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Comparison of the extended linear σ model and chiral perturbation theory

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The pion-nucleon-scattering amplitudes are calculated in tree approximation with the use of the extended linear sigma model (ELSM) as well as heavy-baryon chiral perturbation theory (HBχPT), and the nonrelativistic forms of the ELSM results are compared with those of HBχPT. We find that the amplitudes obtained in ELSM do not agree with those derived from the more fundamental effective approach, HBχPT.

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The linear sigma model [1], which provides an illuminating example of spontaneous chiral symmetry breaking in strong interactions, has been studied extensively in the literature. Some of the consequences of this model, however, are known to be in conflict with observation. Notably, the isoscalar pion-nucleon- (πN) scattering length predicted by the model is larger than the experimental value by an order of magnitude. Furthermore, the model predicts the axial coupling constant g_A to be unity, whereas empirically $g_A \approx 1.26$. Despite the known limitations of the model, its simplicity has invited many authors to use it for exploring the consequences of chiral symmetry in nuclear physics; see, e.g., Ref. [2]. Nauenberg and Bjorken [3] and Lee [4] introduced an extended linear sigma model (ELSM) by adding a pair of extra terms (which jointly preserve chiral symmetry) to the original linear sigma model Lagrangian. An important feature of ELSM is that g_A is no longer restricted to be unity. Furthermore, via chiral rotations of the fields, ELSM leads to the nonlinear chiral Lagrangian of Weinberg (with $g_A \neq 1$) in the limit of an infinitely massive scalar field. Recently ELSM has been used to investigate the g_A dependence of the πN-scattering lengths and the πN sigma term Σ_N [5]. It has been found in Ref. [5] that ELSM can reproduce the very small experimental value of the πN isoscalar scattering length, $a^{(+)}_{\pi N}$, and furthermore the same model can reproduce the large empirical value of the πN sigma term, Σ_N, without invoking any $\bar{s}s$ component of the nucleon.

Meanwhile, low-energy hadronic physics can be described by an effective field theory (EFT) of QCD known as "chiral perturbation theory" (χPT) [6,7]. The χPT Lagrangian, $\mathcal{L}_{\chi\mathrm{PT}}$, reflects the symmetries and the pattern of symmetry breaking of the underlying QCD. $\mathcal{L}_{\chi\mathrm{PT}}$ is expanded in powers of $Q/\Lambda_\chi \ll 1$, where Q denotes the typical four-momentum of the process in question or the pion mass, m_π, which represents the small explicit chiral symmetry breaking scale; $\Lambda_\chi \simeq 4\pi f_\pi \simeq 1$ GeV, is the chiral scale. The parameters appearing in $\mathcal{L}_{\chi\mathrm{PT}}$, called the *low-energy constants* (LECs), effectively subsume the high-energy physics that has been integrated out. These LECs could in principle be determined from the underlying theory, but in practice they are fixed phenomenologically from experimental data. Once the LECs are determined, $\mathcal{L}_{\chi\mathrm{PT}}$ represents a complete and hence model independent Lagrangian up to a specified chiral order. Furthermore, starting from $\mathcal{L}_{\chi\mathrm{PT}}$, one can develop, for the amplitude of a given process, a well-defined perturbation scheme by organizing the relevant Feynman diagrams according to powers in Q/Λ_χ. If all the Feynman diagrams up to a given power, ν, in Q/Λ_χ are taken into account, then the results are model independent up to this order, with the contributions of higher order terms suppressed by an extra power of Q/Λ_χ. A problem one encounters in extending χPT to the nucleon sector is that, as the nucleon mass m_N is comparable to the cutoff scale Λ_χ, a straightforward application of expansion in Q/Λ becomes difficult [8]. This difficulty can be circumvented by employing heavy-baryon chiral perturbation theory (HBχPT) [9], which essentially consists in shifting the reference point of the nucleon energy from 0 to m_N and in integrating out the small component of the nucleon field as well as the antinucleonic degrees of freedom. An effective Lagrangian in HBχPT therefore involves as explicit degrees of freedom the pions and the large components of the redefined nucleon field. The expansion parameters in HBχPT are Q/Λ_χ, m_π/Λ_χ, and Q/m_N. Because $m_N \approx \Lambda_\chi$, it is convenient to combine chiral and heavy-baryon expansions and introduce the chiral index $\bar{\nu}$ defined by $\bar{\nu} = d + (n/2) - 2$. Here n is the number of fermion lines that participate in a given vertex, and d is the number of derivatives (with m_π counted as one derivative). A similar power counting scheme can also be introduced for Feynman diagrams as well [7,10]. HBχPT has been used with great success to the one-nucleon sector, see, e.g., Ref. [11].

We therefore consider it informative to compare the predictions of ELSM with those of HBχPT. [1] As an example of this comparison, we consider here the tree-level πN-scattering amplitudes calculated in ELSM and HBχPT to lowest order corrections in Q/Λ_χ.

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[1]In this connection, it is worth noting that Weinberg's nonlinear chiral Lagrangian is the lowest order lagrangian of χPT, see, e.g., Ref. [8].

The Lagrangian of the extended sigma model (ELSM) consists of the standard linear sigma model Lagrangian plus two pion-nucleon interaction terms with a common coupling constant proportional to $(g_A - 1)$. The additional terms are a vector- and a pseudovector coupling term [3,4]. Thus the Lagrangian of ELSM reads as follows:

$$\begin{aligned}\mathcal{L} &= \bar{\psi} i \partial \psi - g\bar{\psi}[\sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau}]\psi + \frac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2] \\ &+ \frac{1}{2}\mu_0^2[\sigma^2+\vec{\pi}^2] - \frac{\lambda}{4}[\sigma^2+\vec{\pi}^2]^2 + \mathcal{L}_{\chi sb} \\ &+ \left(\frac{g_A-1}{f_\pi^2}\right)\left[\left(\bar{\psi}\gamma_\mu\frac{\vec{\tau}}{2}\psi\right)\cdot(\vec{\pi}\times\partial^\mu\vec{\pi})\right. \\ &+ \left.\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\vec{\tau}}{2}\psi\right)\cdot(\sigma\partial^\mu\vec{\pi} - \vec{\pi}\partial^\mu\sigma)\right], \end{aligned} \quad (1)$$

where the parameters λ and μ_0 are assumed to be real and positive. The last line proportional to $(g_A - 1)$ represents the additional πN coupling terms introduced in [3,4]. As for the explicit chiral symmetry breaking term $\mathcal{L}_{\chi sb}$, we consider three terms (see, e.g., Refs. [12,13]):

$$\mathcal{L}_{\chi sb} = \varepsilon_1\sigma - \varepsilon_2\vec{\pi}^2 - \varepsilon_3\bar{\psi}\psi. \quad (2)$$

The first term is the "standard" chiral symmetry breaking term in the linear sigma model, whereas the second term arises naturally in χPT. The third term proportional to ε_3 was discussed in, e.g., Refs. [5,12,13], and we remark that a term proportional to $\bar{\psi}\psi$ appears in χPT with a coefficient proportional to $m_\pi^2 c_1$, where c_1 is a low-energy constant in χPT [8].

As usual, we redefine the scalar field relative to its vacuum expectation value, $\langle\sigma\rangle_0 = f_\pi$, and introduce the new scalar field s defined by $s = \sigma - f_\pi$. The requirement that the energy is minimum for $\langle\sigma\rangle_0 = f_\pi$ gives the following relation $\mu_0^2 - \lambda f_\pi^2 = -\varepsilon_1/f_\pi$. The pion mass is found to be

$$m_\pi^2 = \varepsilon_1/f_\pi \ + \ 2\varepsilon_2. \quad (3)$$

In what follows we evaluate the πN-scattering amplitude using the Lagrangian in Eq. (1) properly modified to account for the redefinition of the scalar field, $\sigma \to s$, explained above.

The πN-scattering T matrix is conventionally written as follows:

$$T_{\alpha\beta} = T^{(+)}\,\delta_{\alpha\beta} + T^{(-)}\,\tfrac{1}{2}[\tau_\alpha, \tau_\beta], \quad (4)$$

where α and β are the initial and final pion isospin indices, respectively, and $T^{(\pm)}$ are defined as follows:

$$T^{(\pm)} = A^{(\pm)} + B^{(\pm)}\tfrac{1}{2}\gamma_\mu\left(k_1^\mu + k_2^\mu\right). \quad (5)$$

Here k_1 and k_2 are the incoming and outgoing pion momenta, respectively, in the center-of-mass system. It is understood that, to obtain the scattering amplitude, $T_{\alpha\beta}$ should be sandwiched between the relevant Dirac spinors [which, however, are suppressed in Eq. (4)]. To compare the πN amplitudes evaluated in ELSM with the ones obtained in HBχPT, we have to treat the nucleons in ELSM as heavy, nonrelativistic fields of mass m_N. Therefore, the ELSM amplitudes, $A^{(\pm)}$ and $B^{(\pm)}$, in Eq. (5) and the Dirac spinors describing the initial and final nucleons in ELSM need to be expanded in powers of $1/m_N \equiv 1/M$. The corresponding nonrelativistic πN-scattering amplitudes, $g^{(\pm)}$ and $h^{(\pm)}$, are customarily defined by the following:

$$\begin{aligned}\tilde{T}_{\alpha\beta} &= [g^{(+)} + i\vec{\sigma}\cdot(\vec{k}_1\times\vec{k}_2)\,h^{(+)}]\,\delta_{\alpha\beta} \\ &\quad + [g^{(-)} + i\vec{\sigma}\cdot(\vec{k}_1\times\vec{k}_2)\,h^{(-)}]\,\tfrac{1}{2}[\tau_\alpha,\ \tau_\beta]. \end{aligned} \quad (6)$$

It is understood here that $\tilde{T}_{\alpha,\beta}$ is to be sandwiched between the initial and final nucleon Pauli spinors and isospinors to yield the scattering amplitude. The amplitudes, $g^{(\pm)}$ and $h^{(\pm)}$, calculated in ELSM and HBχPT are denoted by $g_{\rm ESM}^{(\pm)}$, $h_{\rm ESM}^{(\pm)}$, $g_{\chi\rm PT},^{(\pm)}$ and $h_{\chi\rm PT}^{(\pm)}$, respectively. Comparison between $g_{\rm ESM}^{(\pm)}$ and $g_{\chi\rm PT}^{(\pm)}$ and between $h_{\rm ESM}^{(\pm)}$ and $h_{\chi\rm PT}^{(\pm)}$ is our main concern in what follows.

In Ref. [5] the elastic πN-scattering amplitude in ELSM was calculated in the tree approximation and the expressions for the four amplitudes, $A^{(+)}$, $A^{(-)}$, $B^{(+)}$, and $B^{(-)}$, are given in Eqs. (17a–d) of Ref. [5].[2] The corresponding $g_{\rm ESM}^{(\pm)}$ and $h_{\rm ESM}^{(\pm)}$ amplitudes were derived in Ref. [14]. The πN-scattering amplitude in HBχPT was evaluated by, e.g., Meissner *et al.* [15]. For our present purposes, we only need the tree approximation amplitudes. The amplitudes, $g_{\chi\rm PT}^{(\pm)}$ and $h_{\chi\rm PT}^{(\pm)}$, were rederived in Ref. [14] and it has been confirmed that in the tree approximation the results agree with those of Ref. [15].

As mentioned, Weinberg's nonlinear sigma model can be derived from ELSM in the limit of $m_\sigma \to \infty$. Therefore, to facilitate comparison with the HBχPT expressions, the amplitudes obtained in ELSM are further simplified by assuming that m_σ is heavy compared to the pion mass and energy and expanding the amplitudes in powers of $1/m_\sigma$. We assume that $m_\sigma \simeq M \simeq \Lambda_\chi \simeq 1$ GeV, whereas m_π, ω, and $\sqrt{t}$ are of order $Q \ll \Lambda_\chi$. We also assume that the chiral symmetry breaking parameters ε_i are of order Q^2. We restrict our comparison to the lowest powers of Q in each amplitude, and we use the fact that the LECs, c_i $(i = 1, 2, 3)$, in HBχPT are of the natural order of magnitude ($c_i\Lambda_\chi \sim 1$).

In comparing the amplitudes obtained in the two approaches under consideration, we find it convenient to introduce the following decompositions:

$$g_{\rm ESM}^{(\pm)} = \tilde{g}^{(\pm)} + \delta g_{\rm ESM}^{(\pm)}\,, \qquad g_{\chi\rm PT}^{(\pm)} = \tilde{g}^{(\pm)} + \delta g_{\chi\rm PT}^{(\pm)}\,, \quad (7)$$

$$h_{\rm ESM}^{(\pm)} = \tilde{h}^{(\pm)} + \delta h_{\rm ESM}^{(\pm)}\,, \qquad h_{\chi\rm PT}^{(\pm)} = \tilde{h}^{(\pm)} + \delta h_{\chi\rm PT}^{(\pm)}\,. \quad (8)$$

In the above, $\tilde{g}^{(\pm)}$ represents the part that has a common analytic expression between $g_{\rm ESM}^{(\pm)}$ [5,14] and $g_{\chi\rm PT}^{(\pm)}$ [14,15], whereas $\delta g_{\rm ESM}^{(\pm)}$ and $\delta g_{\chi\rm PT}^{(\pm)}$ represent the parts that do not have common analytic expressions. Similarly for $\tilde{h}^{(\pm)}$. The terms common between ELSM and HBχPT are given by the

[2] We remark that the overall sign of the amplitude $B^{(-)}$, Eq. (17d) in [5], should be changed.

following:

$$\tilde{g}^{(+)} = \left(\frac{g_A^2}{f_\pi^2}\right)\left(\frac{2\omega^2 m_\pi^2 - \omega^4 + (\vec{k}_1\cdot\vec{k}_2)^2}{4M\omega^2}\right)$$
$$\tilde{g}^{(-)} = \left(\frac{\omega}{2f_\pi^2}\right) - \left(\frac{g_A^2}{f_\pi^2}\right)\frac{\vec{k}_1\cdot\vec{k}_2}{2\omega} + \frac{1}{f_\pi^2}\left(\frac{\omega^4 - m_\pi^2\omega^2 + \omega^2(\vec{k}_1\cdot\vec{k}_2)}{4M\omega^2}\right) + \left(\frac{g_A^2}{f_\pi^2}\right)\frac{1}{4M\omega^2}\big[-2\omega^4 + 2\omega^2 m_\pi^2 - m_\pi^2(\vec{k}_1\cdot\vec{k}_2) - \omega^2(\vec{k}_1\cdot\vec{k}_2) + (\vec{k}_1\cdot\vec{k}_2)^2\big]$$
$$\tilde{h}^{(+)} = -\frac{g_A^2}{2\omega f_\pi^2} - \left(\frac{g_A^2}{f_\pi^2}\right)\left(\frac{\omega^2 + m_\pi^2 - \vec{k}_1\cdot\vec{k}_2}{4M\omega^2}\right)$$
$$\tilde{h}^{(-)} = \frac{1}{f_\pi^2}\left(\frac{\omega^2 - g_A^2(\vec{k}_1\cdot\vec{k}_2)}{4M\omega^2}\right). \tag{9}$$

As for $\delta g^{(\pm)}$ and $\delta h^{(\pm)}$, our ELSM calculation leads to the following results:

$$\delta g^{(+)}_{\rm ESM} = \frac{M}{f_\pi^2}\left\{-\frac{\left[t - m_\pi^2 + 2\varepsilon_2\right]}{m_\sigma^2} - \frac{\varepsilon_3}{M} + \mathcal{O}\left(M^{-2}, m_\sigma^{-2}\right)\right\} \tag{10}$$

$$\delta g^{(-)}_{\rm ESM} = \frac{g_A}{f_\pi^2}\left[\frac{\varepsilon_3\left(t - 2m_\pi^2\right)}{2M\omega} + \cdots\right] \tag{11}$$

$$\delta h^{(+)}_{\rm ESM} = -\frac{1}{f_\pi^2}\left(\frac{t - m_\pi^2 + 2\varepsilon_2}{4Mm_\sigma^2} - \frac{g_A\varepsilon_3}{M\omega}\right) + \cdots \tag{12}$$

$$\delta h^{(-)}_{\rm ESM} = 0. \tag{13}$$

Meanwhile, an HBχPT calculation (in tree approximation) gives, to the order Q^2/Λ_χ^2, the following results:

$$\delta g^{(+)}_{\chi\rm PT} = \left(c_3\left[t - 2m_\pi^2\right] + 4m_\pi^2\, c_1 - 2\omega^2\, c_2\right)\frac{1}{f_\pi^2} \tag{14}$$

$$\delta g^{(-)}_{\chi\rm PT} = 0 \tag{15}$$

$$\delta h^{(+)}_{\chi\rm PT} = 0 \tag{16}$$

$$\delta h^{(-)}_{\chi\rm PT} = -\frac{c_4}{f_\pi^2}, \tag{17}$$

where $t = (k_1 - k_2)^2$ is the four-momentum transfer [14].

We now discuss to what extent the sigma model (ELSM in our case) simulates the effective field theory (here HBχPT). We first look at the results for $g^{(+)}$. It is informative to examine what values ELSM gives to the LECs, c_i $(i = 1, \cdots, 4)$, that appear in HBχPT. To this end, let us impose the requirement $\delta g^{(+)}_{\rm ESM} = \delta g^{(+)}_{\chi\rm PT}$. Comparison of the momentum transfer (t) and energy (ω) dependences in Eq. (10) and Eq. (14) leads us to identify the following:

$$c_3 \sim -\frac{M}{m_\sigma^2}, \quad c_1 \sim -\frac{M}{m_\sigma^2}\left(\frac{m_\pi^2 + 2\varepsilon_2}{4m_\pi^2}\right) - \frac{\varepsilon_3}{4m_\pi^2} \text{ and } c_2 = 0. \tag{18}$$

The result $c_2 = 0$ means that ELSM fails to generate the energy dependence of $g^{(+)}$ required by HBχPT. With the use of the sigma-meson mass scale, $m_\sigma \sim M \sim 1$ GeV, we find $c_3 \sim c_1 \sim -1\ \text{GeV}^{-1}$. These results are not inconsistent with those found in Ref. [16] based on the resonance saturation assumption. There it was shown that the Δ resonance gives a major contribution to c_2 and c_3, whereas the empirical value of c_1 can be explained by a scalar resonance contribution in the two-pion channel. Furthermore, this scalar-meson resonance was found to give a ∼30% contribution to c_3 [16]. These features are compatible with our finding that ELSM, which contains no Δ field, leads to $c_2 = 0$ and to the value of c_3 that is significantly smaller than the empirically determined value. As for $g^{(-)}$, we notice that $\delta g^{(-)}_{\rm ESM}$ in Eq. (11) is of order $\mathcal{O}(Q^3)$, i.e., this amplitude has no terms of order $\mathcal{O}(Q^2)$. This feature is consistent with Eq. (15). Regarding the "spin-flip" amplitudes $h^{(\pm)}$, we note that $h^{(\pm)}$ in Eq. (6) are accompanied by a factor of $\mathcal{O}(Q^2)$. This means that, to the chiral order under consideration, the comparison of the ELSM and HBχPT results for $h^{(\pm)}$ should be limited to the $\mathcal{O}(1)$ terms. Comparison between $\delta h^{(-)}_{\rm ESM}$ in Eq. (13) and $\delta h^{(-)}_{\chi\rm PT}$ in Eq. (17) leads to the conclusion that $c_4 = 0$. This implies that ELSM cannot generate the isovector, spin-dependent term in the πN-scattering amplitude predicted by HBχPT. Again we refer to Ref. [16], where it is shown that the empirical value of c_4 can be explained, within the resonance saturation assumption, by dominant contributions from the Δ resonance and the ρ meson. Because none of these hadrons are included in ELSM, it should come as no surprise that $c_4 = 0$ in ELSM. As for $\delta h^{(+)}$, Eq. (12) indicates that $\delta h^{(+)}_{\rm ESM}$ is of $\mathcal{O}(Q)$, i.e., it has no contribution of $\mathcal{O}(1)$. This feature is consistent with the fact that HBχPT generates no $\delta h^{(+)}$ amplitude of chiral orders lower than Q^2 [see Eq. (16)]. We remark en passant that, if we take the limit $m_\sigma \to \infty$ and require $\varepsilon_3 = 0$, then ELSM leads to $c_1 = c_2 = c_3 = c_4 = 0$, and ELSM and HB$\chi$PT give identical tree approximation πN-scattering amplitudes, $g^{(\pm)}_{\rm ESM} = g^{(\pm)}_{\chi\rm PT} = \tilde{g}^{(\pm)}$ and $h^{(\pm)}_{\rm ESM} = h^{(\pm)}_{\chi\rm PT} = \tilde{h}^{(\pm)}$. This is, however, a very special case.

The above comparison indicates that, in general, the ELSM fails to reproduce some of the πN-scattering amplitude properties (e.g., the energy dependence) that are predicted by HBχPT. It is well known that Δ degrees of freedom play a role in describing πN scattering even at very low energies. In HBχPT, although only the pion and nucleon are explicit degrees of freedom, the effects of the Δ resonance are subsumed in the LECs, c_2, c_3, and c_4. By contrast, the Δ field is normally not included in sigma models. This difference seems to be the main cause of the failure for ELSM to reproduce the HBχPT results. A lesson we learn from the present study is that, although the linear sigma model (either in its original version or in the form of ELSM) is often used as a convenient tool for exploring consequences of chiral symmetry in nuclear physics, conclusions obtained in such studies should be taken with caution.

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