

## Two-pion-exchange contributions to the $pp \rightarrow pp\pi^0$ reaction

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### Abstract

Our previous study of the near-threshold  $pp \rightarrow pp\pi^0$  reaction based on a hybrid nuclear effective field theory is further elaborated by examining the momentum dependence of the relevant transition operators. We show that the two-pion-exchange diagrams give much larger contributions than the one-pion-exchange diagram, even though the former is of higher order in the Weinberg counting scheme. The relation between our results and an alternative counting scheme, the momentum counting scheme, is also discussed.

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In the standard nuclear physics approach (SNPA), a nuclear reaction amplitude is calculated with the use of the transition operator derived from a phenomenological Lagrangian and nuclear wave functions generated by a high-precision phenomenological  $NN$  potential. SNPA has been enormously successful in explaining a vast range of nuclear phenomena. Meanwhile, a nuclear chiral perturbation approach based on heavy-baryon chiral perturbation theory (HB $\chi$ PT) is gaining ground as a powerful tool for addressing issues that cannot be readily settled in SNPA. HB $\chi$ PT is a low-energy effective field theory of QCD, based on a systematic expansion in terms of the expansion parameter  $\epsilon \equiv Q/\Lambda_\chi \ll 1$ , where  $Q$  is a typical energy–momentum involved in a process under study or the pion mass  $m_\pi$ , and the chiral scale  $\Lambda_\chi \simeq 4\pi f_\pi \simeq 1$  GeV. HB $\chi$ PT has been applied with great success to low-energy processes including, e.g., pion–nucleon scattering and electroweak reactions on a nucleon and in few-nucleon systems. Our present work is concerned with a HB $\chi$ PT study of the near-threshold  $pp \rightarrow pp\pi^0$  reaction. A motivation of this study may be stated in reference to the generic  $NN \rightarrow NN\pi$  processes near threshold. Although HB $\chi$ PT presupposes the small size of

its expansion parameter  $Q/\Lambda_\chi$ , the pion-production reactions involve somewhat large energy– and three-momentum transfers even at threshold. Therefore the application of HB $\chi$ PT to the  $NN \rightarrow NN\pi$  reactions may involve some delicate aspects, but this also means that these processes may serve as a good test case for probing the limit of applicability of HB $\chi$ PT. Apart from this general issue to be investigated, a specific aspect of the  $pp \rightarrow pp\pi^0$  reaction makes its study particularly interesting. For most isospin channels, the  $NN \rightarrow NN\pi$  amplitude near threshold is dominated by the pion rescattering diagram where the  $\pi N$  scattering vertex is given by the Weinberg–Tomozawa term, which represents the lowest chiral order contribution. However, a quantitatively reliable description of the  $NN \rightarrow NN\pi$  reactions obviously requires detailed examinations of the corrections to this dominant amplitude. Meanwhile, since the Weinberg–Tomozawa vertex does not contribute to the pion–nucleon rescattering diagram for  $pp \rightarrow pp\pi^0$ , this reaction is particularly sensitive to higher chiral-order contributions and hence its study is expected to provide valuable information to guide us in formulating a quantitative description of all the  $NN \rightarrow NN\pi$  reactions (including the channels that involve a deuteron).

The first HB $\chi$ PT-based study of the near-threshold  $pp \rightarrow pp\pi^0$  reaction was made in Refs. [1,2]. In HB $\chi$ PT one naturally expects a small cross section for this reaction since, for  $s$ -wave pion production, the pion–nucleon vertex in the impulse

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approximation (IA) diagram and the pion-rescattering vertex in the one-pion-exchange rescattering ( $1\pi$ -Resc) diagram arise from the next-to-leading-order (NLO) chiral Lagrangian. A remarkable feature found in Refs. [1,2] is that a drastic cancellation between the IA and  $1\pi$ -Resc amplitudes leads to the suppression of the  $pp \rightarrow pp\pi^0$  amplitude far beyond the above-mentioned naturally expected level. This destructive interference is in sharp contrast with the constructive interference reported in SNPA-based calculations [3,4]. It is to be recalled that the  $pp \rightarrow pp\pi^0$  cross section obtained in Refs. [3,4] was significantly smaller (by a factor of  $\sim 5$ ) than the experimental value [5]. The drastic cancellation between the IA and  $1\pi$ -Resc terms found in the HB $\chi$ PT calculations [1,2] leads to even more pronounced disagreement between theory and experiment. In this connection it is worth noting that, according to Lee and Riska [6], the heavy-meson ( $\sigma$  and  $\omega$ ) exchanges can strongly enhance the  $pp \rightarrow pp\pi^0$  amplitude. It is also to be noted that  $\sigma$ -meson-exchange introduced in many  $NN$  potentials is more properly described by correlated two-pion-exchange (see, e.g., Refs. [7,8]), and that there have been substantial developments in deriving a two-pion exchange  $NN$  potential using HB $\chi$ PT, see, e.g., [9]. These developments were conducive to a HB $\chi$ PT study of two-pion-exchange (TPE) contributions to the  $pp \rightarrow pp\pi^0$  reaction [10,11]. In the plane-wave approximation it was found [10] that TPE contributions are indeed very large (as compared to the  $1\pi$ -Resc amplitude), a result that is in line with the finding in Ref. [6]. A subsequent DWBA calculation [11] indicates that this feature remains essentially unchanged when the initial- and final-state interactions are taken into account. More recent investigations [12–15], however, have raised a number of important issues that call for further investigations, and the purpose of our present Letter is to address these issues.

In Ref. [10], to be referred to as DKMS, were derived all the transition operators for  $pp \rightarrow pp\pi^0$  belonging to next-to-next-to-leading order (NNLO) in the Weinberg counting, and these operators were categorized into types I–VII, according to the patterns of the corresponding Feynman diagrams; see Figs. 2–5 in DKMS. Types I, II, III and IV belong to diagrams of the two-pion exchange (TPE) type, while types V, VI and VII arise from diagrams of the vertex correction type. A notable feature pointed out in DKMS is that the contributions of types II–IV are by far the largest, and that they even exceed those of the  $1\pi$ -Resc amplitude, which is formally of lower chiral order. On the other hand, the possibility of strong cancellation among the TPE diagrams was pointed out in Refs. [12,13]. This motivates us to make here a further study of the behavior of the TPE diagrams.<sup>1</sup>

A remark is in order here on a counting scheme to be used. At the  $NN \rightarrow NN\pi$  threshold the nucleon three-momentum must change from the initial value  $p \sim \sqrt{m_\pi m_N}$  to zero, entailing a rather large momentum transfer. To take this large momentum transfer into account, Cohen et al. [2] proposed a new counting scheme, to be called the momentum counting scheme (MCS); see Ref. [13] for a detailed review. In

MCS the expansion parameter is  $\tilde{\epsilon} \equiv p/m_N \simeq (m_\pi/m_N)^{1/2}$ , which is larger than the usual HB $\chi$ PT expansion parameter  $\epsilon \simeq m_\pi/m_N$ . A study based on MCS [13] indicates that the  $1\pi$ -Resc diagram for  $pp \rightarrow pp\pi^0$  is higher order in  $\tilde{\epsilon}$  (and hence less important) than a certain class of TPE diagrams, called “leading order loop diagrams”, and that MCS is consistent with the estimates of the TPE and other diagrams reported in DKMS. Furthermore, according to Hanhart and Kaiser (HK) [12], the “leading parts” (see below) of these MCS “leading order” diagrams exhibit exact cancellation among themselves<sup>2</sup>; see also Lensky et al. [14]. Although these studies are illuminating, we consider it important to examine the behavior of the “sub-leading” parts (in MCS counting) of these TPE diagrams in order to see whether they can be still as large as indicated by the phenomenological success of the Lee–Riska heavy-meson exchange mechanism. In what follows we shall demonstrate that this is indeed the case.

Analytic expressions for the  $pp \rightarrow pp\pi^0$  transition operators to NNLO in HB $\chi$ PT were given in DKMS. Although these expressions are valid for arbitrary kinematics, we find it illuminating to concentrate here on their simplified forms obtained with the use of fixed kinematics approximation (FKA), wherein the energies associated with particle propagators are “frozen” at their threshold values. In FKA, the TPE operator corresponding to each of the above-mentioned types I–IV can be written as:

$$T = \left( \frac{g_A}{f_\pi} \right) (\vec{S} \cdot \vec{k}) t(p, p', x), \quad (1)$$

where  $\vec{p}$  ( $\vec{p}'$ ) is the relative three-momentum in the initial (final)  $pp$  state ( $\vec{p}_1 - \vec{p}_2 = 2\vec{p}$ ,  $\vec{p}'_1 - \vec{p}'_2 = 2\vec{p}'$ ),  $\vec{k} \equiv \vec{p} - \vec{p}'$ ,  $x = \hat{p} \cdot \hat{p}'$ , and  $\vec{S} = \frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2)$ . The function  $t(p, p', x)$  diverges as  $k \rightarrow \infty$ , and it is useful to decompose  $t(p, p', x)$  into terms that have definite  $k$ -dependence as  $k \rightarrow \infty$ . It turns out [18] that  $t(p, p', x)$  can be expressed as

$$t(p, p', x) \stackrel{k \rightarrow \infty}{\sim} t_1 (g_A / (8f_\pi^2))^2 |\vec{k}| + t_2 (\ln\{|\vec{k}|^2 / \Lambda^2\}) + t_3 + \delta t(p, p', x), \quad (2)$$

where  $t_3$  is asymptotically  $k$ -independent, and  $\delta t(p, p', x)$  is  $\mathcal{O}(k^{-1})$ . For each of types I–IV, analytic expressions for  $t_i$ 's ( $i = 1, 2, 3$ ) can be extracted [18] from the amplitudes  $T$  given in DKMS [10]. The first term with  $t_1$  in Eq. (2) is the leading part in MCS discussed by HK [12], whereas the remaining terms, which we refer to as the “sub-leading” terms, were not considered by HK. The study of these sub-leading terms is an important theme in what follows. Table 1 shows the value of  $t_1$  for type K (K = I–IV) extracted from the results given in DKMS. The third row in Table 1 gives the ratio  $R_K = T_K / T_{\text{Resc}}$ , where  $T_K$  is the plane-wave matrix element of  $T$  in Eq. (1) for type K (K = I–IV) and  $T_{\text{Resc}}$  is the plane-wave matrix element of the  $1\pi$ -Resc diagram. The fourth row in Table 1 gives  $R_K^* = T_K^* / T_{\text{Resc}}$ , where  $T_K^*$  is the plane-wave matrix element of  $T$  with the  $t_1$  term in Eq. (2) subtracted. We can see from the table that the most divergent  $t_1$  terms of the TPE

<sup>2</sup> HK [12] pointed out that the sign of the contribution of Type II in Ref. [10] should be reversed; we have confirmed the necessity of this correction.

<sup>1</sup> For a brief report on this study, see Ref. [16].

Table 1

For the four types of TPE diagrams, K = I, II, III and IV, the second row gives the value of  $t_1$  defined in Eq. (2), and the third row gives the ratio  $R_K = T_K/T_{\text{Resc}}$ , where  $T_K$  is the plane-wave matrix element of  $T$  in Eq. (1) for type K, and  $T_{\text{Resc}}$  is the  $1\pi$ -Resc amplitude. The last row gives  $R_K^* = T_K^*/T_{\text{Resc}}$ , where  $T_K^*$  is the plane-wave matrix element of  $T$  in Eq. (1) with the  $t_1$  term in Eq. (2) subtracted

Type of diagrams: K =	I	II	III	IV
$(t_1)_K$	0	1	1/2	-3/2
$R_K$	-0.70	-6.54	-6.60	9.19
$R_K^*$	-0.70	-0.82	-3.73	0.61

diagrams add up to zero, confirming the result of Ref. [12]. However, this does not necessarily mean that the TPE diagrams are unimportant, because we still need to examine the contributions of the “sub-leading” terms (the  $t_2$ ,  $t_3$  and  $\delta t$  terms) in Eq. (2). Comparison of  $R_K$  and  $R_K^*$  indicates that the subtraction of the  $t_1$  term reduces the magnitude of  $T_K$  drastically (except for type I which has no  $t_1$  term), but the fact that  $|R_K^*|$  is of the order of unity (types I, II and IV) or larger than 1 (type III) suggests that the TPE contributions can be quite important. The sum of the contributions of types I–IV is

$$\sum_K R_K^* = \left( \sum_K R_K \right) = -4.65, \quad (3)$$

which indicates that, at least in plane-wave approximation, the TPE contributions are more important than the  $1\pi$ -Resc contribution.

Next we investigate the behavior of the TPE diagrams as we go beyond the plane-wave approximation by using distorted waves (DW) for the initial- and final-state  $NN$  wave functions. For formal consistency we should use the  $NN$  potential derived from HB $\chi$ PT. Although there has been great progress in deriving  $NN$  potentials based on HB $\chi$ PT [20,21], we adopt here a “hybrid EFT” approach and use phenomenological potentials. Apart from the issue of formal consistency, an additional problem in adopting this hybrid approach is that, whereas the TPE transition operators derived in HB $\chi$ PT are valid only for a momentum range sufficiently lower than  $\Lambda_\chi \sim 1$  GeV, a phenomenological  $NN$  potential can in principle contain any momentum components.<sup>3</sup> To stay close to the spirit of HB $\chi$ PT, we therefore introduce a Gaussian momentum regulator,  $\exp(-p^2/\Lambda_G^2)$ , in the initial and final distorted wave integrals, suppressing thereby the high momentum components of the phenomenological  $NN$  potentials; this is similar to the MEEFT method used in Ref. [19].  $\Lambda_G$  should be larger than the characteristic momentum scale of the  $pp \rightarrow pp\pi^0$  reaction,  $p \simeq \sqrt{m_N m_\pi} \simeq 360$  MeV/ $c$ , but it should not exceed the chiral scale  $\Lambda_\chi$ ; in the present study we shall consider the range,  $500$  MeV  $< \Lambda_G < 1$  GeV. As high-precision phenomenological  $NN$  potentials, we consider the Bonn-B potential [22],

the CD-Bonn potential [23], and the Nijm93 potential of the Nijmegen group [24].

It is worth noting here that several groups [25,26] have developed a systematic approach to construct from a phenomenological potential an effective  $NN$  potential, called  $V_{\text{low-}k}$ , that resides within a model space which only contains momentum components below a specified cutoff scale  $\Lambda_{\text{low-}k}$ . In this work we will use  $V_{\text{low-}k}$  as derived by the Stony Brook group [26]. It is conceptually natural to use  $V_{\text{low-}k}$  in conjunction with transition operators derived from HBChPT [27]. A problem however is  $V_{\text{low-}k}$  [26], primarily meant for describing sub-pion-threshold phenomena, was obtained with the use of a rather low cutoff,  $\Lambda_{\text{low-}k} \sim 2$  fm<sup>-1</sup>. This cutoff is perhaps too close to the characteristic momentum scale  $p \sim 360$  MeV/ $c$  for the pion production reaction. It therefore seems worthwhile to “rederive”  $V_{\text{low-}k}$  employing a momentum cut-off higher than  $2$  fm<sup>-1</sup> and use it in the present DWBA calculation. Below we will use  $V_{\text{low-}k}$  generated from the CD-Bonn potential for  $\Lambda_{\text{low-}k} = 4$  and  $5$  fm<sup>-1</sup>. We remark that, as is well known,  $V_{\text{low-}k}$ ’s generated from any realistic phenomenological potentials lead to practically equivalent half-off-shell  $NN$  K-matrices and hence the same  $NN$  wave function.

We evaluate the TPE contributions in DWBA for a typical case of  $T_{\text{lab}} = 281$  MeV. Since the  $t_1$  terms in Eq. (2) add up to zero, we drop the  $t_1$  terms in our calculation.<sup>4</sup> Thus, in Eq. (1), we use  $t^*(p, p', x)$  instead of  $t(p, p', x)$ , where  $t^*(p, p', x)$  is obtained from  $t(p, p', x)$  by suppressing the  $t_1$  term. The partial-wave projected form of  $t^*(p, p', x)$  in a DWBA calculation is written as:

$$J = - \left( \frac{m_N m_\pi}{8\pi} \right) \int_0^\infty p^2 dp p'^2 dp' \int_{-1}^1 dx \psi_{1S_0}(p') \times t^*(p, p', x)(p - p'x)\psi_{3P_0}(p). \quad (4)$$

Here  $\psi_\alpha(p)$  is a distorted two-nucleon relative wave function in the  $\alpha$  partial-wave ( $^1S_0$  for the initial state and  $^3P_0$  for the final state) given by

$$\psi_\alpha(p) = \cos(\delta_\alpha) \left[ \delta(p - p_{on})/p^2 + \mathcal{P} \left( \frac{K_\alpha(p, p_{on})}{(E - E_p)} \right) \right], \quad (5)$$

where  $\delta_\alpha$  is the phase-shift for the  $\alpha$  partial wave, and  $K_\alpha(p, p_{on})$  is the partial-wave K-matrix pertaining to the asymptotic on-shell momentum  $p_{on}$ . The plane-wave approximation corresponds to the use of the wave functions of the generic form:

$$\psi(p) = \delta(p - p_{on})/p^2. \quad (6)$$

We show in Table 2 the values of  $J$ , Eq. (4), for the TPE operators of types I–IV, calculated at  $T_{\text{lab}} = 281$  MeV, with the use of the Nijm93 potential of the Nijmegen group [24]<sup>5</sup> and  $V_{\text{low-}k}$ . For the Nijm93 potential case, we present the results

<sup>3</sup> A pragmatic problem associated with this conceptual issue is that, in a momentum-space calculation of the matrix elements of the TPE operators sandwiched between distorted  $pp$  wave-functions generated by a phenomenological  $NN$  potential, the convergence of momentum integrations is found to be extremely slow [17,18].

<sup>4</sup> Removing the  $t_1$  term lessens the severity of the convergence problem in our momentum integration mentioned in footnote 3.

<sup>5</sup> We have checked the results obtained using the Bonn-B and CD-Bonn  $NN$  potentials are very similar to those for the Nijm93 potential case, which we show here as a representative case.

Table 2

The values of  $J$ , Eq. (4), corresponding to the TPE diagrams of types I–IV, evaluated in a DWBA calculation for  $T_{\text{lab}} = 281$  MeV. The column labeled “Sum” gives the combined contributions of types I–IV, and the last column gives the value of  $J$  for  $1\pi$ -Resc. For the Nijm93 potential case, the results for five different choices of  $\Lambda_G$  are shown. For the case with  $V_{\text{low-}k}$ , CD-4 (CD-5) represents  $V_{\text{low-}k}$  generated from the CD-Bonn potential with a momentum cut-off  $\Lambda_{\text{low-}k} = 4 \text{ fm}^{-1}$  ( $5 \text{ fm}^{-1}$ ). The last row gives the results obtained in plane-wave approximation

	I	II	III	IV	Sum	$1\pi$ -Resc
$V_{\text{Nijm}}: \Lambda_G = 500 \text{ MeV}/c$	-0.11	-0.12	-0.55	0.08	-0.70	0.18
$V_{\text{Nijm}}: \Lambda_G = 600 \text{ MeV}/c$	-0.12	-0.12	-0.57	0.07	-0.74	0.20
$V_{\text{Nijm}}: \Lambda_G = 700 \text{ MeV}/c$	-0.12	-0.11	-0.57	0.06	-0.74	0.21
$V_{\text{Nijm}}: \Lambda_G = 800 \text{ MeV}/c$	-0.12	-0.11	-0.55	0.04	-0.74	0.22
$V_{\text{Nijm}}: \Lambda_G = 1000 \text{ MeV}/c$	-0.12	-0.10	-0.52	0.03	-0.71	0.23
$V_{\text{low-}k}$ (CD-4)	-0.12	-0.09	-0.46	0.03	-0.65	0.23
$V_{\text{low-}k}$ (CD-5)	-0.09	-0.06	-0.30	-0.01	-0.46	0.22
Plane-wave	-0.06	-0.07	-0.30	0.05	-0.37	0.080

for five different values of  $\Lambda_G$  between 500 and 1000 MeV/ $c$ . For the  $V_{\text{low-}k}$  case, the results for two choices of  $\Lambda_{\text{low-}k}$  are shown:  $\Lambda_{\text{low-}k} = 4$  and  $5 \text{ fm}^{-1}$ . For comparison, the values of  $J$  corresponding to plane-wave approximation are also shown (bottom row). From Table 2 we learn the following: (1) The results for the Nijm93 potential with the Gaussian cutoff  $\Lambda_G$  are stable against the variation of  $\Lambda_G$  within a reasonable range (500–1000 MeV/ $c$ ); (2) There is semi-quantitative agreement between the results for the Nijm93 potential and those for  $V_{\text{low-}k}$ ; (3) A semi-quantitative agreement is also seen between the DWBA and PWBA calculations; (4) The feature found in the plane-wave approximation that the contributions of the TPE diagrams are more important than the  $1\pi$ -Resc contribution remains unchanged in the DWBA calculation; the summed contribution of the TPE operators is larger (in magnitude) than that of  $1\pi$ -Resc by a factor of 2–3.5.

We now discuss the above results in the context of MCS [13]. A subtlety in MCS is that a loop diagram of a given order  $\nu$  in  $\tilde{\epsilon}$  not only contains a contribution of order  $\nu$  (“leading part”) but, in principle, can also involve contributions of higher orders in  $\tilde{\epsilon}$  (“sub-leading part”) due to the non-analytic functions generated by the loop integral. As mentioned, however, HK [12] considered only the leading part, which correspond to the  $t_1$  term in Eq. (2). According to MCS, for the reaction  $pp \rightarrow pp\pi^0$ , the loop diagrams corresponding to our types II, III and IV diagrams belong to NLO in the  $\tilde{\epsilon}$  parameter, whereas those corresponding to type I and the  $1\pi$ -Resc tree diagram are next order in  $\tilde{\epsilon}$  (NNLO); see Table 11 in Ref. [13]. Meanwhile, as discussed earlier, the sum of the “leading parts” of the NLO diagrams vanishes, and therefore, in calculating  $J$ ’s in Table 2, we have dropped the  $t_1$  term contribution, retaining only the “sub-leading” parts of these NLO diagrams. This means that all the entries in Table 2 represent “sub-leading contributions” (NNLO) in MCS. If we look at Table 2 from this perspective, we note that the order-of-magnitude behavior of our numerical results is in rough agreement with MCS, although type IV tends to be rather visibly smaller (in magnitude) than the others. However, it is striking that  $J$  for type III is significantly (if not by an order of magnitude) larger than the other sub-leading contribu-

tions. (A similar feature was also seen in  $R^*$  in Table 1.) In view of the fact that type III arises from crossed-box TPE diagrams [10], there is a possibility that the enhancement of the type III diagrams may be related to the strong attractive scalar  $NN$  potential that is known to arise from TPE crossed-box-diagrams [7,8].

We have studied the “sub-leading” parts, which are of NNLO in the momentum counting scheme (MCS) [13], of the TPE amplitudes for the  $pp \rightarrow pp\pi^0$  reaction in both PWBA and DWBA calculations. We have shown in fixed kinematics approximation (FKA) that, even though the leading parts of the TPE amplitudes cancel among themselves [12,14], the contributions of the sub-leading parts are quite significant. They are in general comparable to the  $1\pi$ -Resc amplitude, and the sub-leading part of the type III diagrams is even significantly larger than the  $1\pi$ -Resc diagram. The total contribution of the TPE diagrams is larger (in magnitude) than that of the  $1\pi$ -Resc diagram by a factor of  $\sim 5$  (PWBA) or  $2 \sim 3$  (DWBA). According to MCS, the sub-leading parts of the TPE diagrams and  $1\pi$ -Resc diagram are both NNLO in  $\tilde{\epsilon} \equiv \sqrt{\frac{m_\pi}{m_N}} \sim 1/3$ . The results of our DWBA calculation are not inconsistent with MCS, if we take the viewpoint that two quantities that differ by a factor of 2–3 can be considered to be of the same order. This viewpoint is certainly valid when the expansion parameter is sufficiently small but, in the present case,  $3 \simeq \tilde{\epsilon}^{-1}$ , and hence one might take the above-mentioned difference by a factor of  $\sim 3$  as an indication of a possible problem with MCS. This issue warrants further studies including the examination of  $N^3\text{LO}$  contributions. We have focused here on the TPE loop diagrams but, to obtain a theoretical cross section for  $pp \rightarrow pp\pi^0$  that can be directly compared with the experimental value, we must consider the other diagrams discussed in DKMS as well as the relevant counter terms. These will be discussed in a forthcoming article [28]. Since, as mentioned, our present calculation is based on hybrid HB $\chi$ PT, it is desirable and important to carry out a similar calculation with the use of  $NN$  potentials that have been derived from HB $\chi$ PT [20,21]. This is also relegated to our future study.

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