THE SPIN CONTENT OF THE PROTON IN THE CHIRAL BAG ☆

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We analyze the sum-rules of the nucleon spin structure functions using a two-phase chiral invariant quark model. Due to the axial anomaly, the flavor SU(3) singlet component outside the quark confinement cavity is suppressed. This singlet is also reduced inside the cavity due to (non-strange) intermediate $qqqq\bar{q}$ states gluon exchange current corrections. Chiral symmetry requires Goldstone pions outside which generate part of the proton spin.

The spin structure function $g_1^p(x)$ of the polarized proton has recently been measured by the European Muon Collaboration (EMC) [1]. According to their analysis EMC concluded that the quarks might contribute only a tiny fraction to the spin of the nucleon [1]. This is a most astonishing result since in non-relativistic spectroscopy the valence quarks carry all of the spin. The EMC results are so surprising that their analysis has been called in doubt [2]. We shall assume in this note that the EMC values are correct, i.e. that the Ellis–Jaffe [3] sum rule is strongly broken. This means the valence quarks do not saturate the sum-rule. The most radical solution has been presented by Brodsky et al. [4] who use the skyrmion model for the baryon. Another proposal concentrates on the effects of the anomaly of the flavor singlet axial current to explain the data [5,6]. In fact, Jaffe [5] favored the assumption that the flavor singlet axial current would play a negligible role in the sum-rules.

In this work we shall show how a similar phenomenon takes place in a class of models that for a long time have been much discussed in hadron spectroscopy. These are the chiral bag models where space is divided into two regions using the arguments of Brown and Rho [7]. In one region inside a cavity of radius R $(r<R)$ chiral symmetry is realized in the Wigner–Weyl model while in the outer region $r>R$ the symmetry is realized in the Nambu–Goldstone model. Since the Bjorken sum rule is derived from quark current algebra it is important to have chiral invariant low energy models to discuss the sum rules. We shall show that we can explain the EMC data without including any explicit $s\bar{s}$ component in the wave functions of the nucleons by invoking very reasonable physical assumptions how we calculate matrix elements of currents in chiral bag models.

A most important ingredient in our approach is the existence of massless Goldstone bosons transforming as a flavor octet outside the bag. Due to the axial anomaly, the corresponding flavor singlet will be massive. In the class of chiral bag models under discussion, we assume that there is a sharp (spherical) boundary at $r=R$ separating the spatial region where

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quark fields are the dynamical variables and the region where the pseudoscalar fields are the dynamical variables. A baryon sum rule is usually written in terms of quark operators and fields ψ.

$$\Gamma(\mathrm{B}) = \int_0^1 \mathrm{d}x\, g_1^{\mathrm{B}}(x) = \langle B\uparrow | \sum_{i=1}^{B} \tfrac{1}{2} Q_i^2 \bar{\psi}\gamma_5\gamma_3\psi \,|\mathrm{B}\uparrow\rangle . \quad (1)$$

The baryon state of spin-up is denoted as $|\mathrm{B}\rangle$. The quark charge-operator Q can be written in terms of flavor U(3) operators as

$$\tfrac{1}{2}Q^2 = \tfrac{1}{12}[\lambda_3 + (1/\sqrt{3})\lambda_8 + 2\sqrt{\tfrac{2}{3}}\lambda_0], \quad (2)$$

where the λ are the usual Gell-Mann matrices. Eq. (2) inserted into (1) permits us to write the axial current operator in terms of its transformation properties under flavor SU(3).

$$\Gamma(\mathrm{B}) = \tfrac{1}{12}\langle \mathrm{B}\uparrow | \sum_i \bar{\psi}\gamma_5\gamma_3 \times [\lambda_3 + (1/\sqrt{3})\lambda_8 + 2\sqrt{\tfrac{2}{3}}\lambda_0]\psi |\mathrm{B}\rightarrow\rangle . \quad (3)$$

We now introduce the quark axial currents

$$A_3^k = \bar{\psi}\gamma_5\gamma_3\lambda^k\psi . \quad (4)$$

If we compute the matrix elements of A^k in a bag model, the current A^k is discontinuous on the surface of the bag. To restore chiral symmetry in bag models we replace the quark axial currents in eq. (3) by

$$A_\mu^k = \bar{\psi}\gamma_5\gamma_\mu\lambda^k\psi\theta(r-R) - f_k\,\partial_\mu \Phi^k\theta(R-r), \quad (5)$$

where Φ^k is an octet of pseudoscalar fields and f_k their decay constant. We demand that this axial current is continuous for $r=R$ [7]. We furthermore make the assumption that the axial current on the right-hand side of eq. (3) should be replaced by the general axial current in eq. (5) when the quark fields are confined. (For the moment we shall forget about the U(1) problem and work as if we have a nonet of axial currents in the linear combinations of eq. (3).) The famous Bjorken sum rule [8]

$$\Gamma(\mathrm{p}) - \Gamma(\mathrm{n}) = \tfrac{1}{6} g_{\mathrm{A}}/g_{\mathrm{v}}, \quad (6)$$

where g_{A} is the measured axial charge of neutron decay, is unchanged when we evaluate the axial current matrix elements as above.

Before continuing we shall recall how g_{A} for neutron decay is evaluated in chiral bag models. If the pseudoscalar field Φ^k is continuous through the bag surface (the cloudy bag model) g_{A} is essentially given by the valence quarks [9]. If there is a two-phase model such that Φ^k is discontinuous at a radius R (which usually equals the bag radius), then there is an important contribution to g_{A} from Φ^k (in the neutron decay case the pionic field Φ^π). In the chiral limit (all masses are zero) the pionic contribution is precisely half of the quark contribution [10] to g_{A} [#1],

$$g_{\mathrm{A}} = g_{\mathrm{A}}^{\mathrm{Q}} + g_{\mathrm{A}}^{\pi} + G = \tfrac{3}{2} g_{\mathrm{A}}^{\mathrm{Q}} + G. \quad (7)$$

Here the quark contribution is

$$g_{\mathrm{A}}^{\mathrm{Q}} = \frac{5}{3}\int_0^R \mathrm{d}r\, r^2 (|F|^2 - \tfrac{1}{3}|G|^2), \quad (8)$$

where the quark wave function in $(r<R)$ the cavity is

$$\psi = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} \mathrm{i}F \\ G\sigma\cdot\hat{r} \end{pmatrix} \chi^{[1/2]} . \quad (9)$$

The pionic contribution is determined by the pionic axial current $(r>R)$ at zero momentum transfer (see eq. (5)), and is for a massive pseudoscalar field found to be [11]

$$g_{\mathrm{A}}^{\pi}(\mu) = \tfrac{5}{9} f_\pi [g_{\pi\mathrm{q}}(\mu)/M](1+\mu R)\exp(-\mu R). \quad (10)$$

Of course, $\mu = 0$ MeV in the chiral limit under discussion, and $g_{\pi\mathrm{q}}(0)$ is the coupling to the quarks determined by requiring the axial current to be continuous for $r=R$. The last contribution to g_{A} in eq. (7) is the gluon exchange current correction to g_{A} [12–14].

From this example it is evident how we shall decompose the contributions to the sum rule of different flavor symmetry into mainly two parts: the quark and the pseudoscalar contributions. The two parts have the same transformation properties in flavor SU(3). We evaluate the sum rules for the proton and neutron in this manner and get

$$\Gamma(\mathrm{p}) = \tfrac{1}{6}\{\tfrac{5}{6}[B(3)+C(3)] + \tfrac{1}{6}[B(8)+C(8)] + \tfrac{2}{3}[B(0)+C(0)] - 2G\}, \quad (11\mathrm{a})$$

[#1] The value of g_{A} is too large with free quarks inside the bag. However, any model with a residual quark interaction inside the bag which reduced sufficiently the quark wave function on the bag surface $(r=R)$ to give the correct pseudoscalar–nucleon coupling constant, gives the correct value for g_{A} according to the Goldberger–Treiman relation (see example later).

$$\Gamma(\mathrm{n}) = \tfrac{1}{6}\{-\tfrac{5}{6}[B(3)+C(3)] + \tfrac{1}{6}[B(8)+C(8)] + \tfrac{2}{3}[B(0)+C(0)] - 3G\}. \quad (11b)$$

Here the quark and the pseudoscalar contributions are

$$B(k) = \int_0^R \mathrm{d}r\, r^2(|F|^2 - \tfrac{1}{3}|G|^2) \quad (12a)$$

and

$$C(k) = \tfrac{1}{3} f_\mathrm{K}(g_{\mathrm{Kq}}/M)[1+\mu(k)R] \exp[-\mu(k)R]. \quad (12b)$$

Since we assume equal masses for u and d quarks and no s (or $\bar{\mathrm{s}}$) in the nucleon's wave function probed at very low momentum transfer we have that all quark integrals are equal

$$B = B(3) = B(8) = B(0). \quad (13)$$

Above the g_{Kq} are the pseudoscalar meson Φ^k–quark couplings and $\mu(k)$ the meson masses. In the chiral limit $\mu(3) = \mu(8) = 0$ MeV. This pionic correction $C(3)$ to the sum rules is much larger than the one from the cloudy bag model [15]. The gluonic correction $G \cong 0.05$ to the sum rules has been evaluated earlier [16]. As shown [12–14], this correction explains the experimentally measured ratio $\Lambda \to \mathrm{pe}\bar{\nu}/\Sigma \to \mathrm{ne}\bar{\nu} \cong 2$ (the SU(6) value is 3), it explains the baryon magnetic moments, e.g. $\mu_{\Xi^-} < \mu_\Lambda = -0.61$ n.m. and it easily restores in the chiral bag models the magnetic moment ratio $\mu_\mathrm{p}/\mu_\mathrm{n} \cong -\tfrac{3}{2}$ [14].

From eqs. (11) we see immediately that the quark terms cancel in the neutron case. In the Bjorken sum rule

$$\Gamma(\mathrm{p}) - \Gamma(\mathrm{n}) = \tfrac{10}{36}[B(3)+C(3)] + \tfrac{1}{6}G = \tfrac{1}{6} g_\mathrm{A}/g_\mathrm{v} \quad (14)$$

we recognize $\tfrac{10}{6}[B(3)+C(3)]$ as $g_\mathrm{A}^\mathrm{Q} + g_\mathrm{A}^\pi$, the quark and pion contributions to the axial charge of neutron decay. We now invoke the axial anomaly which makes the flavor SU(3) singlet Φ^0 very massive, i.e. $C(0) \cong 0$ using eq. (12b). As stated earlier, in the chiral limit

$$C(3) = C(8) = \tfrac{1}{2}B, \quad (15)$$

which means

$$6\Gamma(\mathrm{p}) = \tfrac{13}{6}B - 2G \quad (16a)$$

and

$$6\Gamma(\mathrm{n}) = -\tfrac{1}{3}B - 3G. \quad (16b)$$

From the Bjorken sum rule (14) we get $B = 0.48$ which means

$$\Gamma(\mathrm{p}) = 0.157 \quad (17a)$$

and

$$\Gamma(\mathrm{n}) = -0.052. \quad (17b)$$

Since the EMC experiment was performed at finite $Q^2 \cong 10.7$ GeV/c we have to include the QCD corrections of the structure function in order to compare with the EMC result [1]

$$\Gamma(\mathrm{p}) = 0.114 \pm 0.012 \pm 0.026. \quad (18)$$

We then find 0.146 using three flavors when we use $\alpha_\mathrm{S}(Q^2 \cong 10\ \mathrm{GeV}^2/c^2) = 0.25$. Our theoretical estimate is well within the experimental errors.

The analysis above has been performed in the chiral limit. In nature chiral symmetry is broken and we proceed as if this symmetry is explicitly broken by adding a mass to the quarks which in turn results in a mass on the pseudoscalar fields. In this case also the $C(8)$ contribution to the sum rules, eqs. (11), is much reduced due to the large masses of the η and η' compared to the pionic contribution $C(3)$.

As an example of a model where we will perform explicit calculations we will use the LAPP version [17] of the MIT bag model. This model is characterized by letting the quarks experience a central attractive potential to simulate that the quarks do not move freely inside the bag. To permit analytic solutions of the Dirac equation a Coulomb-type potential

$$V(r) = \beta(1/R - 1/r), \quad r \leqslant R \quad (19)$$

is chosen. In the present context we have used a chiral invariant formulation of the LAPP bag and performed calculations of observables following ref. [18]. One set of parameters that gives $g_\mathrm{A} = 1.25$ for neutron decay is $B^{1/4} = 110$ MeV, $Z_0 = -2.5$, $\alpha_\mathrm{s} = 2.4$, $\beta = 0.93$ and $m = 15$ MeV. Here B is the bag pressure. Z_0 denotes the "zero point energy" and α_s the quark–gluon coupling constant which together with pionic contributions gives the correct nucleon–Δ mass difference. With these parameters the additional observables we have calculated are: the nucleon mass $M = 938$ MeV, the proton magnetic moment and RMS radius are 2.63 n.m. [#2] and 0.93 fm [#2], respectively, and the neutron magnetic moment is -1.73 n.m. [#2]. In this calculation we find the pion–nucleon coupling constant to be $g_{\pi N}^2/4\pi = 14.2$. All of these values are quite reasonable and we find in this model calculation the bag integral, eq. (15), to be $B=0.528$ and the contribution to g_A from the pion cloud is 0.33. With these numbers we find, using the QCD correction $(1-\alpha_s/\pi)$ for the flavor three and eight components and three flavors for the singlet correction

$$\Gamma(\mathrm{p}) \times \mathrm{QCD\ correction} = 0.147 \tag{20a}$$

and

$$\Gamma(\mathrm{n}) \times \mathrm{QCD\ correction} = -0.046. \tag{20b}$$

These values are surprisingly similar to our earlier estimates using the chiral limit. The fact that the pion contributions to $\Gamma(\mathrm{p})$ and $\Gamma(\mathrm{n})$ are reduced compared to the chiral limit case ($\mu=0$ MeV) is almost exactly balanced by the large reduction of the isosinglet flavor octet meson contribution in eqs. (11) when keeping g_A the same.

In our calculations we have obtained results for integrals over structure functions which are approximately half-way between the naive SU(6) results and Jaffe's result [5] when he suggested that (due to the axial anomaly) the flavor singlet amplitude is paying an insignificant part in $\Gamma(\mathrm{p})$ and $\Gamma(\mathrm{n})$. Our argument that the flavor SU(3) pseudoscalar singlet meson η_0 is heavy, due to the axial anomaly, reduces the (naive) SU(3) singlet contribution by a factor $\frac{2}{3}$. Analyzing the flavor effect of the "perturbative" gluon exchange correction (also responsible for most of the N–Δ mass difference in our model), the remaining flavor SU(3) singlet amplitude is reduced further by approximately one-third. This one-gluon exchange contribution also influences the flavor octet amplitudes.

As our analysis rests on a specific model calculation we can therefore identify the spin content of the proton. A large part of the spin comes from the valence quarks and the remainder comes partly from the intermediate states containing one P-wave antiquark and four S-wave quarks through the "perturbative" one-gluon exchange, and partly from the faction of the state where the nucleon is dissociated into one nucleon (or delta) and one pion.

[#2] These numbers are only the SU(6) quark wave function values in this model calculation.

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