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THE BARYON MASSES AND THE CHIRAL QUARK BAG MODEL

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Abstract: The baryon octet and decuplet mass spectrum is calculated within the chiral bag model. To zeroth order, this reduces to the MIT bag model. To the next order, baryon self-energies coming from coupling to pions enter. These are about as important as terms from gluon exchange; their inclusion decreases somewhat the colour coupling constant α_s needed to fit the empirical baryon splittings. We show that mass splittings resulting from our calculation obey well the Gell-Mann–Okubo mass formulae, although certain terms included would individually violate them. Also the absolute value of the Λ magnetic moment comes out very well.

In our theory, part of the mass splitting is a manifestation of the spontaneous breaking of chiral symmetry in the region outside the bag, resulting from the Goldstone pion which is introduced to conserve the axial current at the bag boundary. With inclusion of pionic interactions, the phenomenological zero-point energy introduced through the constant Z_0 in the MIT work is much smaller in order to fit the empirical masses, the pionic self-energies replacing most of it.

1. Introduction

In QCD the forces between coloured objects are very strong at long distances. The QCD vacuum does not allow for colour electric fields so we cannot have colour separations which are large compared to the QCD scale parameter Λ^{-1}. In practice this means that the coloured quarks and gluons can move but are confined inside small colour singlet bubbles of radius $R \approx \Lambda^{-1}$. Within each bubble gluons and quarks move freely and interact only perturbatively. This type of model, which says quark confinement can be described as a phase transition, has been considered by several authors [1]), and it leads naturally to a quark model like the MIT bag model, e.g. ref. [2]), which is very successful in explaining static properties of hadrons.

The QCD lagrangian is invariant under chiral transformation in the limit of zero quark masses. The MIT bag model assumes that u- and d-quarks are massless and

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will easily produce colour separation inside the bag. But quark quantum numbers are prevented from moving out of the bag by a boundary condition on the quark current

$$n_\mu \cdot \overline{q_a} \gamma_\mu q_b = 0 \quad \text{for} \quad r = R\,, \tag{1}$$

where a and b are colour indices on the quark wave function q which exists for $r \leq R$. This boundary condition is also chiral invariant. The original MIT bag model chose a linear realization of eq. (1) which implies that the axial current is discontinuous on the bag surface, e.g. ref. [3]). This led Callan *et al.* [1]) to introduce a pion field outside the bag so as to conserve the axial vector current, a line that was developed further with the so-called "little" or "chiral" bag [4]).

Inside the bag, chiral symmetry is realized in the Wigner–Weyl mode. Massless u- and d-quarks move in a perturbative vacuum and they produce chiral multiplets. Their wave functions satisfy a chirally invariant boundary condition on the bag surface

$$n_\mu \gamma_\mu q(\boldsymbol{r}) = e^{i\vec{\tau}\cdot\vec{\pi}\gamma_5} q(\boldsymbol{r})\,, \qquad r = R\,, \tag{2}$$

which implies eq. (1). Here $\vec{\pi}$ represents the pion field which exists only in the region outside the bag where the chiral symmetry is realized in the Goldstone mode. Johnson [5]) has connected the pion field with a chiral phase linkage on the surfaces of empty bags which make up the physical vacuum. Here we shall treat the pion as a phenomenological field, neglecting its $q\bar{q}$ substructure. This should be adequate for pions of long wavelength; since we shall deal with large bags as pion sources, only relatively low-momentum pions are allowed by the large sources.

We deal here only with perturbative corrections to the MIT bag. This procedure is appropriate for bags of large radius, although we shall include the pion mean field in the zero-order energy. It has been argued [4]) that in the case of the nucleon and the isobar, the pion mean field can become so strong for small bag radii as to allow for a "little bag" solution. These lie outside the framework of perturbation theory, so we are forced in this paper to consider the nucleon and Δ as large bags. The strange baryons, which have much smaller pion clouds, would, in any case, be large in size. Even if the nucleon and Δ bags are small, we hope that most of the features found here follow from symmetries, etc., and will still be at least approximately applicable there.

In this work we will use a model to describe the Goldstone pions outside the bag. We assume that the Goldstone pions carry the axial current outside the bag and we demand $\partial_\mu A_\mu = 0$ in all space. The Goldstone pions are described by a non-linear chiral lagrangian [6,7]). Working with a chiral model, we automatically have the soft-pion predictions; see, e.g., Adler and Dashen [8]). From the Goldberger–Treiman relation, we can fix the pion–nucleon coupling at the bag surface in terms of the strong-interaction constant f. This relates the asymptotic pion field back to the usual Yukawa one [4]).

In sect. 2 we will present the equation of motion for the model and calculate the Goldstone pion self-energy correction of the baryons to first order in the pion field. We argue in sect. 3 that this first-order correction which is a second-order perturbation calculation has to be modified when we include intermediate state energy differences. This leads us to our final result for the pion self-energies.

2. The chiral quark bag model

The chiral quark bag is an extension of the MIT bag model [3]). On the surface of the static, spherical bag the long-range QCD forces are approximated by a boundary condition which confines quark quantum numbers to the bag. The quarks satisfy the Dirac equation inside the bag which, for the massless u- and d-quarks, is

$$-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}q(r)=\omega q(r)\,,\qquad r<R\,,\tag{2.1}$$

where $\boldsymbol{\alpha}$ is the Dirac matrix and $q(r)$ the quark wave function and its energy ω. The boundary condition imposed on $q(r)$ on the bag surface is [2])

$$\hat{\boldsymbol{r}}\cdot\bar{q}\boldsymbol{\gamma}q=0\,,\qquad r=R\,.\tag{2.2}$$

In lowest order, as we shall outline, the linear condition which implies (2.2) is used [2])

$$\hat{\boldsymbol{r}}\cdot\boldsymbol{\gamma}q=q\,,\qquad r=R\,.\tag{2.3}$$

From this one finds the S-state quark kinetic energy $\omega_0=2.04/R$ and the wave function

$$q_0(r)=\frac{N}{\sqrt{4\pi}}\begin{pmatrix} ij_0(\omega_0 r) & u \\ -j_1(\omega_0 r) & \boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}}u\end{pmatrix},$$

where $N^{-2}=R^3 j_0^2(x_0)2(x_0-1)/x_0$, $x_0=\omega_0 R=2.04$ and j_l are the spherical Bessel functions.

As discussed in the introduction eqs. (2.1) and (2.2) are invariant under the chiral transformation

$$q_0(r)\to q_0'(r)=\mathrm{e}^{i\alpha\gamma_5}q_0(r)\,,\qquad r<R\,,$$

whereas a linear boundary condition, e.g. eq. (2.3), breaks chiral symmetry. From the quark wave function we can construct an axial current

$$A_\mu(r)=-i\bar{q}(r)\gamma_5\gamma_\mu q(r)\,,\qquad r<R\,,\tag{2.4}$$

and we find [3,4])

$$n_\mu A_\mu(r)\neq 0 \quad \text{for} \quad r=R\,.$$

Since the quark wave functions do not exist outside the bag our $A_\mu(r)$ is discontinuous on the bag surface, i.e. it is not even approximately conserved on the surface.

The most general linear boundary condition in SU(2)×SU(2) corresponding to massless u- and d-quarks is

$$\hat{r}\cdot\gamma q(r)=\mathrm{e}^{i\vec{\tau}\cdot\vec{\pi}\gamma_5/f_\pi}q(r)\,,\qquad r=R\,. \tag{2.5}$$

As argued in the introduction we expect that there exists a Goldstone pion field $\vec{\pi}$ outside the bag. We assume that this π-field satisfies a non-linear chiral invariant lagrangian [6]) outside the bag ($r>R$). We will use the expressions given by Jaffe [7]). The π-field equation of motion is

$$\Box\vec{\pi}+\partial^\mu\left(1-\frac{\sin 2x}{2x}\right)\hat{\pi}\times(\hat{\pi}\times\partial_\mu\vec{\pi})=0\,,\qquad r>R\,, \tag{2.6}$$

where $x=|\vec{\pi}|/f_\pi$, and the π-fields sources on the quarkish bag surface is

$$\hat{r}\cdot\nabla\pi_i=\left[\delta_{ij}-\left(1-\frac{2x}{\sin 2x}\right)(\delta_{ij}-\hat{\pi}_i\hat{\pi}_j)\right]$$

$$\times\frac{i}{2f_\pi}\bar{q}\tau_j\gamma_5\,\mathrm{e}^{i\vec{\tau}\cdot\vec{\pi}\gamma_5/f_\pi}q\,,\qquad r=R\,, \tag{2.7}$$

$$\tfrac{1}{2}\hat{r}\cdot\nabla\bar{q}\,\mathrm{e}^{i\vec{\tau}\cdot\vec{\pi}\gamma_5/f_\pi}q+\tfrac{1}{2}(D_\mu\vec{\pi})^2=B\,,\qquad r=R\,, \tag{2.8a}$$

where the covariant derivative is

$$D_{\mu ij}=\left[\delta_{ij}-\left(1-\frac{\sin x}{x}\right)(\delta_{ij}-\hat{\pi}_i\hat{\pi}_j)\right]\partial_\mu\,. \tag{2.8b}$$

The latin subscripts refer to isospin components.

With this $\vec{\pi}$-field we have constructed an axial current outside the bag:

$$\vec{A}_\mu(r)=f_\pi D_\mu\vec{\pi}(r)\,,\qquad r>R\,. \tag{2.9}$$

The π-field source, eq. (2.8), follows from the requirement that we demand the axial current, eq. (2.9) and eq. (2.4), to be continuous in all space. This is the key assumption in this model, and the π-fields cluster outside the bag in a way demanded by continuity of the axial current. The sources of the π-field on the bag surface are the quarks inside the bag as seen from eq. (2.7).

Now we have a chiral quark bag model where inside the bag chiral symmetry is realized in the Wigner mode; outside it is spontaneously broken, giving rise to the π-field.

We shall now see how the π-field modifies the static properties of the quark bag and we will expand eqs. (2.1), (2.5)–(2.9) in increasing orders in the π-field where our expansion parameter will be proportional to $(f_\pi R)^{-2}$. In sect. 4 we will indicate for what radii R this perturbation expansion is valid.

Following Vento [9]) and Jaffe [7]) we will write the quark wave function and energy as

$$q=q_0+f_\pi^{-2}q_1+f_\pi^{-4}q_2+\cdots\,, \tag{2.10a}$$

$$\omega = \omega_0 + f_\pi^{-2}\omega_1 + f_\pi^{-4}\omega_2 + \cdots, \tag{2.10b}$$

and the pion field as

$$\vec{\pi} = f_\pi^{-1}\vec{\pi}_1 + f_\pi^{-3}\vec{\pi}_2 + \cdots. \tag{2.10c}$$

To zeroth order in f_π we obtain the MIT bag equations

$$-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} q_0 = \omega_0 q_0, \qquad r<R, \tag{2.11a}$$

$$\hat{\boldsymbol{r}}\cdot\boldsymbol{\gamma} q_0 = q_0, \qquad r=R. \tag{2.11b}$$

The latter is eq. (2.3) and the solution to this is given below that equation. To next order in f_π we find the corrections to the quark wave functions q_1 satisfying

$$-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} q_1 = \omega_0 q_1 + \omega_1 q_0, \qquad r<R, \tag{2.12a}$$

$$\hat{\boldsymbol{r}}\cdot\boldsymbol{\gamma} q_1 = q_1 + i\vec{\tau}\cdot\vec{\pi}_1\gamma_5 q_0, \qquad r=R, \tag{2.12b}$$

where the static pion field in eq. (2.12) is given by expanding eqs. (2.6) and (2.7):

$$\nabla^2\vec{\pi}_1 = 0, \qquad r>R, \tag{2.13a}$$

$$\hat{\boldsymbol{r}}\cdot\boldsymbol{\nabla}\vec{\pi}_1 = \tfrac{1}{2}i\bar{q}_0\vec{\tau}\gamma_5 q_0, \qquad r=R. \tag{2.13b}$$

And finally to order f_π^{-4} we find

$$-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} q_2 = \omega_0 q_2 + \omega_1 q_1 + \omega_2 q_0, \qquad r<R, \tag{2.14a}$$

$$\hat{\boldsymbol{r}}\cdot\boldsymbol{\gamma} q_2 = q_2 + i\vec{\tau}\cdot\vec{\pi}_1\gamma_5 q_1 + i\vec{\tau}\cdot\vec{\pi}_2\gamma_5 q_0 - \tfrac{1}{2}\vec{\pi}_1^2 q_0, \qquad r=R, \tag{2.14b}$$

where the $\vec{\pi}_2$ field is given by

$$\nabla^2\vec{\pi}_2 = 0, \qquad r>R, \tag{2.15a}$$

$$\hat{\boldsymbol{r}}\cdot\boldsymbol{\nabla}\vec{\pi}_2 = \tfrac{1}{2}i\bar{q}_1\vec{\tau}\gamma_5 q_0 + \tfrac{1}{2}i\bar{q}_0\vec{\tau}\gamma_5 q_1, \qquad r=R. \tag{2.15b}$$

From eqs. (2.13) we find the pion field to lowest order $\vec{\pi}_1$ from all quarks, j, in the bag to be

$$\vec{\pi}_1 = -\frac{1}{16\pi}\frac{x_0}{x_0-1}\sum_j \frac{\langle|\boldsymbol{\sigma}(j)\cdot\hat{\boldsymbol{r}}\vec{\tau}(j)|\rangle}{r^2}, \tag{2.16}$$

where the bra and kets are the quark j's spin–isospin wave function in the bag, see refs. [7,9]). We also find from eqs. (2.12) the correction to the quark, j, kinetic energy due to the Goldstone pion field to be

$$\omega_1(j) = R^2\int_{r=R} \mathrm{d}\hat{r}\,\vec{\pi}_1\cdot\tfrac{1}{2}i\bar{q}_0(j)\vec{\tau}(j)\gamma_5 q_0(j), \tag{2.17}$$

where $q_0(j)$ is the quark j's wave function and $\vec{\tau}(j)$ is the isospin operator of quark j, and the integral is over all 4π angles on the bag surface, $r=R$.

The first-order correction to the energy E_0 of the MIT bag due to the Goldstone pions outside is then

$$E_1 - E_0 = f_\pi^{-2} 3\omega_1 + f_\pi^{-2} \cdot \tfrac{1}{2} \int_{r>R} \mathrm{d}^3 r \, (\nabla \vec{\pi}_1)^2 \,. \tag{2.18a}$$

Ref. [15]) included energy in the pion field only to lowest order in f_π and forgot the surface term which is twice as large and of opposite sign; the latter changes the effects markedly. Inserting the expression above, we find that

$$E_1 - E_0 = \tfrac{1}{2} f_\pi^{-2} 3\omega_1 = -\frac{1}{48\pi f_\pi^2 R^3} \left(\frac{x_0}{2(x_0 - 1)} \right)^2 \Sigma \,, \tag{2.18b}$$

where†

$$\Sigma = \sum_{i,j} \langle H | (\boldsymbol{\sigma}(i) \vec{\tau}(i)) (\boldsymbol{\sigma}(j) \vec{\tau}(j)) | H \rangle \,. \tag{2.18c}$$

The sum is over all quarks i and j in the bag making up the hadron state $|H\rangle$. Using the Goldberger–Treiman relation and the value of g_A in the chiral bag [7]) we find an expression for $E_1 - E_0$ containing only strong interaction parameters. We evaluate this expression using the experimentally known pion–nucleon coupling constant $f = 1$. We obtain

$$E_1 - E_0 = -\frac{0.44}{m_n^2 R^3} \Sigma \,, \tag{2.19}$$

where m_n is the nucleon mass.

The matrix elements of the spin–isospin operator Σ, eq. (2.18c), are listed in table 1. These numbers satisfy, to 3%, the Gell-Mann–Okubo octet mass formula

$$\tfrac{1}{2}(m_\Xi + m_n) = \tfrac{3}{4} m_\Lambda + \tfrac{1}{4} m_\Sigma \,,$$

although we know of no *a priori* reason that they should. Of course, the Goldstone

TABLE 1

The values for the spin–isospin matrix elements Σ for different hadrons that enter the Goldstone pion mass self-energy term, eq. (2.19)

Baryon	N	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω^-
Σ	57	36	20	9	33	20	9	0
Meson	ρ	ω	K*	ϕ	K			
Σ	16	24	9	0	9			

† One should note that the ith quark operator $\boldsymbol{\sigma}(i)\vec{\tau}(i)$ changes sign when quark is replaced by antiquark.

pion is coupled only to non-strange quarks, so insofar as the same quark emits and reabsorbs a pion, the corrections will look like a mass correction common to the two non-strange quarks, and this will obey the mass formula. On the other hand, terms in which one quark emits a pion and another quark absorbs it are just as large as the one-body terms, so the fact that the above mass relation is closely obeyed appears somewhat as a coincidence. The equal mass splitting of the decuplet is violated by ~15% by the numbers in table 1, although this equal splitting formula will be more accurately obeyed after inclusion of other corrections; this again appears as a coincidence.

In comparing the numbers in table 1 we have assumed that the baryons all have the same radii R. This is not so as shown in ref. [2]), and as will also be shown in the next section. The values for Σ must also be corrected from the value given in table 1. We have assumed when calculating the pion mass self-energy term, eq. (2.18), that the baryons in the intermediate state have the same masses as the baryon in consideration. This is not so because, e.g., the gluon exchange between quarks will split the N- and Δ-masses. We shall adopt the following procedure. We shall include the diagonal self-energy elements; e.g., in the case of the nucleon, the 25 of table 3, in the zero-order equation for $E(R)$, and treat the off-diagonal elements connecting nucleon and isobar by perturbation theory. We do this firstly as a matter of principle; the pion mean field is the agency producing non-perturbative solutions [11]). As a practical matter, this allows us to include simply corrections for nucleon–isobar energy splittings, etc., and we find that net effects from the off-diagonal elements are small. Leaving the corrections to the numbers in table 1 to the next section, we can already now point out some interesting features the Goldstone pions have on the baryon mass spectra.

The mass of the baryon is calculated from the energy functional given by ref. [2]) for N and the Δ:

$$E(R)=\tfrac{4}{3}\pi BR^3+\frac{3\times 2.04-0.75}{R}+\frac{E'_{\rm M}}{R}-\frac{0.44}{m_{\rm n}^2R^3}\Sigma_{\rm diag}\,, \tag{2.20}$$

where the first term is the bag energy required to excite a bubble from the QCD vacuum to the free vacuum state. The next two terms are the kinetic energy of the three quarks minus their c.m. energy [7,10]). The next term is the magnetic gluon-exchange energy [see ref. [2]) and their fig. 3]

$$E'_{\rm M}(m_{\rm s}R)=\tfrac{2}{3}\alpha_{\rm s}[a_{00}0.175+a_{0{\rm s}}(0.175-0.023m_{\rm s}R)+a_{\rm ss}(0.175-0.043m_{\rm s}R)]\,, \tag{2.21}$$

where we have approximated the curves in fig. 3, ref. [2]) with a linear $m_{\rm s}R$ dependence ($m_{\rm s}$ is the mass of the strange quark), $\alpha_{\rm s}$ is the colour fine-structure coupling constant and the a's are tabulated in table 2. The last term in eq. (2.20) is the diagonal part of the Goldstone mass self-energy contribution, eq. (2.19).

TABLE 2

The values of the colour-spin coefficient entering the gluon magnetic interaction in eq. (2.21) taken from ref. [2])

Baryon	N	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω^-
a_{oo}	−3	−3	1	0	3	1	0	0
a_{os}	0	0	−4	−4	0	2	2	0
a_{ss}	0	0	0	1	0	0	1	3

The quark kinetic energy equal to $2.04/R$ in eq. (2.20) will in the case of strange quarks ($m_s \neq 0$) become equal to $[x^2+(m_sR)^2]^{1/2}/R$ where $x \simeq 2.04+0.36m_sR$ for $m_sR \leq 1.5$[†], see fig. 2 of DeGrand *et al.* [2]).

Only a small part of the zero-point energy $-z_0/r$ has been included; our $z_0=0.15$, an order of magnitude smaller than that of ref. [2]). Part of it is summarized in the c.m. correction [10]) $-0.75/R$ in eq. (2.20). A mass ratio m_n/m_B, where m_B is the physical mass of the baryon considered, multiplies the 0.75 of eq. (2.20) in our actual calculations in order to account for the fact that the c.m. corrections must be smaller as m_B increases. The role of most of the remainder is taken over by our pion self-energy terms. Johnson [5]) argues that gluon interactions provide additional attractive R^{-1} terms. Our fits will tolerate small contributions of this type.

As in ref. [2]) we require the pressures on the bag's surface to balance, i.e. the massless quarks will balance the pressure from the surrounding QCD vacuum as well as the pressure from the Goldstone pions outside. We require

$$\partial E/\partial R = 0 \tag{2.22}$$

and find for the N and Δ

$$E(R)=\frac{4}{3}\,\frac{3\times 2.04-0.75+E'_M(0)}{R}-2\frac{0.44}{m_n^2R^3}\Sigma\,, \tag{2.23}$$

where the radius of the bag R is given by the bag constant B via the equation

$$R^6-\frac{3\times 2.04-0.75+E'_M(0)}{4\pi B}R^2+3\frac{0.44}{m_n^2}\frac{\Sigma}{4\pi B}=0\,. \tag{2.24}$$

The solution to eq. (2.24) for the nucleon and the Δ (with Σ a parameter) is shown in fig. 1. As is seen and as was shown earlier by Vento [9]) and by Vento *et al.*[11]) for the hedgehog solution, this model puts an upper limit on the value of the bag constant B.

Now using the values for Σ of table 1 we can make a first estimate of the effect the last term in eq. (2.23) will have on the N − Δ mass difference. We find, assuming $R=1$ fm, that $m_n-m_\Delta \simeq -184$ MeV which is a sizeable fraction of the measured N − Δ mass splitting. This already tells us that when we fit the colour fine-structure coupling constant α_s to this mass difference our α_s will be smaller than the value

[†] This is correct for $m_sR \simeq 1.5$; it is not a Taylor expansion.

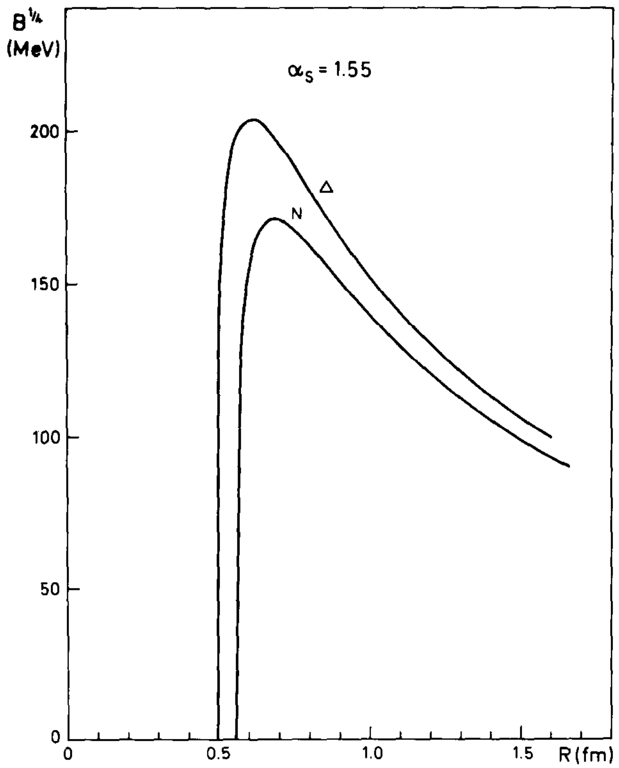

Fig. 1. The bag constant B as a function of the bag radius R resulting from the pressure balance equation $\partial E/\partial R = 0$. Only the mean Goldstone-pion self-energy is included in E; i.e., $\Sigma = 25$ for the nucleon and the Δ, see table 2. The colour fine structure coupling constant $\alpha_s = 1.55$.

$\alpha_s = 2.2$ found by ref. [2]), who blamed the whole $\Delta - \mathrm{N}$ splitting on the gluon exchange. The estimate from the Goldstone pion self-energy above would require $\alpha_s \simeq 0.85$ provided $R_{\mathrm{N}} = R_{\Delta} = 1$ fm, but we will find that a more careful treatment gives a much smaller decrease in α_s.

Finally, we like to stress an improvement in the baryon spectrum we find with our Goldstone pion term. The measured $\Sigma^0 - \Lambda$ mass difference is 77 MeV and this is not very well described by the MIT model who obtain 39 MeV for this difference with $\alpha_s = 2.2$. In our chiral quark bag model the Goldstone pions give a $\Sigma^0 - \Lambda$ mass difference of 123 MeV with $R_{\Sigma} = R_{\Lambda} = 1$ fm. This would say that the $\Sigma^0 - \Lambda$ mass difference comes mainly from the Goldstone pion. As we will show in the next section these numbers will be reduced somewhat when we treat this Goldstone pion self-energy term more properly, but we can already draw the conclusion that a part of the $\Sigma^0 - \Lambda$ mass splitting is a manifestation of the spontaneously broken chiral symmetry, the rest coming from the short-range magnetic gluon exchange interaction.

3. The Goldstone pion self-energy corrections

In the previous section we calculated the Goldstone pion self-energy corrections to the baryon masses. However, we did not consider that this self-energy, eq. (2.18), receives contributions from both octet and decuplet intermediate baryon states M and that the colour gluon exchange splits the octet and decuplet masses. These mass differences exist already in the chiral limit. Here we will include this mass difference and calculate a corrected spin–isospin matrix element Σ' using an effective Yukawa model of Brown *et al.* [4]), also used by Jaffe [12]) in his calculation of the pion–nucleon sigma term. We can do this since we are calculating a correction to a correction term and the final result does not depend strongly on this intermediate mass correction.

We now write the spin–isospin matrix element Σ, eq. (2.18c), as

$$\Sigma = \sum_{\mathrm{M}} \sum_{i,j} \langle \mathrm{H}|\boldsymbol{\sigma}(i)\vec{\tau}(i)|\mathrm{M}\rangle\langle \mathrm{M}|\boldsymbol{\sigma}(j)\vec{\tau}(j)|\mathrm{H}\rangle . \qquad (3.1)$$

Fig. 2. Goldstone pion self-energy diagrams for the nucleon with an intermediate nucleon (a) giving the pion mean-field or with an intermediate Δ (b).

We calculate the self-energy diagrams for, e.g., the proton in fig. 2, when the mass of baryon M, m_{M}, can be different from m_{H}. The mass correction to the diagram in the effective Yukawa model is proportional to [see Jaffe [12])]

$$\delta(\mathrm{M}) = \int_0^K \frac{q^4\,\mathrm{d}q}{\omega_\pi(\omega_\pi + m_{\mathrm{M}} - m_{\mathrm{H}})} \Big/ \int_0^K \frac{q^4\,\mathrm{d}q}{\omega_\pi^2} , \qquad (3.2)$$

where q is the pion momentum and $\omega_\pi = (\mu^2 + q^2)^{1/2}$. The cut-off is provided by the finite source of the pion-field $\sim 1/R$. We determine the value of K for the proton by calculating $\delta_{\mathrm{H}}(\mathrm{M})$ for a nucleon intermediate state, i.e. $m_{\mathrm{M}} = m_{\mathrm{H}} = m_{\mathrm{n}}$, and require that this mass correction and the correction of eq. (2.19) with only an intermediate nucleon state in eq. (3.1) should give the same result, i.e. $\delta_{\mathrm{N}}(\mathrm{N}) = 1$. This gives $K = 500\ \mathrm{MeV}/c$. Our result differs from Jaffe's $400\ \mathrm{MeV}/c$. We then calculate the mass correction to the proton from the Δ intermediate state, fig. 2b, using eq. (3.2) with baryon mass splittings due to gluon exchange ($\alpha_s = 1.55$) and find the correction factor $\delta = \delta_{\mathrm{H}}(\mathrm{M})$, $\mathrm{M} \neq \mathrm{H}$, which multiply the value of the corresponding spin–isospin matrix element of eq. (3.1). The results for all baryons are given in table 3 together with the values of the different terms in eq. (3.1). The corrected matrix element Σ' which replaces Σ in eqs. (2.19)–(2.24), are also given in table 3. As can be seen from this table, the factors δ work so as to reduce the nucleon–Δ splitting which arises from coupling to the pion, so that our final value of α_s will be closer to the MIT value than

TABLE 3

Elements in the corrected Goldstone pion mass self-energies. The matrix element is given in eq. (3.1). The other quantities are defined in the text. The energy $E_1 - E_0$ is calculated assuming $R = 1$ fm

Baryon H	Intermediate baryon, M	Matrix element	Σ	Mass correction δ	Σ'	$E_1 - E_0$ (MeV)
N	N	25	57	1	48.4	−185.6
	Δ	32		0.73		
Λ	Σ	12	36	1.11	32.0	−122.7
	Σ^*	24		0.78		
Σ	Σ	$\frac{32}{3}$	20	1	19.0	−72.9
	Λ	4		0.99		
	Σ^*	$\frac{16}{3}$		0.82		
Ξ	Ξ	1	9	1	7.2	−27.6
	Ξ^*	8		0.78		
Δ	Δ	25	33	1	41.8	−160.3
	N	8		2.1		
Σ^*	Σ^*	$\frac{40}{3}$	20	1	26.1	−100.1
	Σ	$\frac{8}{3}$		1.8		
	Λ	4		2.0		
Ξ^*	Ξ^*	5	9	1	12.2	−46.8
	Ξ	4		1.8		

one would estimate from the numbers in the last section. The final column in table 3 gives the Goldstone pion self-energy correction assuming $R = 1$ fm, and as can be seen, this correction contributes in some cases significantly to the splittings in the baryon spectrum, e.g. the $\Sigma^0 - \Delta$ mass splitting. As we will find in table 4 $R_\Sigma \simeq R_\Lambda = 1.1$ fm, so this splitting will be reduced.

When we include this corrected Goldstone pion self-energy in eq. (2.20), Σ' replaces Σ, and we impose eq. (2.22) for all baryons we find the baryon masses and their radii, given in table 4. As is seen from table 4 the reproductions of the masses are reasonable. The value of the bag constant B is fixed, from trying to get the best over-all fit.

With these remarks we now return to the numbers in table 4. We find that the SU(3) mass relations are very well satisfied with our chiral quark bag. The decuplet mass splitting is a constant; even the Goldstone pion self-energy, $E_1 - E_0$, alone satisfies this difference fairly well; the differences are 152 : 160 : 167 as seen in table 4 which is the same trend as observed experimentally. As for the $\Sigma^0 - \Lambda$ difference we find a value of 62 MeV in table 4. The contribution from gluon exchange is here 19 MeV and from the Goldstone pion self-energy is 35 MeV. The SU(6) relation $\Xi^* - \Sigma^* = \Xi - \Sigma$ is trivially satisfied [13]) for an arbitrary two-body interaction. In our model the slightly different parameters for the relevant particles do not upset this.

We have also made this calculation for the vector meson octet and the kaon with the parameters used for the baryons and we find reasonable results. Without the intermediate energy correction the ω will be lower than ρ by about 30 MeV, see table

TABLE 4

Masses of the baryons and their bag sizes R when $\partial E/\partial R=0$ is imposed on eq. (2.20)

Baryon	Mass (exp) MeV	Mass (model) MeV	R fm	E_{bag} MeV	E_Q MeV	E_M MeV	E_1-E_0 (mean field) MeV	E_1-E_0 (quantum fluctuation) MeV
N	938	938	1.04	216.0	990.4	−102.9	−85.4	−79.8
Λ	1116	1149	1.11	262.6	1073	−96.4	0.0	−90.0
Σ	1189	1211	1.10	255.6	1088	−77.4	−30.8	−24.1
Ξ	1321	1369	1.11	262.6	1213	−85.8	−2.8	−17.5
Δ	1236	1205	1.14	284.5	934.9	93.9	−64.8	−43.6
Σ^*	1385	1371	1.15	292.1	1062	82.1	−33.7	−32.3
Ξ^*	1533	1531	1.17	307.6	1180	72.2	−12.0	−17.3
Ω^-	1672	1683	1.17	307.6	1312	63.5	0.0	0.0

Also shown are the values of the individual terms in eq. (2.20) for $B^{1/4}=137$ MeV; $\alpha_s=1.55$ and $m_s=210$ MeV. The c.m. correction having the correct mass dependence is incorporated into E_Q which also contain a zero point energy of $-0.15/R$.

1. But ω has no δ-correction since $m_\omega \approx m_\rho$ whereas ρ will have a $\delta > 1$ from the 2π intermediate state and this will produce almost degenerate ω- and ρ-masses. Also the K and K* will be split in the right direction due to the intermediate mass corrections. At $\partial E/\partial R = 0$ we find all radii to be just above 1 fm except the kaon ($R_K = 0.82$ fm). If we look at the π- and the η-meson we find that the Goldstone field provides an attraction for the π in addition to the colour interaction making the π much lighter than the MIT one whereas this Goldstone π has *zero* effect on the η-meson.

As we would like to emphasize again, here we do not try to produce an excellent fit to the baryon spectra, but to show the effect the Goldstone pion self-energy has on this spectrum. Since this term alone satisfies the SU(3) mass relation (not obvious *a priori*) and contributes significantly to the $\Sigma^0-\Lambda$ mass splitting we conclude that our chiral quark bag describes very satisfactorily the baryon mass spectrum. This new contribution to the $\Sigma^0-\Lambda$ mass splitting should be more than welcomed in light of the results of ref. [2]).

As discussed before, the zero-point energy Z_0/R of the MIT model [2]) has mostly been replaced by the c.m. correction and the pion mean field in our calculation. Since the different baryons have different pion mean fields this means in particular that the radius of the Λ found from $\delta E/\delta R = 0$, $R_\Lambda = 1.1$ fm, is larger in our calculation than what the MIT group finds [R_Λ(MIT) = 0.98 fm]. Since the magnetic moment of the Λ-particle is given by the strange quark (u and d are in a spin $S = 0$ state), we find a larger absolute magnetic moment of the Λ. Our value is

$$2m_n\mu_\Lambda \simeq -0.564(1+\langle p^2\rangle/2m_\Lambda^2)\,.$$

The $\langle p^2\rangle/2m_\Lambda^2$ term constitutes the c.m. correction [10]), which we make in the same way as in eq. (2.23) where we calculate the masses. We find $2m_n\mu_\Lambda \simeq -0.62$ which is very close to the experimental value, $2m_n\mu_\Lambda = -0.6138 \pm 0.0047$ [ref. [14])]. If in the c.m. correction for the masses and μ_Λ we use the physical masses m and $\langle p^2 \rangle = 3(2.04/R)^2$ we find $R_\Lambda = 1.02$ fm and $2m_n\mu_\Lambda \approx -0.63$, i.e. as long as we treat the c.m. correction consistently we obtain nearly the same value for μ_Λ.

In the above treatment we have not included effects from the coupling of the K-meson to the strange quark which in this model is massive and therefore explicitly breaks chiral symmetry. We expect effects from this to be small, because of the large K-mass. We plan to investigate these corrections. The theoretical value of the Λ magnetic moments depends chiefly on the Λ radius R_Λ and, to a lesser extent, on the strange quark mass m_s. The good agreement between theoretical and empirical values of the moment seems to us to be a good indication that we have the correct radius for the Λ.

Giving the pion a mass = 140 MeV will only change the $E_1 - E_0$ results by 10% and introducing quark masses, $m_u = m_d = 10$ MeV, gives negligible effects so we have left these corrections out.

4. Conclusions

We have shown that the chiral quark bag and even the pion self-energy corrections satisfy the SU(3) and SU(6) mass relations for the baryons and in addition we find that this Goldstone pion gives a significant contribution to the $\Sigma^0 - \Lambda$ mass splitting. This mass splitting is then a manifestation of the spontaneous broken chiral symmetry of QCD. Inclusion of pion self-energies allows us to lower the colour coupling constant α_s, which is determined from the fit to the baryon masses, from the original MIT value of 2.2 to 1.55.

The chiral bag model has $\partial_\mu A_\mu = 0$ so we can take over the soft-pion theorems, breaking explicitly the chiral symmetry by giving the pion a mass $m_\pi \neq 0$ (which also means $m_u = m_d \neq 0$). The chiral bag model then enables us to find the Yukawa forces between nucleons due to pion exchange [4]).

Calculations of the second-order pion self-energy correction[16] indicates that the lowest-order correction should be sufficiently accurate for large bags. For bags of radius ~0.7 fm, the second-order correction becomes comparable with the first-order one, and perturbation theory can no longer be trusted.

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