Elastic pp and $p\bar{p}\to\pi\pi$ reactions in short- and middle-distance QCD

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Arguments are presented for the expected behavior of the reaction $p\bar{p}\to\pi^+\pi^-$ at high energy and large scattering angle. This reaction can have a sizable analyzing power, and both it and the scaled cross section $s^8 d\sigma/dt$ will oscillate with energy since the QCD phase difference between the independent scattering (Landshoff) and short-distance processes is energy dependent at high but not superasymptotic energies. The argument follows studies of pp elastic scattering, where we update the fits to its scaled cross section and show that the behavior of the double polarization quantity A_{nn} can be explained simultaneously. As we will see, $p\bar{p}\to\pi^+\pi^-$ is theoretically simpler than pp elastic scattering because it has only two helicity amplitudes. Furthermore, it would be possible to measure this reaction (and also $p\bar{p}\to\pi^0\pi^0$ and K^+K^-) at a future SuperLEAR or KAON facility.

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I. INTRODUCTION

Considerations of how QCD works for hadron-hadron exclusive reactions have been greatly stimulated by both theoretical and experimental studies of pp elastic scattering at high momentum transfer. Possibly an even better process to study, and the subject of this work, is $p\bar{p}\to\pi^+\pi^-$ (or K^+K^-). It is, in several ways, simpler theoretically than pp elastic scattering; it could show more strikingly phenomena seen but inadequately mapped out in the pp cases, and it may be possible to measure both the cross section $d\sigma/dt$ at high momentum transfer and the analyzing power A_{0n} over the full kinematic range.

The reaction $p\bar{p}\to\pi^+\pi^-$ is simpler than pp elastic scattering if only because it has fewer helicity amplitudes. A consequence of this is that there is less chance of averaging out the oscillatory or polarization effects that we shall discuss. Further, QCD perturbation theory diagrams will involve fewer lines. An experimental consideration is that one expects the cross sections $d\sigma/dt$ at wide angles to fall roughly like a power law, s^{-n}, where s is the square of the center-of-mass energy, and n is around 10 for pp elastic scattering and around 8 for $p\bar{p}\to\pi^+\pi^-$. Hence, annihilation into pion pairs has a bigger cross section at high enough energy than pp elastic scattering.[1] Also, polarized gas jet targets or other means may allow one to measure single polarization quantities even when the cross section is getting small, so that A_{0n} could be measured over a wide range of high energies and wide angles.

The phenomena of interest observed in pp elastic scattering is first the differential cross section $d\sigma/dt$ at 90°, which oscillates with energy about an otherwise smooth power-law falloff [1–3]. In addition, pp elastic scattering has a significant analyzing power A_{0n} even at fairly high momentum transfers [4,5]. We will present some details of one explanation (other explanations are also possible [6]) of these measurements which involves both short-distance ("hard") QCD amplitudes and Landshoff amplitudes [7] (which have multiple independent scatterings) contributing to the total amplitude. This explanation is known [8] but requires some reworking in light of recent developments [9]. Examples of graphs leading to similar "hard" QCD amplitudes and to Landshoff amplitudes for the $p\bar{p}\to\pi^+\pi^-$ case are shown in Fig. 1. The "hard" QCD amplitude is exemplified by the graph in Fig. 1(a) which is completely connected, even without the hadron wave functions. In Fig. 1(a), all of the internal propagators are far off shell for high energy, wide angle scattering (except if one or more of the quark momentum fractions approaches zero, in which case factors in the hadronic wave functions squelch this particular contribution). Hence, all the interaction vertices must be spatially near each other; i.e., one has a short-distance process. An example of a graph contributing to the Landshoff amplitude is illustrated in Fig. 1(b), which has two unconnected pieces corresponding to two independent scatterings. While each scattering is itself a short-distance process, there is no immediate argument that one independent scattering need be close to the other. It has been shown that the size of the hadron will

[1] Using this power law to extrapolate to $p_{\rm lab}=15$ GeV/c we find $d\sigma/dt$ at wide angles (90°) to be about 0.03 nb/GeV^2 for $\bar{p}p\to\pi^+\pi^-$ which is about a factor 30 smaller that $d\sigma/dt\approx 1$ nb/GeV^2 for the measured elastic pp scattering.

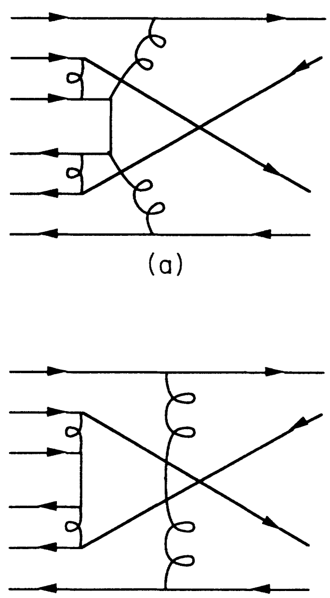

FIG. 1. (a) An example of short-distance QCD diagrams to order $\alpha_s^4(s)$ for the process $p\bar{p}\to\pi^+\pi^-$. The diagram has an s^{-3} dependence. (b) An example of diagrams for large-angle Landshoff process for the reaction $p\bar{p}\to\pi^+\pi^-$ of order $\alpha_s^3(s)$. The timelike gluon and one quark are off shell and the diagram gives an $s^{-5/2}$ behavior when we neglect radiative corrections.

determine the likely separation of these two independent scatterings [9]. The independent scattering diagram as drawn is lower order in perturbation theory and also has different energy dependence than the short-distance process. Before discussing the annihilation reaction $p\bar{p}\to\pi^+\pi^-$ further, we will, in the next section, discuss pp elastic scattering, where relevant data exist. Then we will return to our discussion of the theoretically simpler case of $p\bar{p}\to\pi^+\pi^-$.

II. SCENARIO FOR *pp* ELASTIC SCATTERING

The pp elastic-scattering amplitude for a hard QCD process falls like s^{-4}, leading to a $d\sigma/dt$ (90°) that decreases like s^{-10}. The naive counting of the Landshoff amplitude gives an s^{-3} behavior, but radiative corrections change the exponent to about (-3.8) for three flavors of quark [9,10]. The crucial realization is that the radiative corrections also give the quark-quark scattering amplitude a phase, which is calculable in perturbative QCD [11]. In hadronic processes, the form for the phase that follows from calculations is [9]

$$a \ln\left[\frac{\ln(s/\Lambda^2)}{\ln(1/b^2\Lambda^2)}\right]+\text{const}\ , \tag{1}$$

where b is some mean impact parameter measuring the spatial separation between two independent scatterings. The radiative corrections encourage the independent scatterings to come closer together, and give [9]

$$b=\frac{1}{\Lambda}\left[\frac{\sqrt{s}}{\Lambda'}\right]^{-\tau}\ , \tag{2}$$

where τ is about 0.70 for three flavors of quark. Hence, one sees an energy dependence of the phase at what one might call medium-high energy, although asymptotically the phase obviously becomes energy independent as stressed in Refs. [9,12].

This suggests writing the five pp elastic scattering amplitudes as

$$\varphi_i \propto s^{-4}M_i = s^{-4}(B_i+C_i s^{0.2}e^{i[\psi_i(s)+\delta_i]})\ , \tag{3}$$

where B_i, C_i, and δ_i are real constants, $i=1,\ldots,5$, and

$$\psi_i(s)=a\ln\left[\frac{\ln(s/\Lambda^2)}{\ln(s/\Lambda_i^2)}\right]\ . \tag{4}$$

For 90° c.m. scattering, one has $\varphi_5=0$ and $\varphi_4=-\varphi_3$, and we find it convenient to work with the sum and difference $\varphi_\pm=(\varphi_1\pm\varphi_2)/\sqrt{2}$. Then,

$$s^{10}\frac{d\sigma}{dt}\propto |M_+|^2+|M_-|^2+2|M_3|^2\equiv R\ . \tag{5}$$

One can fit the above result to the data above some value of s using a variety of parameters. One set, whose result is illustrated along with the data in Fig. 2, is summarized in Table I. The values of B_i and C_i in the table imply dominance of the Landshoff process at energies where perturbative QCD is working. The fit in Fig. 2 is mainly to illustrate how the concept could work. Other parametrizations are possible and we have not done an exhaustive search. One expects the fit to fail at low s and for the present fit this seems to mean s below about 10 GeV^2, which is not unreasonable. For the values of s used here, the values of b are well below 1 fm, prompting us to use the phrase "middle-distance QCD." The form of the phase is gotten from the asymptotic calculation. Botts [12] has shown that the region where the asymptotic calculation is numerically applicable is at somewhat higher energy than we consider. In using these ideas, we are supposing that important subasymptotic contributions have similar behavior, and in this vein we allow the coefficient a, which determines the energy periodicity, to be larger than suggested in Refs. [9,12].

The pp data indicate a significant analyzing power at high momentum transfers [4,5]. This is noteworthy because, if short-distance processes dominate, large values of the analyzing power are not permitted according to

TABLE I. Parameters for the fits to the pp elastic-scattering data whose results are shown in Figs. 2 and 3. B_i are given in arbitrary units, C_i are in units of GeV$^{-0.2}$ relative to B_i, and Λ_i are in units of GeV. We use $\Lambda=\Lambda_{QCD}=0.2$ GeV.

$i=$	+	−	3
a	4π	4π	4π
B_i	1.34	1.34	0.375
C_i	0.20	0.14	0.375
Λ_i	2.2	2.1	1.4
δ_i	0.93π	-0.55π	0.40π

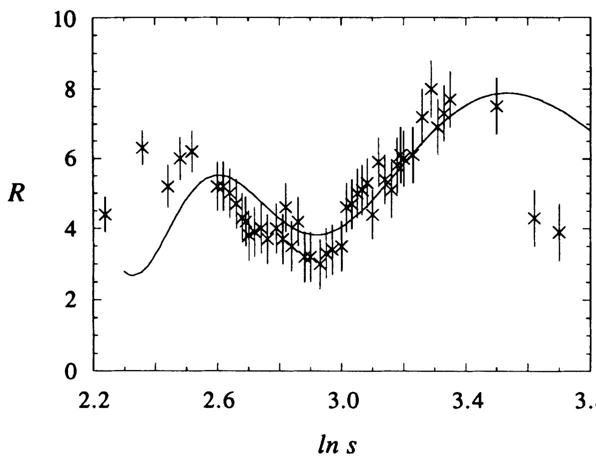

FIG. 2. The scaled pp elastic differential cross section, Eq. (5), as a function of ln$s$ (in GeV^2) is drawn with parameters given in Table I. The experimental points are taken from [1]. We have not tried very hard to fit the two highest-energy points. For a better, though simpler, fit see [26].

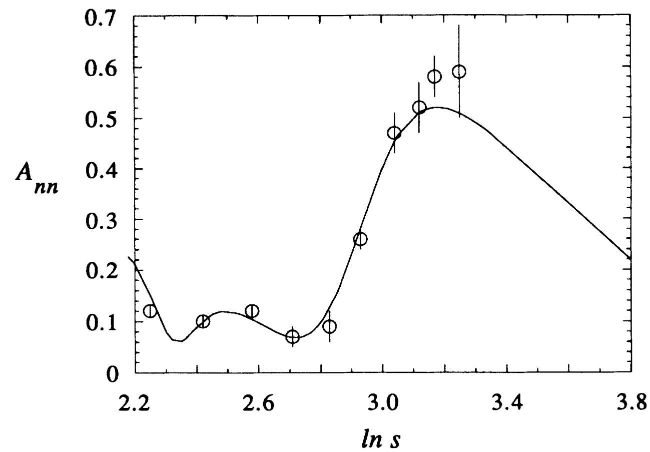

FIG. 3. The beam-target spin correlation A_{nn} at 90° c.m. for pp elastic scattering, Eq. (6), as a function of ln$s$ (in GeV^2) is drawn with parameters given in Table I. The experimental points are taken from [16].

the following arguments [13,14]: in high momentum transfer processes, the quark helicity is conserved. Furthermore, in a short-distance reaction, the limited transverse quark momentum in a hadronic bound state leads to the quarks having no orbital angular momentum along the direction of motion. Consequently, quark helicity conservation translates into hadron helicity conservation, which, in turn, leads generally to zero analyzing power. On the other hand, for the relatively long-distance process of Landshoff, the product of quark transverse momentum and transverse distance is sufficient to vitiate the hadron helicity conservation arguments [15]. Hence, the Landshoff process could give significant analyzing power even at high momentum transfer. The available data [4,5] on the pp elastic analyzing power at high momentum transfer is not sufficient to check this claim of oscillations in A_{0n}. However, because of the possible different values of the δ_i, the oscillatory behavior of A_{0n} might be averaged out in this case [8].

There are good data on the beam-target spin correlation A_{nn} at 90° c.m. for pp elastic scattering up to $s=26$ GeV^2 [16]. A_{nn} is not predicted to be zero even if the short-distance processes dominate [17,18], but one would not expect large variations of A_{nn} in a regime where one can use perturbative QCD unless there were interference effects. In terms of our amplitudes, we have

$$A_{nn}=(|M_+|^2-|M_-|^2+2|M_3|^2)/R \tag{6}$$

and the result of using the Table I parameters is shown in Fig. 3 along with the data. If we can use perturbative QCD phases at these energies,the data on R and A_{nn} require at least two helicity amplitudes to have different energy periods, as reflected in the parameters shown in Table I.

The energy behavior of A_{nn} at 90° has also been addressed by Brodsky and de Teramond [6] who postulated the existence of a type of dibaryon resonance, and more recently by Ramsay and Sivers [19] whose approach is close to ours. They introduce a relative normalization of the "hard" QCD and Landshoff amplitudes via certain "form factors" and adjust the form factors to fit the data. They do not have an energy-dependent oscillating phase as we do. One may compare their Fig. 10 for A_{nn} to our Fig. 3.

III. THE REACTION $p\bar{p}\to\pi^+\pi^-$

There are several reasons why we want to consider another exclusive hadronic reaction to test our understanding of the perturbative QCD mechanisms in their short-distance and multiple independent scattering guises, particularly if $\bar{p}$ capabilities with sufficient energy, intensity, and polarization options are available. As discussed, elastic scattering of pp is described by five helicity amplitudes, which complicates the analysis [8]. In contrast, the reaction $p\bar{p}\to\pi^+\pi^-$ (and also $p\bar{p}\to\pi^0\pi^0$ and K^+K^-) is described by only two helicity amplitudes f_{++} and f_{+-}, where the subscripts give the helicity of p and $\bar{p}$. Further, the short-distance amplitude is real and only its contribution to f_{+-} is asymptotically nozero. The Landshoff process, in general, contributes to both f_{++} and f_{+-}. We will therefore have a signal for the presence of two different quark processes by establishing as a function of energy an oscillatory behavior in the cross section (about a smooth power-law falloff), which is due to the interference of the amplitudes. In addition, if the Landshoff contribution to f_{++} is comparable to the short-distance contribution to f_{+-}, then the analyzing power A_{0n} of this reaction will not only be nonzero but will oscillate with energy for fixed wide angle scattering. The beauty of this is that both observables can be measured at the SuperLEAR or KAON facilities if built.

For $p\bar{p}\to\pi^+\pi^-$ we expect that the short- and "medium"-distance amplitudes will have the following s dependences:

$$f_S\propto s^{-3} \quad \text{and} \quad f_L\propto s^{-2.85}\ , \tag{7}$$

where the naive power count for the Landshoff process is

$s^{-5/2}$ and the radiative corrections have the same effect as for $\pi\pi$ elastic scattering [9,10] since the number of independent scatterings is the same. The slower falloff with increasing s for this reaction compared to pp elastic scattering is consistent with measurements up to $p_{\rm lab}=6.2$ GeV/c or $s=13.5$ GeV^2 [20,21]. The QCD phase is basically determined by the large-angle quark-quark scattering in the Landshoff process, and we expect the period of oscillation to be of the same order of magnitude for $p\bar{p}\to\pi^+\pi^-$ wide angle scattering as for both elastic pp or elastic $p\bar{p}$ scattering. We further anticipate that the oscillations will commence at about the same $p_{\rm lab}$ for all three processes.

Below $p_{\rm lab}$, approximately 3 GeV/c, or s below about 7.7 GeV^2, there are indications of "resonance activity" in the s channel of the $p\bar{p}\to\pi^+\pi^-$ reaction [22]. These "resonances" were seen earlier in a simple analysis of $d\sigma/d\Omega$ for this reaction [23]. There has been a more recent analysis of this reaction and of the reaction $p\bar{p}\to K\bar{K}$, including consideration of the very large measured asymmetry of both reactions. The data for $p_{\rm lab}$ up to 2.2 GeV were explained using a diffraction model assuming that the partial-wave helicity flip amplitudes f_{+-} are given as the impact parameter derivative of the helicity noflip amplitude f_{++} [24]. This analysis has no apparent resonances and describes the data reasonably well. In any case, the physics questions addressed in the present work concern the reactions at much higher energies than those in the "resonance region."

Since the annihilation reaction is given by only two helicity amplitudes, and since the short-distance contribution to f_{++} is zero, we have a simpler situation for examining experimentally the multiple independent scattering (Landshoff) process and its phase. In Fig. 4, we have plotted the present experimental data [20,21,25] (we have interpolated $d\sigma/dt$ between two angles at the two highest energies [20,21]) for the differential cross section of $p\bar{p}\to\pi^+\pi^-$ scaled by s^8. Unfortunately, the measurements are not at high enough energy to clearly see the oscillatory behavior of R, although the general trend accords with the naive scaling law. The asymmetry, which should also show an oscillatory behavior, is given by

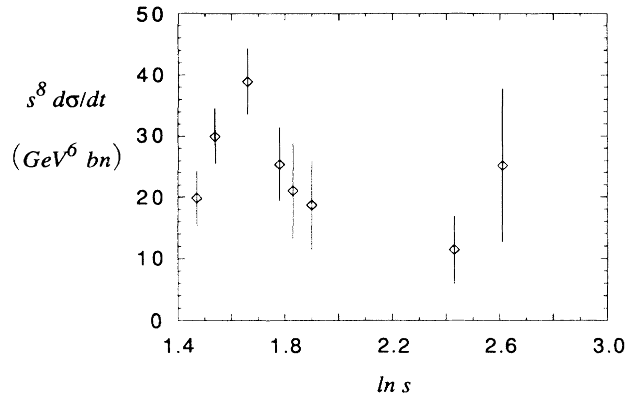

FIG. 4. The ration $R=s^8 d\sigma/dt(90°)$ for the reaction $p\bar{p}\to\pi^+\pi^-$ as a function of ln$s$ (in GeV^2). The experimental points are taken from [20,21,25]; we have interpolated $d\sigma/dt$ between two angles at the two highest energies [20,21].

$$A_{0n}=2\frac{{\rm Im}(f^*_{++}f_{+-})}{|f_{++}|^2+|f_{+-}|^2}\,, \tag{8}$$

where f_{++} will have only f_L, while f_{+-} is $(f_{S+-}+f_{L+-}\exp\{i[\psi_{+-}(s)+\delta_{+-}]\})$, where f_{S+-} and f_{L+-} are real. One has

$$A_{0n}=-2\frac{f_{L++}f_{S+-}\sin(\psi_{++}+\delta_{++})+f_{L++}f_{L+-}\sin(\Delta\psi+\Delta\delta)}{f^2_{S+-}+f^2_{L+-}+f^2_{L++}+2f_{S+-}f_{L+-}\cos(\psi_{+-}+\delta_{+-})}\,, \tag{9}$$

where $\Delta\psi=\psi_{++}-\psi_{+-}$ and similarly for $\Delta\delta$. Given the larger $\bar{p}$ intensity anticipated at new facilities, the reaction $p\bar{p}\to\pi^+\pi^-$ can be investigated in detail at large angles, and due to the slower falloff with s compared to pp elastic scattering, it should be possible to measure this reaction at 90° for the range $2<p_{\rm lab}<15$ GeV/c and beyond. With only two helicity amplitudes one can expect to disentangle completely the phases and energy dependence of both of them.

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