

Fixed-Point Analysis of the Low-Energy Constants in the Pion-Nucleon Chiral Lagrangian

Youngman KIM,^{1,2} Fred MYHRER² and Kuniharu KUBODERA²

¹*School of Physics, Seoul National University, Seoul 151-742, Korea*

²*Department of Physics and Astronomy, University of South Carolina, Columbia,
South Carolina 29208, USA*

(Received March 2, 2004)

In the framework of heavy-baryon chiral perturbation theory, we investigate the fixed point structure of renormalization group equations (RGE) for the ratios of the renormalized low energy constants (LECs) that feature in the pion-nucleon chiral Lagrangian. The ratios of the LECs deduced from our RGE analysis are found to be in semi-quantitative agreement with those obtained from direct fit to the experimental data. The naturalness of this agreement is discussed using a simple dimensional analysis combined with Wilsonian RGEs.

§1. Introduction

As is well known, the long-distance or low-energy behaviour of QCD is not amenable to perturbative approaches. Chiral perturbation theory (ChPT) provides a highly useful framework for correlating hadronic observables in the low-energy regime.¹⁾ In ChPT, possible terms in the effective Lagrangian, \mathcal{L}_{eff} , are classified in terms of the chiral index ν ; the contribution of a term with index ν carries the factor $(Q/\Lambda)^\nu$, where Q is a typical energy-momentum scale involved in a given process, and Λ is the scale of short-range processes that have been integrated out. Since ChPT is a non-renormalizable theory, one must introduce, for each given order ν , new counter-terms containing unknown coupling constants, called the low energy constants (LECs). These LECs reflect short-distance physics that has been integrated out. After renormalization at each chiral order, the finite part of the LECs is to be determined from empirical data, see e.g., Refs. 2) and 3). Once the LECs are determined, \mathcal{L}_{eff} can be used to make predictions on those observables which have not been used as input.

The present work is concerned with the renormalization group properties of the LECs in the pion-nucleon sector. Our work is motivated by a similar study for the pion sector by Atance and Schrempp.⁴⁾ In Ref. 4) it was discussed that there exist non-trivial infrared fixed points in the renormalization group equations (RGEs) for the ratios of LECs in the pion sector. These fixed points exist in the limit $\mu \rightarrow 0$, where μ is a renormalization scale.^{*)} Atance and Schrempp estimated, for

^{*)} Whether or not the fixed points in the limit $\mu \rightarrow 0$ govern low energy physics of the system in question is a subtle question, reflecting the arbitrariness of the renormalization scale μ in ChPT. This is in contrast to the case of renormalizable theory such as QCD, where one must choose μ as a typical energy of the process in question to keep perturbation theory meaningful, see e.g. Ref. 13) for details.

the $\nu = 4$ and $\nu = 6$ mesonic chiral Lagrangians, the LEC ratios at the relevant infrared fixed points. The resulting values of the ratios were found to be in reasonable agreement with those determined from data: qualitative agreement in the numerical values and perfect agreement in the signs. We describe here an application of a similar fixed-point analysis to the pion-nucleon Lagrangian. We show that, by studying the infrared fixed points in the RGEs for the ratios of LECs, one can estimate the LEC ratios pertaining to the pion-nucleon sector and that the results are in semi-quantitative agreement with those obtained from direct fitting to the experimental data. We argue that this agreement can be understood by invoking a simple dimensional analysis, and we discuss our results in the context of the Wilsonian renormalization scheme. The analysis done in this work is related to the method of coupling constant reduction^{5),6)} in the sense that both generate the renormalization group invariant relations between coupling parameters (LECs in the present case).*)

§2. RGE fixed points and ratios of LECs

ChPT is based on a low-energy effective Lagrangian, \mathcal{L}_{eff} , which is the most general possible Lagrangian consistent with spontaneously broken chiral symmetry of QCD. \mathcal{L}_{eff} is written as an expansion in $(Q/\Lambda)^\nu$ (see e.g. Refs. 2), 7) and 8) for details):

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \dots, \quad (2.1)$$

where the superscript denotes the chiral order ν , and “...” stands for higher order terms in the expansion. In this work we focus on the third-order pion-nucleon Lagrangian, $\mathcal{L}_{\pi N}^{(3)}$. Using heavy-baryon chiral perturbation theory, and adopting the notational conventions of Ref. 2), we write

$$\mathcal{L}_{\pi N}^{(3)} = \mathcal{L}_{\pi N}^{(3),\text{fixed}} + \sum_{i=1}^{23} d_i \bar{H} \tilde{O}_i H + \sum_{i=24}^{31} \tilde{d}_i \bar{H} \tilde{O}_i^{\text{div}} H. \quad (2.2)$$

Here H is the heavy nucleon field, and \tilde{O}_i and \tilde{O}_i^{div} are operators constrained by chiral symmetry considerations. We refer to Ref. 2) for a complete list of \tilde{O}_i and \tilde{O}_i^{div} . The LECs, d_i and \tilde{d}_i , cannot be constrained by symmetry, and their behaviour is our main concern here. The ultraviolet (UV) divergences of loop-diagrams (if they exist) are absorbed in d_i via dimensional regularization as

$$d_i = d_i^r(\mu) + \frac{(4\pi)^2}{(4\pi F)^2} \kappa_i L(\mu), \quad (2.3)$$

where μ is a renormalization scale, and

$$L(\mu) = \frac{\mu^{d-4}}{(4\pi)^2} \left(\frac{1}{d-4} - \frac{1}{2} [\log 4\pi + 1 + \Gamma'(1)] \right). \quad (2.4)$$

*) We thank the referee for pointing this out to us.

The constant F is the leading term in the quark-mass expansion of the pion decay constant F_π . A complete list of analytic expressions for κ_i can be found in Refs. 2) and 8). The values of the finite renormalized LECs, $d_i^r(\mu)$, are to be either fixed by phenomenology or estimated with the use of models.

From Eq. (2.3) one can derive⁸⁾ a renormalization group equation (RGE) for the renormalized LEC, $d_i^r(\mu)$,

$$\mu \frac{d}{d\mu} d_i^r(\mu) = -\frac{\kappa_i}{(4\pi F)^2}. \quad (2.5)$$

Note that Eq. (2.5) implies that there are no fixed points for the renormalized LEC. We integrate Eq. (2.5) to obtain

$$d_i^r(\mu) = d_i^r(\mu_0) - \frac{\kappa_i}{(4\pi F)^2} \log \frac{\mu}{\mu_0}. \quad (2.6)$$

Equation (2.5) leads to the RGE for the ratio d_i^r/d_j^r Ref. 4),

$$\mu \frac{d}{d\mu} \left(\frac{d_i^r(\mu)}{d_j^r(\mu)} \right) = \frac{1}{(4\pi F)^2} \frac{\kappa_j}{d_j^r(\mu)} \left(\frac{d_i^r(\mu)}{d_j^r(\mu)} - \frac{\kappa_i}{\kappa_j} \right). \quad (2.7)$$

We emphasize that these fixed points exist only for the ratios of LECs and therefore it might be better to refer to them as “infrared quasi-fixed points”⁹⁾ or as “Pendleton-Ross infrared fixed points.”⁶⁾ However, for the sake of simplicity, we shall continue to use the word “infrared fixed points”. We note that, as can be easily seen from Eq. (2.6), $\kappa_j/d_j^r(\mu)$ in Eq. (2.7) is positive in the limit $\mu \rightarrow 0$. Therefore Eq. (2.7) gives a non-trivial stable fixed point (f.p.) in the variable d_i^r/d_j^r in the limit $\mu \rightarrow 0$,^{*})

$$\left. \frac{d_i^r}{d_j^r} \right|_{\text{f.p.}} = \frac{\kappa_i}{\kappa_j} \quad \text{for } \kappa_i, \kappa_j \neq 0. \quad (2.8)$$

We note that the derivation of this equation involves only few assumptions. We shall study the consequences of Eq. (2.8) in what follows. Since κ_i 's in the pion-nucleon sector are known, we can predict the l.h.s. of Eq. (2.8). Meanwhile, the value of the l.h.s. can also be obtained from the LECs that have been deduced directly from the experimental data.^{2),3)} The comparison of these two determinations will check the validity of Eq. (2.8).

For this comparison, it is useful to consider the scale-independent LEC, \bar{d}_i , defined by

$$d_i = \bar{d}_i + \frac{\kappa_i}{(4\pi F)^2} \left((4\pi)^2 L(\mu) - \log \frac{\mu}{M} \right), \quad (2.9)$$

where M is the leading term in the quark mass expansion of the pion mass. The \bar{d}_i 's have been determined phenomenologically in Ref. 2). Equations (2.3) and (2.9) lead to a relation between \bar{d}_i and $d_i^r(\mu)$:

$$\bar{d}_i = d_i^r(\mu) + \frac{\kappa_i}{(4\pi F)^2} \log \frac{\mu}{M}, \quad (2.10)$$

^{*}) Equation (2.7) has a non-trivial stable f.p. in the limit $\mu \rightarrow \infty$ as well. However, we will focus on the f.p.'s in the limit $\mu \rightarrow 0$ for a reason to be discussed in §3.

which implies

$$d_i^r(M) = \bar{d}_i. \quad (2.11)$$

Since the pion mass M is a small scale in ChPT, we presume that the ratio of LECs at $\mu = M$, $d_i^r(M)/d_j^r(M)$, is close to its fixed-point value:

$$\frac{d_i^r}{d_j^r} \Big|_{\text{f.p.}} \approx d_i^r(M)/d_j^r(M). \quad (2.12)$$

From this equation and Eq. (2.8), we arrive at our key relation

$$\kappa_i/\kappa_j \approx \bar{d}_i/\bar{d}_j. \quad (2.13)$$

This relation allows us to constrain the ratios of LECs without any input from experiments. However, a cautionary remark may be in order here. The present analysis is not applicable to a case in which the β function of an LEC vanishes, because in such a case the RGEs for the ratios of LECs involving the LEC with the vanishing β function do not show any non-trivial fixed-point structure like the one in Eq. (2.8). In general, this could limit the applicability of a method described here. But, in a particular case of the third-order pion-nucleon chiral Lagrangian $\mathcal{L}_{\pi N}^{(3)}$, all the β functions of the LECs as listed in Ref. 8) are non-zero. In Ref. 2), the authors use a basis different from Ref. 8) for the chiral Lagrangian $\mathcal{L}_{\pi N}^{(3)}$ and they list most of the β functions for the LECs in that basis. We can see that the β functions listed in Ref. 2) are non-zero. Therefore, our analysis is in practice free from the problem of a vanishing β function. More discussion about the relation in Eq. (2.13) will be given later in the text. In the following, we compare the left- and right-hand sides of Eq. (2.13).

Experimental information available for this comparison may be summarized as follows. In Refs. 2) and 3), \bar{d}_i 's of $\mathcal{L}_{\pi N}^{(3)}$ have been determined by analyzing pion-nucleon scattering. Table I gives the values of $\bar{d}_i = d_i^r(\mu = M)$ determined by Fettes et al.^{2),3)} In Ref. 2), the authors investigated pion-nucleon scattering in a $\nu=3$ ChPT calculation assuming exact isospin symmetry. A more general ChPT $\nu=3$ analysis of pion-nucleon scattering was implemented in Ref. 3), wherein electromagnetic corrections as well as $\nu = 2$ isospin violating terms of $\mathcal{L}_{\pi N}^{(2)}$ were taken into account. Since the terms in $\mathcal{L}_{\pi N}^{(3)}$ are higher order correction to $\mathcal{L}_{\pi N}^{(2)}$, one naturally expects that the determination of the counter-terms in $\mathcal{L}_{\pi N}^{(3)}$ is strongly influenced by the improvement in the treatment of $\mathcal{L}_{\pi N}^{(2)}$. Indeed, Table I shows drastic differences between the values of the LECs obtained in Refs. 3) and 2). In the following we shall therefore be primarily concerned with the $\nu = 3$ LECs obtained in the more general analysis in Ref. 3).

In addition to the LECs presented in Table I, Eq. (2.2) contains one more $\nu=3$ LEC determined from experiment. In Ref. 10), $d_6^r(\mu)$ was deduced from the isovector charge radius at $\mu = \overset{\circ}{m}$, where $\overset{\circ}{m}$ is the value of the nucleon mass in the chiral limit; the result is $d_6^r(\overset{\circ}{m}) = -0.13 \text{ GeV}^{-2}$. For our present purposes, we need to

Table I. Phenomenologically determined values of the LECs (in GeV^{-2}) in $\mathcal{L}_{\pi N}^{(3)}$. The values in the rows labelled EXP(Fit 1), EXP(Fit 2) and EXP(Fit 3) are taken from Ref. 2), while those in the row EXP(2001) are taken from Ref. 3).

	$\bar{d}_1 + \bar{d}_2$	\bar{d}_3	\bar{d}_5	$\bar{d}_{14} - \bar{d}_{15}$	\bar{d}_{18}
EXP(Fit 1)	3.06 ± 0.21	-3.27 ± 0.73	0.45 ± 0.42	-5.65 ± 0.41	-1.40 ± 0.24
EXP(Fit 2)	3.31 ± 0.14	-2.75 ± 0.18	-0.48 ± 0.06	-5.69 ± 0.28	-0.78 ± 0.27
EXP(Fit 3)	2.68 ± 0.15	-3.11 ± 0.79	0.43 ± 0.49	-5.74 ± 0.29	-0.83 ± 0.06
EXP(2001)	-2.24 ± 0.16	0.81 ± 0.16	0.67 ± 0.11	-0.63 ± 0.75	-10.14 ± 0.45

scale $d_6^r(\mu)$ down to $\mu = M$. Carrying out this rescaling using Eq. (2.6), we obtain $d_6^r(M) = \bar{d}_6 = -2.96 \text{ GeV}^{-2}$. This value of $d_6^r(M)$ will also be considered below.

We now examine the two sides of Eq. (2.13). Using the analytic expressions of κ_i given in Refs. 2) and 8) and $g_A = 1.26$, we obtain $\kappa_1 + \kappa_2 = -0.42$, $\kappa_3 = 0.18$, $\kappa_5 = 0.37$ and $\kappa_6 = -1.49$. The use of these values of κ_i in Eq. (2.13) leads to RGE-based predictions of the LEC ratios at the scale $\mu = M$. The results are given in the last column (labelled ‘‘RGE’’) in Table II. This table also shows the LEC ratios determined from the experimental information summarized in Table I. (For simplicity and because of the semi-quantitative nature of the present study, we do not quote errors in the experimental data.) Since \bar{d}_{15} and \bar{d}_{18} are finite LECs and independent of μ ,²⁾ the ratios involving these LECs cannot be determined from our analysis based on infrared fixed points.

In Table II we note that the ratios given in the column labeled EXP (2001) and those obtained in our RGE analysis show semi-quantitative agreement; the signs are all in agreement and the magnitudes exhibit a similar general tendency. We note that the results of our RGE analysis show definite disagreement with those of the earlier (presumably less reliable) empirical determinations of the LECs. This may be

Table II. The ratios of the LECs in $\mathcal{L}_{\pi N}^{(3)}$. The data in the columns labeled EXP(Fit 1), EXP(Fit 2) and EXP(Fit 3) are taken from Ref. 2), which ignored the isospin breaking term in $\mathcal{L}_{\pi N}^{(2)}$. The results in the column labeled EXP(2001) were obtained with the use of the LECs in the last row in Table I. These LECs were determined in Ref. 3) in an analysis that includes all the relevant terms in $\mathcal{L}_{\pi N}^{(2)}$.

Ratio of LECs	EXP (Fit 1)	EXP (Fit 2)	EXP (Fit 3)	EXP (2001)	RGE
$(\bar{d}_1 + \bar{d}_2)/\bar{d}_3$	-0.94	-1.23	-0.86	-2.77	-2.33
$(\bar{d}_1 + \bar{d}_2)/\bar{d}_5$	6.8	-6.9	6.23	-3.34	-1.14
\bar{d}_3/\bar{d}_5	-7.27	5.79	-7.23	1.21	0.49
$(\bar{d}_1 + \bar{d}_2)/\bar{d}_6$	-1.03	-1.12	-0.91	0.76	0.28
\bar{d}_3/\bar{d}_6	1.1	0.93	1.1	-0.27	-0.12
\bar{d}_5/\bar{d}_6	-0.15	0.16	-0.15	-0.23	-0.25

Table III. Examples of the predicted ratios of the LECs that appear in $\mathcal{L}_{\pi N}^{(3), (2)}$. The symbol $d_i^{(j)}$ is defined as $d_i^{(j)} \equiv \bar{d}_i/\bar{d}_j$.

$d_8^{(5)}$	1.35	$d_{10}^{(5)}$	0.63	$d_8^{(6)}$	-0.34	$d_{10}^{(6)}$	-0.16
$d_{11}^{(5)}$	-0.29	$d_{12}^{(5)}$	-2.33	$d_{11}^{(6)}$	0.07	$d_{12}^{(6)}$	0.58
$d_{13}^{(5)}$	-0.72	$d_{14}^{(5)}$	-1.7	$d_{13}^{(6)}$	0.18	$d_{14}^{(6)}$	0.42
$d_{16}^{(5)}$	1.03	$d_{20}^{(5)}$	3.41	$d_{16}^{(6)}$	-0.26	$d_{20}^{(6)}$	-0.85

taken as an indication that we can profitably use a fixed-point analysis like the one described here as a constraint in determining the LECs from data. To illustrate this point, we present here predictions on the ratios of LECs that involve LECs which have not so far been determined by experiments. Although the number of ratios that can be predicted is in fact quite large, we consider here only some examples. Predictions for these selected cases are given in Table III.

§3. Discussion and summary

We observed that the RGE for the ratio d_i^r/d_j^r of the renormalized LEC has a non-trivial infrared fixed point given by Eq. (2.8). We now discuss the implication of Eq. (2.8) in the context of a simple dimensional analysis.^{11),12)} To this end, we first briefly review the naive dimensional analysis given in Ref. 11). Consider the π - π scattering amplitude at order Q^4 . The amplitude of a one-loop diagram involving the lowest order Lagrangian $\mathcal{L}_{\pi\pi}^{(2)}$ is given by

$$\frac{Q^4}{F^4} \frac{1}{(4\pi)^2} \log \mu + \dots \quad (3.1)$$

The necessary counter-terms come from the $\nu=4$ Lagrangian $\mathcal{L}_{\pi\pi}^{(4)}$. A term like

$$\frac{F^2}{\mu_{\chi SB}^2} \text{tr}(\partial^\mu \Sigma \partial^\nu \Sigma \partial_\mu \Sigma^\dagger \partial_\nu \Sigma^\dagger) \quad (3.2)$$

gives a scattering amplitude of order cQ^4/F^4 , where c is given by $c = F^2/\mu_{\chi SB}^2$. Since the total π - π scattering amplitude should not depend on the renormalization scale μ , a shift in μ should be compensated by a shift in c . Therefore a change in μ of order one produces a change in c of order $\delta c \sim 1/(4\pi)^2$. Then, barring the accidental fine tuning of the parameters, c must be of the order of δc ,

$$c \sim \delta c \sim \frac{1}{(4\pi)^2}. \quad (3.3)$$

This implies that $\mu_{\chi SB} \sim 4\pi F$.¹¹⁾ Now, an advantage of using the RGE is that one can calculate δc explicitly. To show this point in our case, we return to Eq. (2.6). As stated, observables should be independent of μ . From Eq. (2.6) we can see that

a change in μ produces a change in the LEC $d_i^r(\mu)$:

$$\begin{aligned} \delta d_i^r &\equiv d_i^r(\mu) - d_i^r(\mu_0) \\ &= -\frac{\kappa_i}{(4\pi F)^2} \log \frac{\mu}{\mu_0}. \end{aligned} \tag{3.4}$$

As discussed above, from $\delta d_i^r(\mu) \sim d_i^r(\mu)$, we may infer

$$\frac{d_i^r(\mu)}{d_j^r(\mu)} \sim \frac{\kappa_i}{\kappa_j}. \tag{3.5}$$

We now discuss, in the context of a Wilsonian renormalization group, whether the fixed points featuring in Eq. (2.8) are relevant to low energy physics.^{*)} To do this, we follow Ref. 14) and derive a Wilsonian RGE that includes quadratic divergences as well as logarithmic divergences. To preserve chiral symmetry, dimensional regularization is adopted, and the quadratic divergences are identified by the following replacement:^{14), 15)}

$$\int \frac{d^n k}{i(2\pi)^n} \frac{1}{-k^2} \rightarrow \frac{\Lambda^2}{(4\pi)^2}, \quad \int \frac{d^n k}{i(2\pi)^n} \frac{k_\mu k_\nu}{[-k^2]^2} \rightarrow -\frac{\Lambda^2}{2(4\pi)^2} g_{\mu\nu}, \tag{3.6}$$

where the scale Λ has the meaning of a naive cutoff. These replacements are known to preserve chiral symmetry at the one-loop order.¹⁶⁾ For illustration, we again use the one-loop diagram of the π - π scattering. Since the logarithmic divergences in the π - π scattering are well known, we focus on the quadratic divergences. Let us take a one-loop diagram with one vertex from $\mathcal{L}_{\pi\pi}^{(2)}$ and the other one from $\mathcal{L}_{\pi\pi}^{(4)}$. The vertex from $\mathcal{L}_{\pi\pi}^{(2)}$ is of the form p^2/F^2 and the one from $\mathcal{L}_{\pi\pi}^{(4)}$ has the form $L_i p^4/F^4$, where L_i is a LEC in $\mathcal{L}_{\pi\pi}^{(4)}$. We consider the contribution of a particular term in which only two of the six derivatives act on the internal lines. If we apply Eq. (3.6) to this term, its quadratic divergence is given schematically by

$$L_i \frac{p^4}{F^6} \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2} \rightarrow L_i \frac{p^4}{F^4} \frac{\Lambda^2}{(4\pi F)^2}. \tag{3.7}$$

We may assume that there are similar quadratic divergences in the π - N sector. We can then generalize the RGE in Eq. (2.5) and write:

$$\mu \frac{d}{d\mu} d_i^r(\mu) = -\frac{\kappa_i}{\Lambda_{\chi SB}^2} + g_i \frac{\mu^2}{\Lambda_{\chi SB}^2} d_i^r(\mu) + h_i \frac{\mu^2}{\Lambda_{\chi SB}^2}, \tag{3.8}$$

where $\Lambda_{\chi SB} = 4\pi F$ and g_i and h_i are calculable constants. The term with g_i in Eq. (3.8) is similar to the quadratic divergence in Eq. (3.7), and the term with h_i is a possible quadratic divergence with all vertices coming from $\mathcal{L}_{\pi N}^{(1)}$ and $\mathcal{L}_{\pi N}^{(2)}$. It is easy to obtain from Eq. (3.8) a modified RGE for the ratio of the LECs

$$\begin{aligned} \mu \frac{d}{d\mu} \left(\frac{d_i^r(\mu)}{d_j^r(\mu)} \right) &= \frac{\kappa_j}{d_j^r(\mu)} \left[\frac{1}{\Lambda_{\chi SB}^2} \left(\frac{d_i^r(\mu)}{d_j^r(\mu)} - \frac{\kappa_i}{\kappa_j} \right) \right. \\ &\quad \left. + \frac{1}{\kappa_j} \frac{\mu^2}{\Lambda_{\chi SB}^2} \left(g_i d_i^r - g_j d_j^r + h_i - h_j \frac{d_i^r(\mu)}{d_j^r(\mu)} \right) \right]. \end{aligned} \tag{3.9}$$

^{*)} We thank M. Rho for suggesting this viewpoint.

In this Wilsonian RGE, the terms with $\mu^2/\Lambda_{\chi SB}^2$ become negligible in a low-energy regime ($\mu^2 \ll \Lambda_{\chi SB}^2$), and, as a result, we recover the RGE in Eq. (2.7). This feature suggests that the infrared fixed points in Eq. (2.8) govern low energy physics qualitatively up to $\mu^2/\Lambda_{\chi SB}^2$ corrections.

Finally we discuss the significance of our infrared fixed points in relation to QCD. As is well known, the form of a ChPT Lagrangian is chosen using the chiral symmetry properties of QCD without invoking detailed dynamical information on QCD. This feature, combined with the fact that a fixed point (if any) of a given field theory is generally independent of the initial conditions, may make one wonder whether there is any physical relation between QCD and the infrared fixed points considered in the present work. Although a formal treatment of this issue goes beyond the scope of this article, we present a plausibility argument that indicates that these fixed points are likely to have physical significance. First, we invoke the general tenet of ChPT that, if the LECs appearing in ChPT are determined to reproduce the relevant low-energy observables, these LECs effectively subsume the short-distance physics of QCD. The phenomenological success of ChPT indicates the basic soundness of this tenet; in this phenomenological sense ChPT is physically related to QCD. Meanwhile, once the ChPT Lagrangian is specified, its renormalization group properties and fixed points are automatic consequences of the structure of the given Lagrangian. These two aspects lead us to expect that the infrared fixed points considered in the present work are physically (if not formally) related to QCD. Admittedly, this is just a plausibility argument. In this connection, there exists a very illuminating study by Harada and Yamawaki.¹⁷⁾ These authors investigated a hidden local symmetry (HLS) approach as an effective theory of QCD, by matching HLS and QCD at a matching scale $\Lambda \sim 1$ GeV. They found that in HLS there occur three fixed points and one fixed line in the physical region but that only one of them, a so-called vector manifestation (VM) point, has correspondence with QCD. We recognize that it is not possible to make a *direct* connection between the results reported in Ref. 17) and those obtained in our present study, because Ref. 17) discusses the fixed points in the LECs themselves whereas we are considering here the ratios of LECs. After stating this warning, we may add the following speculative remark. In our consideration of the RGE for ChPT, we have found only one fixed point for each ratio of the LECs. It therefore seems natural to expect that the fixed points determined in the present work have bearing upon QCD. Again, this is just an intuitive argument, but we believe that, with this caveat kept in mind, the results given in this work can be of practical use.

In summary, we have estimated the ratios of the scale invariant LECs, \bar{d}_i/\bar{d}_j , by studying the fixed point structure of the RGE for the ratios of the renormalized LECs. We have found that the ratios of the LECs determined from experiments^{3),10)} and the ones estimated by our RGE analysis agree semi-quantitatively. We have given a plausible explanation for this agreement, invoking a simple dimensional analysis and the Wilsonian RGE. To address the subtle problem in ChPT, whether or not the limit $\mu \rightarrow 0$ can be associated with a low-energy scale of the system, we have resorted

to the Wilsonian RGE, Eq. (3·9), and argued that the fixed points in Eq. (2·8) govern low-energy physics qualitatively up to $\mu^2/\Lambda_{\chi SB}^2$ corrections. This argument combined with the fact that the pion mass M is a small scale in ChPT leads us to presume that the ratio of the LECs at $\mu = M$, $d_i^r(M)/d_j^r(M)$, is close to its infrared fixed-point value. We should perhaps repeat here our cautionary remark (see §2) that what we call “infrared fixed point” should, to be precise, be referred to as “infrared quasi-fixed point”⁹⁾ or as “Pendleton-Ross infrared fixed point.”⁶⁾ It is hoped that an analysis like the one described here may be useful in placing constraints on the values of other LECs in ChPT as well.

Acknowledgements

We thank S. Ando for discussion. Our thanks are also due to M. Harada, U.-G. Meissner and M. Rho for their helpful comments. This work is supported in part by the U.S. National Science Foundation, Grant Nos. PHY-0140214 and INT-9730847. The work of YK is supported in part by the Brain Korea 21 Project of the Ministry of Education, by the KOSEF R01-1999-000-00017-0 and by the KRF 2001-015-DP0085.

References

- 1) S. Weinberg, *Physica A* **96** (1979), 327.
For reviews, see e.g., G. Colangelo and G. Isidori, hep-ph/0101264.
U.-G. Meissner, *Rep. Prog. Phys.* **56** (1993), 903; hep-ph/9302247.
- 2) N. Fettes, U.-G. Meissner and S. Steininger, *Nucl. Phys. A* **640** (1998), 199; hep-ph/9803266.
- 3) N. Fettes and U.-G. Meissner, *Nucl. Phys. A* **693** (2001), 693; hep-ph/0101030.
- 4) M. Atance and B. Schrempp, hep-ph/9912335; hep-ph/0009069.
- 5) R. Oehme, *Lecture Notes in Phys.* **558** (Springer, 2000), p. 136; hep-th/9903092.
- 6) J. Kubo, *Lecture Notes in Phys.* **558** (Springer, 2000), p. 106; hep-ph/9903482.
- 7) N. Fettes, U.-G. Meissner, M. Mojzsis and S. Steininger, *Ann. of Phys.* **283** (2000), 273 [Errata; **288** (2001), 249]; hep-ph/0001308.
- 8) G. Ecker, *Phys. Lett. B* **336** (1994), 508; hep-ph/9402337.
- 9) M. Lanzagorta and G. G. Ross, *Phys. Lett. B* **349** (1995), 319; hep-ph/9501394.
- 10) V. Bernard, N. Kaiser and U.-G. Meissner, *Int. J. Mod. Phys. E* **4** (1995), 193; hep-ph/9501384.
- 11) A. Manohar and H. Georgi, *Nucl. Phys. B* **234** (1984), 189.
- 12) A. V. Manohar, hep-ph/9606222.
- 13) S. Weinberg, *The Quantum Theory of Fields*, Vol. II (Cambridge University Press, 1996).
- 14) M. Harada and K. Yamawaki, *Phys. Rev. Lett.* **83** (1999), 3374; hep-ph/9906445; *Phys. Rev. D* **64** (2001), 014023; hep-ph/0009163.
- 15) M. Veltman, *Acta. Phys. Pol. B* **12** (1981), 437.
- 16) M. Harada, private communication.
- 17) M. Harada and K. Yamawaki, *Phys. Rev. Lett.* **87** (2001), 152001; hep-ph/0105335.