A Simple Model for Proton-Antiproton Scattering at Low Energies.

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Summary. — The $\bar{p}p$ scattering data at low energies are very well reproduced with the one-boson exchange potential (OBEP) and with the annihilation described by a boundary condition at a certain radius. Our only free parameter is the boundary radius. We show that the elastic $\bar{p}p$ forward peak is not a diffractive peak. Its slope as well as the antishrinkage are explained by the OBEP alone.

1. – Introduction.

It has been shown that the one-boson exchange potential (OBEP) which fits nucleon-nucleon scattering data, see *e.g.* ref. (1), predicts many nucleon-antinucleon bound states and resonances (2) (quasi-nuclear–type states). Experimentally, a bump is found in the $\bar{p}p$ cross-section at 1940 MeV with a width

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(1) R. A. BRYAN and B. L. SCOTT: *Phys. Rev.*, **177**, 1435 (1968).
(2) I. S. SHAPIRO: *Sov. Phys. Usp.*, **16**, 173 (1973); L. N. BOGDANOVA, O. D. DALKAROV and I. S. SHAPIRO: *Ann. Phys.*, **84**, 261 (1974); C. DOVER: *Proceedings of the IV International Symposium on Nucleon-Antinucleon Interactions*, edited by T. E. KALOGEROPOULOS and K. C. WALI, Vol. **2** (Syracuse, N. Y., 1975), p. VIII, 37; J. M. RICHARD, M. LACOMBE and R. VINH MAU: *Phys. Lett.*, **64** B, 121 (1976).

of about 5 MeV ([3-5]). This bump is a candidate for one of these nucleon-antinucleon resonances. In the dual quark model, one also expects exotic resonances according to the recent argument presented by CHEW ([6]) and by VENEZIANO ([7]).

We will discuss here two questions in the framework of the potential model with $\bar{p}p$ annihilation described by a boundary condition. First we discuss the size of the annihilation region needed to reproduce $\bar{p}p$ scattering data, and second the influence of the annihilation on the resonances predicted by the OBEP ([*]). The latter question has been discussed earlier by MYHRER and GERSTEN ([8]). They used the Bryan-Phillips ([9]) energy-dependent $\mathcal{N}\bar{\mathcal{N}}$ potential which describes annihilation by an imaginary potential. MYHRER and GERSTEN showed that, when the strength of the imaginary potential was made large enough to fit the observed elastic $\bar{p}p$ cross-section, the $\mathcal{N}\bar{\mathcal{N}}$ resonances of the real OBEP disappeared. The reason was that the absorptive potential became so strong that it was felt even at large distances ($\sim$ 1 fm). Therefore, the absorptive potential strongly modified the scattered wave functions from the pure OBEP result.

Here we will describe the annihilation by a boundary condition so as *to avoid* the long tail of the absorptive potential of Bryan and Phillips. We will ask the following question: which radius r_c of the boundary is necessary to describe the observed elastic and absorptive cross-sections and their energy variation? Our model is that, at the boundary of radius r_c, we have only incoming waves, *no* reflected waves. This model is similar to the one used by, *e.g.*, SPERGEL ([10]), for a review see ref. ([11]). He assumed only incoming plane waves at the boundary r_c with the effective wave number K at the boundary as an additional parameter. While he needed two parameters to describe annihilation, we will need only one, the boundary radius r_c. Further, we obtain a simple

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([3]) A. S. CARROLL, I. H. CHIANG, T. F. KYCIA, K. K. LI, P. O. MAZUR, D. N. MICHAEL, P. MOCKETT, D. C. RAHM and R. RUBINSTEIN: *Phys. Rev. Lett.*, **32**, 247 (1974); T. E. KALOGEROPOULOS and G. S. TZANAKOS: *Phys. Rev. Lett.*, **34**, 1047 (1975).
([4]) V. CHALOUPKA, H. DREVERMAN, F. MARZANO, L. MONTANET, P. SCHMID, J. R. FRY, H. ROHRINGER, S. SIMOPOULOU, J. HANTON, F. GRARD, V. P. HENRI, H. JOHNSTAD, J. M. LESCEUX, J. S. SKURA, A. BELLINI, M. CRESTI, L. PERUZZO, P. ROSSI, R. BIZZARI, M. IORI, E. CASTELLI, C. OMERO and P. POROPAT: *Phys. Lett.*, **61** B, 487 (1976).
([5]) W. BRÜCKNER, B. GRANZ, D. INGHAM, K. KILIAN, U. LYNEN, J. NIEWISCH, B. PIETRZYK, B. POVH, H. G. RITTER and H. SCHRÖDER: CERN preprint (1976).
([6]) G. F. CHEW: preprint LBL-5391 (1976); talk at the *III European Symposium on Nucleon-Antinucleon Interactions, Stockholm* (July 1976).
([7]) G. ROSSI and G. VENEZIANO: private communications; G. VENEZIANO talk at *$\mathcal{N}\bar{\mathcal{N}}$ workshop*, CERN (December 6-8, 1976).
([*]) We will not discuss the $\bar{p}p$ bound states from OBEP in this work.
([8]) F. MYHRER and A. GERSTEN: *Nuovo Cimento*, **37** A, 21 (1977).
([9]) R. A. BRYAN and R. J. N. PHILLIPS: *Nucl. Phys.*, **5** B, 201 (1968).
([10]) M. S. SPERGEL: *Nuovo Cimento*, **47** A, 538 (1967).
([11]) R. J. N. PHILLIPS: *Rev. Mod. Phys.*, **39**, 681 (1967).

physical explanation the of other Spergel parameter. Since our $\mathcal{N}\mathcal{N}$ potential is much better than the one used by SPERGEL, we get a good description of the experimental $\bar{p}p$ data.

2. – The boundary model for annihilation.

We describe the $\mathcal{N}\bar{\mathcal{N}}$ scattering by a potential model. The potential is the OBEP taken from Bryan and Scott [1], but with the coupling constants and the cut-off parameter as those used by BRYAN and PHILLIPS [9]. This part of our $\mathcal{N}\bar{\mathcal{N}}$ model has no free parameters. The parameters in the OBEP are all determined from fits to the $\mathcal{N}\mathcal{N}$ phase shifts.

The $\mathcal{N}\bar{\mathcal{N}}$ annihilation is described by the boundary condition of Feshbach and Weisskopf [12]. Their idea is simply that, at the boundary r_c, the scattered wave satisfies a certain condition to be specified. As a consequence, it is not possible to obtain any information about the interior ($r < r_c$). The model of Feshbach and Weisskopf assumes only incoming waves at $r = r_c$, *i.e.* we have no reflections from the boundary.

Using the WKB approximation, we can write the wave function at a boundary $r = r_c$ in terms of incoming and outgoing radial waves as

$$u_l(r) \sim r\big(h_l^{(2)}(Kr) + b h_l^{(1)}(Kr)\big) , \tag{1}$$

where K is the wave number to be defined later. Here $h_l^{(1)}(Kr)$ and $h_l^{(2)}(Kr)$ are Hankel functions describing outgoing and incoming waves, respectively. FESHBACH and WEISSKOPF say that $b = 0$ in eq. (1). Further, they assume that, at the boundary, eq. (1) with $b = 0$ can be described reasonably well by

$$u_l(r) \sim \exp[-iKr] . \tag{2}$$

This boundary condition was used by SPERGEL to describe $\mathcal{N}\bar{\mathcal{N}}$ annihilation. He used r_c and K as two free parameters to fit the data. The effective wave number K was determined such that he had maximum absorption in each partial wave. His condition reads $\partial\sigma_R^l/\partial K = 0$, where σ_R^l is his reaction cross-section for $\bar{p}p$ partial-wave number l.

In our model we use the fact that K is the effective wave number at a distance r. We determine K from the value of the OBEP at this point:

$$K = \sqrt{M(E - V(r))} , \tag{3}$$

where M is the nucleon mass, E is the scattering centre-of-mass energy and

[12] H. FESHBACH and V. F. WEISSKOPF: *Phys. Rev.*, **76**, 1550 (1949).

$V(r)$ is the one-boson exchange potential at distance r. Since $V(r)$ differs for each partial wave, K will also depend upon the $\bar{p}p$ angular-momentum channel (in eq. (3) we only include the diagonal parts of $V(r)$ for coupled channels). Generally speaking, $V(r) < 0$ for $r_c < 1$ fm. In some angular-momentum channels for too small r_c, our energy-dependent $V(r)$ becomes positive. These small values of r_c will not be needed in *our* numerical calculations (with our choice of OBEP). However, we will discuss this point in the conclusions.

We have our free parameter in our calculation, r_c, which we determine by requiring that our model describes the data. We will discuss two boundary conditions: model I with $u_l(r)$ given by eq. (2) and model II with $u_l(r)$ given by eq. (1) and $b = 0$. The only relevant input of this wave function is the logarithmic derivative of u_l at $r = r_c$. From eq. (2) we have

$$\left.\frac{u_l'(r)}{u_l(r)}\right|_{r=r_c} = -iK\,. \tag{4}$$

From eq. (1) we find eq. (4), but with a constant depending on l multiplying the right-hand side of eq. (4). Because Kr_c is fairly large (about $2 \div 4$), this does not change the right-hand side of eq. (4) very much for $l \leqslant 3$. Moreover, since K does not change very rapidly with r_c, *i.e.* the OBEP does not vary drastically for our values of r_c, we can assume that the WKB approximation of u_l, eqs. (1) or (2), is good. With the real nucleon-antinucleon OBEP from ref. [8,9] and with K determined by eq. (3) we solve the coupled-channel Schrödinger equation to obtain the cross-sections.

3. – Results.

We will first discuss the results obtained with model I. We fitted σ_{tot}, σ_{el} and σ_{ex}(ex = $\bar{p}p \to \bar{n}n$) *vs.* energy rather well. The best value of r_c is dependent on the particular σ or energy range, but it is not a strong function of them. This model did not fit σ *vs.* energy as well as, *e.g.*, the Bryan-Phillips potential model. When we looked at $\bar{p}p$ elastic and charge exchange differential cross-sections, a value of r_c equal to 0.5 fm gave the best results at backward angles. On the other hand, this model did not have a pronounced dip and a second maximum in $d\sigma/d\Omega(\bar{p}p \to \bar{n}n)$ as does Bryan and Phillips' [9].

In model II we do not find very great differences from model I. In this case, however, all the cross-sections, *vs.* energy, are well described by a single boundary radius $r_c = 0.5$ fm (see fig. 1 and 2) (*). In addition, a more pro-

(*) The proton-neutron mass difference is neglected in these calculations.

nounced dip bump develops in $d\sigma/d\Omega$ for $\bar{p}p \to \bar{n}n$ at forward angles. However, we cannot reproduce the data of Bogdanski *et al.* [13], who find the ex-

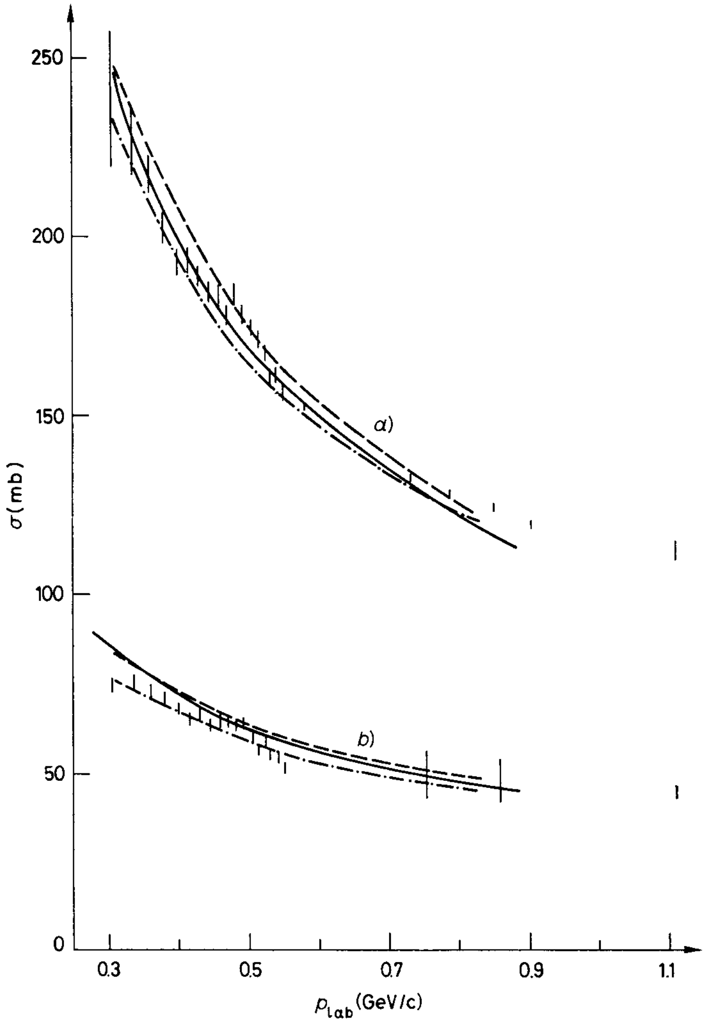

Fig. 1. – Total (a)) and elastic (b)) $\bar{p}p$ cross-sections as functions of laboratory momentum are plotted. The theoretical curves are all from our model II, the fully drawn ones calculated with boundary radius $r_c = 0.5$ fm and OBEP cut-off $\Lambda = 980$ MeV; the dashed curve is for $r_c = 0.5$ fm and $\Lambda = 1100$ MeV and the dash-dotted one for $r_c = 0.6$ fm and $\Lambda = 980$ MeV. The experimental points are taken from ref. [3,4]. The highest-energy points are taken from ref. [14].

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[13] M. Bogdanski, T. Emura, S. N. Ganguli, A. Gurtu, S. Hamada, R. Hamatsu, E. Jeannet, I. Kita, S. Kitamura, J. Kishino, H. Kohno, M. Komatsu, P. K. Malhotra, S. Matsumoto, U. Mehtani, L. Montanet, R. Raghavan, A. Subramanian, H. Takahashi and T. Yamagata: *Phys. Lett.*, **62** B, 117 (1976).

[14] Particle Data Group: *$\mathcal{N}\bar{\mathcal{N}}$ compilation*, LBL-58 (May 1972).

perimental $d\sigma/d\Omega$ at the dip too high compared to the Bryan-Phillips potential. Since the $\bar{p}p \to \bar{n}n$ cross-section in this experiment is higher than that in other experiments (see BOGDANSKI *et al.*, fig. 1), we suspect that a reduction in their value of this cross-section will improve the agreement with theory considerably, Our $d\sigma/d\Omega$ does not differ much from the results of the Bryan-Phillips model. as shown in BOGDANSKI *et al.* [13] fig. 2 (see our fig. 3).

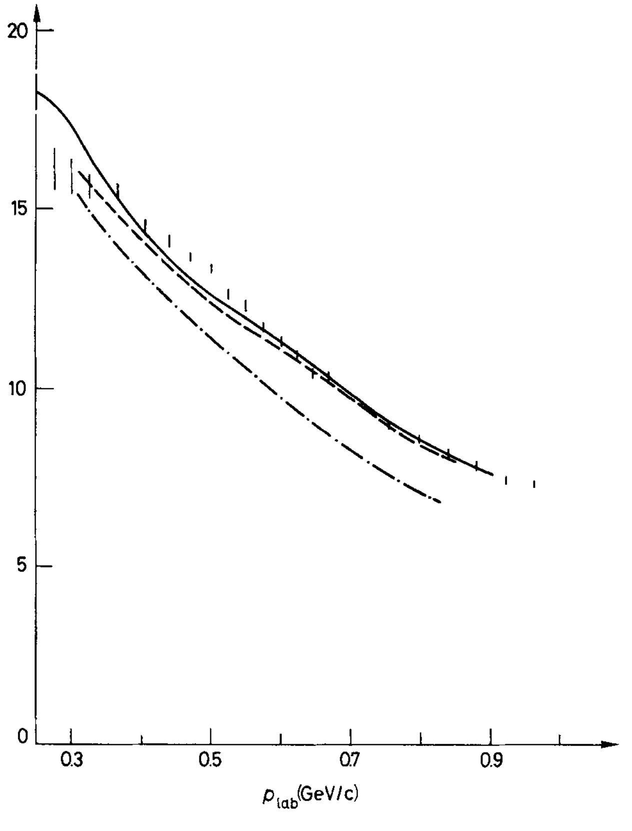

Fig. 2. – The cross-section for $\bar{p}p \to \bar{n}n$ as a function of laboratory momentum is plotted. See fig. 1 for details. The experimental points are taken from ref. [15].

Another interesting fact concerns the forward slope of the $\bar{p}p$ elastic $d\sigma/dt$. At low energies, model II gives $d\sigma/dt$ varying as $\exp[-b|t|]$ for angles up to 60°, and the value of b can be explained by the OBEP alone. This means that the forward peak in $d\sigma/dt$ is *not* a diffractive peak, but rather the result of a delicate interference between different $\bar{p}p$ partial waves. At these energies, the

[15] M. ALSTON-GARNJOST, R. KENNEY, D. POLLARD, R. ROSS, R. TRIPP and H. NICHOLSON: *Phys. Rev. Lett.*, **35**, 1685 (1975).

S, P, D and some F waves are the ones that contribute. One does not need higher partial waves to eplain the $\exp[-b|t|]$. To be precise, we find that, for r_c between 0.3 and 0.8 fm, the value of $b = 24\ (\text{GeV/c})^{-2}$ equals the one from the

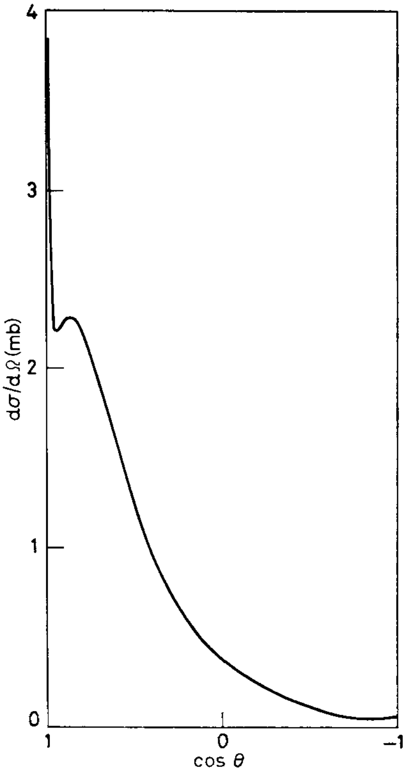

Fig. 3. – The differential cross-section $d\sigma/d\Omega$ for $\bar{p}p \to \bar{n}n$ with $\bar{p}$ laboratory energy of 250 MeV is plotted. The curve is calculated with model II and $r_c = 0.5$ fm.

OBEP alone to within 5% at $p_{lab} = 536$ MeV/c. PHILLIPS ([11]) finds that a pure absorptive potential can describe the forward elastic peak. With our results it is clear that one cannot relate the slope b to the range of the annihilation forces at these energies.

TABLE I. – *The slope b of the elastic $\bar{p}p$ forward peak* $(d\sigma/dt) \propto \exp[-b|t|]$ *as a function of laboratory momentum is given.* The slope b is calculated with our model II, $r_c = 0.5$ fm and OBEP cut-off $\Lambda = 980$ MeV. To find b, we only used values of $d\sigma/d\Omega$ between $1.0 \leqslant \cos\theta^* \leqslant 0.5$.

b $(\text{GeV/c})^{-2}$	42.7	32.6	23	19
p_{lab} (GeV/c)	0.218	0.310	0.536	0.73

In the table we show the energy behaviour of b calculated from model II with $r_c = 0.5$ fm including only points up to 60° cm. From these results it is clear that we have an antishrinkage of the elastic $\bar{p}p$ forward peak ([16]).

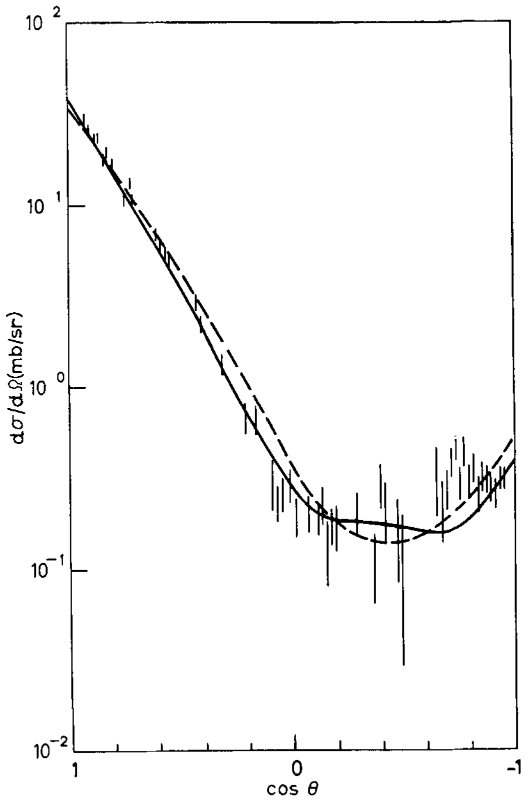

Fig. 4. – The elastic differential cross-section $d\sigma/d\Omega$ with model II and $r_c = 0.5$ fm is calculated. The fully drawn line is for $p_{lab} = 0.73$ GeV/c and the dashed line for $p_{lab} = 0.66$ GeV/c. The experimental data points are from ref. ([17]) and their $p_{lab} = 0.69$ GeV/c.

Figure 4 shows the calculated elastic differential cross-section at two energies from model II compared with the experimental data from EISENHANDLER *et al.* ([17]).

([16]) V. BARGER and D. CLINE: *Nucl. Phys.*, **23** B, 227 (1970).
([17]) E. EISENHANDLER, W. R. GIBSON, C. HOJVAT, P. I. P. KALMUS, L. C. Y. LEE, T. W. PRITCHARD, E. C. USHER, D. T. WILLIAMS, H. HARRISON, W. H. RANGE, M. A. R. KEMP, A. D. RUSH, J. N. WOULDS, G. T. J. ARNISON, A. ASTBURY, D. P. JONES and A. S. L. PARSONS: *Nucl. Phys.*, **113** B, 1 (1976).

Finally in fig. 5 we have plotted the elastic differential cross-section at 180° as a function of incoming momentum. A clear peak around $p_{lab} \simeq$ $\simeq 0.5$ GeV/c is seen.

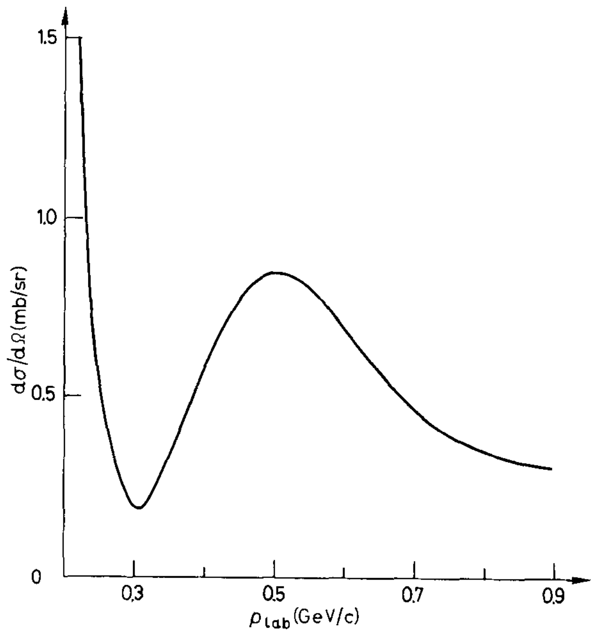

Fig. 5. – The elastic differential cross-section at backward angle $d\sigma/d\Omega(180°)$ is plotted as a function of laboratory momentum for model II with $r_c = 0.5$ fm.

4. – Discussion and conclusions.

We have reproduced the $\bar{p}p$ experimental data at low energies with a real one-boson exchange potential plus a boundary (model II) at $r_c = 0.5$ fm to describe annihilation. The radius r_c is the only free parameter in our calculation.

Our real nucleon-antinucleon OBEP (without annihilation) predicts many $\bar{p}p$ resonances. With an $r_c = 0.1$ fm we still have OBEP resonances, but they disappear very quickly for increasing r_c. With our large value for r_c none survives the annihilation process. The reason is that our boundary condition acts in all partial waves at the same r_c. While this assumption has the advantage of simplicity and economy with parameters, it is certainly not a necessary one. In our boundary condition model we can easily see that, *e.g.*, if the OBEP for some angular momenta becomes repulsive for $r \gg r_c$, then the scattered wave might not reach the annihilation boundary and OBEP resonance(s) will remain. (The OBEP plus the centrifugal barrier is repulsive for several $\bar{p}p$ D-waves below $r \simeq 0.4$ fm). At this point, we should caution that the OBEP from nucleon-nucleon scattering is not known at 0.5 fm. We should stress that our r_c is the overall annihilation radius necessary to fit the data which is a rather crude picture of the annihilation. In this work, we have made no speculations about a possible r_c channel dependence and a possible fit to the $\bar{p}p$ 1940 MeV resonance.

From our calculations we understand Spergel's boundary condition [10]. His effective momentum can be explained by eq. (3) and our K is not too different from his parameter. On the other hand, we do not find that K increases with increasing spin J as his parameter does. We ascribe this difference as well as our much better fit to the $\bar{p}p$ data to our better $\mathcal{N}\mathcal{N}$ potential. Spergel's $\mathcal{N}\mathcal{N}$ potential did not have any explicit ω exchange which produces a strongly attractive $\bar{p}p$ potential. In fact, our fit to the $\bar{p}p$ data is easily comparable in quality to that from the Bryan-Phillips potential [9]. Unlike, the Bryan-Phillips optical potential, our final numbers only depend weakly on the value of the OBEP cut-off parameter Λ [8]. A variation of 10% in this parameter influences our final cross-section very little (see fig. 1 and 2).

There is, however, one problem that has to be faced in a potential approach to the $\bar{p}p$ scattering. The value of the OBEP for $r \leqslant 0.5$ fm is typically -1 GeV or deeper. For such a depth, relativistic effects must be considered. Further, we know from the work of, *e.g.*, Gross [18] that relativistic effects, terms of order v^2/c^2, can introduce short-range repulsion in the $\mathcal{N}\mathcal{N}$ interaction (and, therefore, also in the $\mathcal{N}\overline{\mathcal{N}}$ interaction). But to what extent is it still an open question [19,20].

We show that the forward $\bar{p}p$ elastic peak is *not* a diffractive peak and we explain the antishrinkage of this peak by means of the OBEP alone. Because several partial waves (S, P, D) contribute to the scattering even at very low energies, one does not expect a $1/v$ behaviour for, *e.g.*, $\sigma_{\text{annihilation}}$.

This model has been developed in order to reproduce $\bar{p}p$ scattering data at low energies. A characteristic feature of this specific model is that our annihilation boundary is at relatively large distances compared to the Compton wave-length of the nucleon. Our crude annihilation model with only one free parameter is able to give a surprisingly good reproduction of the bulk of the low-energy proton-antiproton data.

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(18) W. Buck and F. Gross: *Phys. Lett.*, **63** B, 286 (1976).
(19) A. Gersten, R. H. Thompson and A. E. S. Green: *Phys. Rev. D*, **3**, 2076 (1971).
(20) J. Fleischer and J. A. Tjon: *Nucl. Phys.*, **84** B, 375 (1975).

● RIASSUNTO (•)

I dati dello scattering $\bar{p}p$ a basse energie sono molto ben riprodotti con il potenziale di scambio a un bosone (OBEP) e con l'annichilazione descritta da una condizione limite ad un certo raggio. L'unico nostro parametro libero è il raggio di confine. Si mostra che il picco elastico in avanti di $\bar{p}p$ non è un picco difrattivo. La sua pendenza così come il suo antiaccorciamento sono spiegati dal solo OBEP.

(•) *Traduzione a cura della Redazione.*

Простая модель для протон-антипротонного рассеяния при низких энергиях.

Резюме (•). — Данные по протон-антипротонному рассеянию при низких энергиях очень хорошо воспроизводятся с помощью потенциала с однобозонным обменом и с помощью аннигиляции, описываемой посредством граничного условия при определенном радиусе. Единственный свободный параметр представляет граничный радиус. Мы показываем, что упругий пик $\bar{p}p$ рассеяния вперед не является дифракционным пиком. Его наклон, а также анти-сокращение объясняются с помощью потенциала с однобозонным обменом.

(•) *Переведено редакцией.*