tag{17 cont'd}$$

where p_{K-F} is the Fermi momentum of baryonic matter. The presence of the Y^p–N^h states gives repulsive (attractive) contributions above $\omega_{\max}$ (below $\omega_{\min}$). $\omega_{\min}$).

The K^-–p interaction at threshold is known to be rather strongly repulsive ($\mathrm{Re}\, a_{K^-p} = -0.67$ fm [19]). This repulsion comes largely from the subthreshold resonance, $\Lambda^*(1405)$, lying very close to the $\bar{K}$–N threshold. By contrast, the K^-–n interaction is rather weak despite the presence of the $\Sigma^*(1385)$ p-wave resonance below threshold. The sign of the K^-–n scattering length is still controversial [24]. An analysis consistent with the data used in the present calculation indicates attraction in this channel ($\mathrm{Re}\, a_{K^-n} = 0.37$ fm [14]). These features are reflected in the behavior of $m_{K^-}^*$ for very low densities $\rho/\rho_0 < 0.5$; $m_{K^-}^*$ increases in symmetric matter (fig. 2), but decreases in neutron matter and approaches the $\Sigma^*(1385)^p$–N^h "continuum" at $\rho = (0.4 \sim 0.5)\rho_0$ (fig. 3). A new feature that appears in the high density region, is that the attractive Y^p–N^h interactions develop a collective bound state (K_{sc}^-) below $\omega_{\min}^Y$. This is analogous to the π_s^+ mode

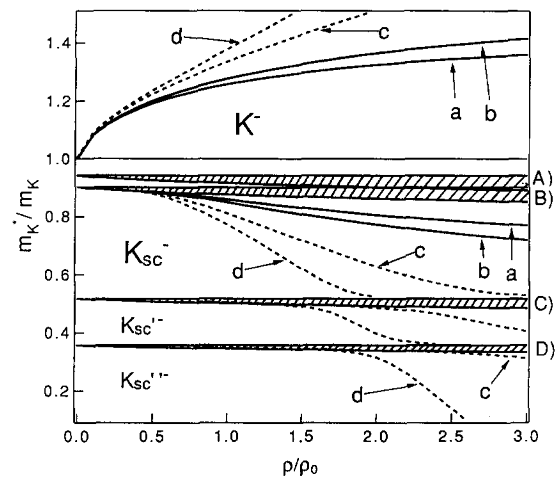

Fig. 2. The effective K^- mass m_K^* and the energies of the collective modes in symmetric matter as functions of ρ. The solid lines are for the positive sigma term: Σ_{KN}=400 MeV (200 MeV) for a (b). The dashed lines are for the negative sigma term: $\Sigma_{KN}=-200$ MeV (-400 MeV) for c (d). Here we take $\Sigma_{Kp}=\Sigma_{Kn}=\Sigma_{KN}$. The hatched areas are the hyperon-particle–nucleon-hole (Y^p–N^h) continua: A) $Y=\Lambda^*(1405)$, B) $Y=\Sigma^*(1385)$, C) $Y=\Sigma(1195)$, and D) $Y=\Lambda(1116)$.

[#3] The s-wave scattering lengths are, $a_{K^+p} = -0.33$ fm and $a_{K^+n} = -0.15$ fm [14].

[#4] For the Weinberg–Tomozawa term effect on the threshold behavior of m_K^*, see [23].

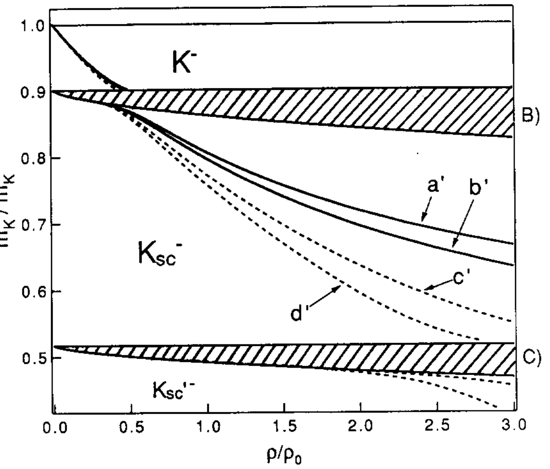

Fig. 3. The effective K^- mass m_K^* and the energies of the collective modes in neutron matter as functions of ρ. The solid lines are for the positive sigma term: $\Sigma_{KN}=400$ MeV (200 MeV) for a′ (b′). The dashed lines are for the negative sigma term: $\Sigma_{KN}=-200$ MeV (-400 MeV) for c′ (d′). The hatched areas are the Y^p–N^h continua: B) $Y=\Sigma^*(1385)$, C) $Y=\Sigma(1195)$.

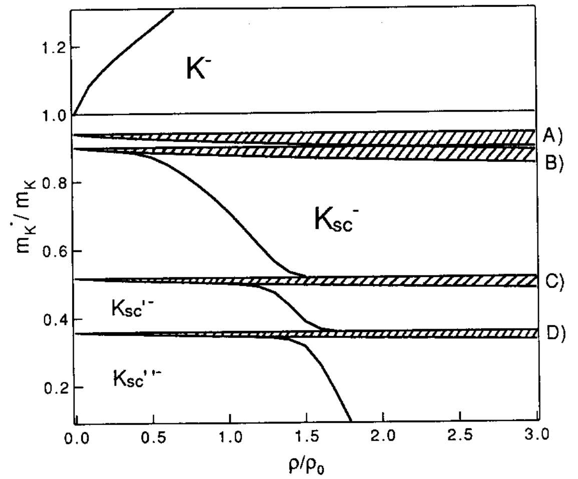

Fig. 4. The effective K^- mass m_K^* and the energies of the collective modes in symmetric matter with $\Sigma_{KN}=-600$ MeV. Here we take $\Sigma_{Kp}=\Sigma_{Kn}=\Sigma_{KN}$. The hatched areas are the Y^p–N^h continua: A) $Y=\Lambda^*(1405)$, B) $Y=\Sigma^*(1385)$, C) $Y=\Sigma(1195)$, and D) $Y=\Lambda(1116)$.

(spin-collective mode) in the pion sector, which is a collective particle–hole excitation with the same quantum number as the pion [12]. This collective state can be seen clearly in symmetric matter (fig. 2), where a K_{sc}^- state appears at $\rho=(0.5\sim0.7)\rho_0$ as a second branch because of the strong attraction caused by $\Lambda^*(1405)$ and $\Sigma^*(1385)$. A similar K^- state appears in neutron matter (fig. 3) starting at $\rho=0$ and continues below the Σ^* $(1385)^p$–N^h band [#5].

The density dependences of K_{sc}^- in symmetric matter (figs. 2 and 4) and K^- in neutron matter (fig. 3) are highly sensitive to $\sim\Sigma_{KN}$. As is evident from eq. (11), the sigma term contribution changes its sign at $\omega=m_K/\sqrt{2}\sim0.7m_K$ so that a *positive* sigma term makes the $m_{K^-}^*$ approach $\sim0.7m_K$ as ρ increases, a feature discussed in detail in [11]. Such a behavior can be seen for K_{sc}^- (lines a, b in fig. 2) and K^- (lines a, b in fig. 3) for a positive Σ_{KN}. However, for a negative value of Σ_{KN}, $m_{K^-}^*$ has a completely different behavior. In this case, Σ_{KN} gives an attractive contribution near the soft-kaon limit so that $m_{K^-}^*$ can be smaller than $0.7m_K$. Furthermore, if the magnitude of a negative Σ_{KN} is large enough, the second collective state $K_{sc}'^-$ can develop below the $\Sigma(1195)$ continuum (lines c, d in fig. 2 and line d in fig. 3) and the third one $K_{sc}''^-$ below the $\Lambda(1116)$ continuum (line d in fig. 2) (see also footnote 5). In fig. 4 we show, for an extreme case of $\Sigma_{KN}=-600$ MeV, the K^- effective mass in symmetric matter.

[#5] It is possible to interpret the interplay between K^- and Y^p–N^h as a level-crossing phenomenon.

Based on the above results, we now discuss the possibility of kaon condensation. We have mentioned that the behavior of m_K^* depends crucially on the value of Σ_{KN}. For positive values of Σ_{KN}, the lowering of m_K^* is always small and therefore neither in symmetric matter nor in neutron matter do we expect kaon condensation. However, if Σ_{KN} is negative, kaon condensation might occur. In symmetric matter, with $\Sigma_{KN}=-400$ MeV, the energy of $K_{sc}''^-$ shows a very sharp drop around $\rho\sim2.7\rho_0$ (fig. 2), indicating the possibility of $m_K^*\to0$ for somewhat higher densities or more negative values of Σ_{KN} (cf. fig. 4). In neutron matter (fig. 3) the lowering of m_K^* with increasing ρ for $K_{sc}'^-$ is less dramatic in the density region studied here. Yet the fact that $m_K^*/m_K\lesssim0.47$ for $\rho\sim3\rho_0$ indicates that kaon condensation may occur in neutron star for $\rho\gtrsim3\rho_0$ with the "help" of the chemical potential (typically, μ of the order of m_π) and the nuclear symmetry energy [8]. A detailed calculation of kaon condensation within the present framework will be reported elsewhere [25].

The Gell-Mann–Oakes–Renner model together with the linear approximation on the s-quark mass [14], gives $\Sigma_{Kp}=148$ MeV, and baryon models suggest positive values [26] of Σ_{KN}. However, as mentioned earlier, the value of Σ_{KN} found using the dispersion theory is quite uncertain. A recent reinvestigation of the t-dependence in $\Sigma_{\pi N}$ has led to a reduced value of $\Sigma_{\pi N}$ [27]. A preliminary calculation based on chiral perturbation theory suggests that the t-dependence in the Σ_{KN} might be very large [28]. Since a t-dependence in Σ_{KN} indicates that fourth order terms in the expansion of eq. (6) are of importance, the error introduced by truncating this series should be investigated. The form of the sigma term contribution in eq. (11) is an automatic consequence of taking only up to second-order terms in the kaon energy–momentum in eq. (6). To examine to what extent this form is modified by including higher order terms seems interesting.

Finally we compare our results with those obtained in chiral perturbation theory by Brown et al. (BLRT) [9]. Using the heavy fermion formalism (HFF), BLRT have calculated up to next-to-leading-order terms ($\nu=2$ in the language of the HFF), and found a significant energy–momentum dependence in the symmetry breaking terms. The off-shell effect found in [9] has a partial overlap with that contained in the $B^{(+)}$ and $C^{(+)}$ terms of our eq. (6). However, the off-shell behavior of the Σ_{KN} term differs drastically between the two works; in our approach the sigma term contribution to the elementary amplitude features as $(k^2+k'^2-m_K^2)/(f_K^2 m_K^2)\Sigma_{KN}$ in eq. (6), whereas in BLRT it has a constant value Σ_{KN}/f_K^2 corresponding to the on-shell expression. This means that, while the Σ_{KN} contribution in our approach turns from attractive to repulsive as one goes from the on-shell kinematics to the Weinberg point, the Σ_{KN} term in [9] always retains its on-shell attraction. It is obviously important to examine the origin of this difference. One may say that, although the methods we used (PCAC, current algebra, and the dispersion relations) are completely standard, the off-shell behavior obtained therewith may not be unique. However, we emphasize that, quite apart from detailed of-shell extrapolation procedures, the key relation $G_N^{(+)}(0, 0, 0, 0)=-\Sigma_{KN}/f_K^2$ [eq. (9a)] is a direct consequence of the generalized Ward–Takahashi identity [18]. Hence, the fact that BLRT's result does not satisfy this constraint seems to indicate that the effective chiral Lagrangian used by BLRT probably needs extra terms. The reasons for this conjecture are: (i) this discrepancy, belonging to order $\nu=2$, cannot be attributed to higher-order terms ($\nu\geqslant 3$) which BLRT excluded in a systematic manner; (ii) for a given chiral Lagrangian, BLRT's calculation is a complete one up to $\nu=2$; (iii) although the "standard" chiral lagrangian used by BLRT is the most general one for on-shell mesons, non-standard terms such as $K(\Box K)\bar{B}B$ can have a non-trivial contribution for off-shell mesons [#6]. For on-shell mesons these additional terms can be absorbed into the standard terms using $\Box\to m^2$, but, for off-shell kinematics, they can give rise to additional k^2-dependence, which is probably what is needed to satisfy the general requirement eqs. (9a) and (8). This point seems to warrant a further study.

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[#6] The addition of these non-standard terms cannot be rewritten as a total derivative term, this latter producing no physical effects.

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